A TEXT-BOOK OF PHYSICS

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BY

R. S. WILLOWS

M.A. (CAMB.), D.SC. (LOND.), F.INST.P.

LATE EXAMINER IN PHYSICS AT LONDON UNIVERSITY,
CHIEF PHYSICIST, TOOTAL BROADHURST LEE CO., LTD.

B. I. PUBLICATIONS
BOMBAY

First published, 1913
Second Edition, 1917; *reprinted* 1918, 1919 (*twice*), 1920, 1921
Third Edition, 1923, *reprinted* 1924, 1926, 1928, 1929, 1931, 1932, 1934, 1936, 1943, 1945 (*twice*), 1947, 1950, 1954, 1956, 1958

First Indian Edition 1962

published by

B. I. PUBLICATIONS
359, D. Naoroji Road,
Bombay-1

by arrangement with

Edward Arnold (Publishers) Ltd.
London, England.

Printed by offset in Great Britain
by William Clowes and Sons, Limited, London and Beccles

Made in India

PREFACE TO THE THIRD EDITION

THE call for a third edition has given me an opportunity to add a chapter on the conduction of electricity through gases. The properties of electrons, X-rays, and radioactive substances are matters of such general interest and scientific importance as to justify this addition. I have to thank Dr. J. A. Crowther for permission to use some of the simpler Figs. drawn for his book on "Ions, Electrons, and Ionizing Radiations."

R. S. W.

MANCHESTER.

PREFACE TO THE FIRST EDITION

Although there is no lack of Text-books of Physics of an advanced character I have frequently found a difficulty in choosing for students a book of a more elementary nature, such as is roughly represented by the standard of the Intermediate Examinations of the various Universities and of the Civil Service Commission. The present book was designed to fill what appeared to be a gap in the literature of the subject. It will be found to contain what a student usually requires at such a stage in Heat, Light, Sound, Magnetism and Electricity. As far as possible the treatment is based on experiment; usually a number of simple phenomena are described which lead up to the enunciation of some general law, and methods are then given by means of which the law may be proved more accurately. A large number of simple exercises which can be readily carried out by the student are shown in smaller type. In classes where the lectures are run concurrently with a laboratory course it is sometimes troublesome to keep the theory ahead of the practical work; the order of treatment which has been followed here is such as to provide a large number of experiments at an early stage in each branch of the subject, so lessening this difficulty in some degree. With the same end in view I have relegated the greater part of the Electrostatics to the end of the course on Electricity. I have found this arrangement to work admirably in my own teaching experience.

For alternative methods of carrying out some of the experiments, or when additional details are desirable, references have been given to Barton and Black's small volume on "Practical Physics."

My best thanks are due to my colleague, Mr. F. J. Harlow, B.Sc., for his care and skill in making the drawings for the figures and also for much helpful criticism. I have also to thank various students for help in verifying the examples, and the authorities of London University for permission to include a number of questions set at the various examinations.

R. S. W.

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MECHANICS

CHAPTER I*

UNITS AND LENGTH MEASUREMENTS

MECHANICS is the science which deals with the simplest effects arising from the application of force to matter. As such it may be regarded as introductory to Physics, in that the forces and the matter on which they act are taken for granted, while it is the object of Physics to study more complicated cases in order to explain not only the origin of the forces, but the structure of matter itself. As the simple must be dealt with before the complex, so must the principles of Mechanics be understood before the student is ready to begin the study of Physics.

The branches of Mechanics with which we shall deal in these introductory pages are (1) *Kinematics*, in which motion is studied without reference to the forces which cause it. (2) *Dynamics*, with its subdivisions *Kinetics* and *Statics*. Of these, Kinetics deals with the motion of bodies, taking into account the forces and masses concerned, and Statics has to do with the conditions for the equilibrium of bodies, or their state of rest. (3) *Hydrostatics*, in which the properties of fluids at rest are the subject of inquiry.

Units.—One of the main objects of physical science, because it is a condition for future progress, is to obtain accurate measurements of the quantities dealt with. As a preliminary, it is necessary to decide in what units the results are to be expressed. The statement that a certain body "weighs 25 grams" contains two ideas: one is the unit—the gram; the other, the measure, states how many times the unit is repeated—in this case 25. Each physical quantity must be expressed in terms of its appropriate unit. It is possible, however, to express the more complicated units in terms of some of the simpler ones, when it will evidently be an advantage to have the relation between them of the simplest kind. For example, when the unit of length is the foot, the simplest unit of area is the square foot, and of volume the cubic foot. A unit like the gallon is not only entirely arbitrary, but bears no simple relation to other

units. The principle can be carried further, inasmuch as it is found that many physical quantities can be expressed in terms of three properly chosen units. These are called the **fundamental units,** while all others bearing a more or less simple relation to them are called **derived units.** Such a system is called an absolute system of units. The fundamental units in most scientific work are those of length, mass and time, and the units taken are the centimetre, the gram and the second; hence the system is referred to as the cm.-gm.-sec. (C.G.S.) system.

The centimetre is the $\frac{1}{100}$th part of a metre, the latter being defined arbitrarily as the length of a certain platinum bar, preserved in Paris, when its temperature is that of melting ice.

The metric standard of mass is the kilogram. It is the mass of a piece of platinum kept in Paris. Originally it was intended to be connected with the standard of length by being defined as the mass of a cubic decimetre[1] of distilled water at a temperature of 4° Centigrade; now it is defined arbitrarily as above.

The gram is $\frac{1}{1000}$th of a kilogram.

The unit of time is the mean solar second, and is the $\frac{1}{86400}$th part of the mean solar day. The solar day is the period between successive transits of the sun across the meridian. For various reasons this interval is not constant, so an average is taken over a whole year, and this is called the mean solar day.

In the British system of units, which is still used by engineers, the units corresponding to the centimetre, gram and second are the foot, pound and second. We shall refer to this system as the F.P.S. system.

Co-ordinates.—The position of a point on a plane is known if its perpendicular distances from two lines at right angles to each other are given. Thus in Fig. 1* the lines OX, OY are called the X and Y axes, O the origin, and the positions of P is known when PN and PM or OM and ON are given. OM is called the abscissa, ON the ordinate of P, and the two together are referred to as the co-ordinates. If OM $= 2$ and ON $= 3$ units, P is referred to as the point (2, 3), the abscissa being written first. If the abscissa is drawn to the left of OY it is taken as negative; similarly the ordinate is negative when drawn below OX. Thus the co-ordinates of P_1 are $(-2, 3)$, of P_2 $(-2, -3)$, and of P_3 $(2, -3)$.

[1] A decimetre is $\frac{1}{10}$th of a metre; 1 cub. decim. $=$ 1 litre $=$ 1000 cu. cms. It follows that a litre of water at 4° Cent. weighs 1000 gms. very nearly. A litre $= 1{\cdot}76$ pints; 1 inch $= 2{\cdot}5400$ cms.; 1 lb $= {\cdot}4536$ kilograms.

Angles.—Two units of angle are in common use, the degree and the radian. If the right angle XOY of Fig. 1* is divided into 90 equal parts, each is called a degree. The degree is further divided into 60 minutes, and each minute into 60 seconds. Nine degrees, six minutes, twelve seconds is written $9° \ 6' \ 12''$. The radian is defined as the angle subtended at the centre of a circle by an arc of length equal to the radius. Thus in Fig. 2* if arc PX = OX, the $\angle \theta = 1$ radian. The number of radians in $\angle$ P′OX, called its circular measure, is found by dividing the arc P′X by the radius

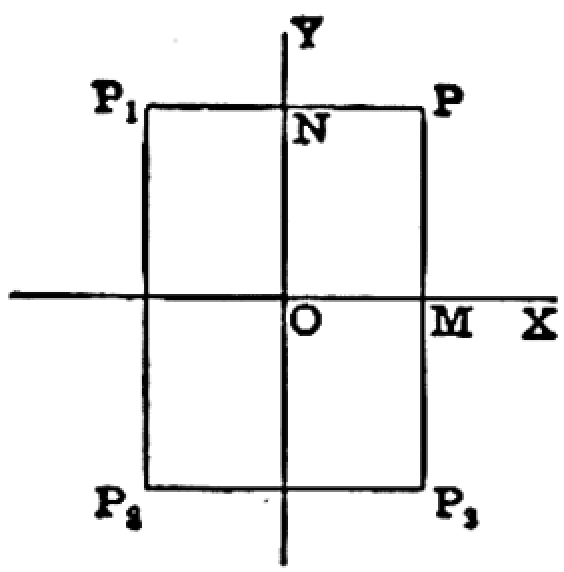


FIG. 1*.—Co-ordinates.

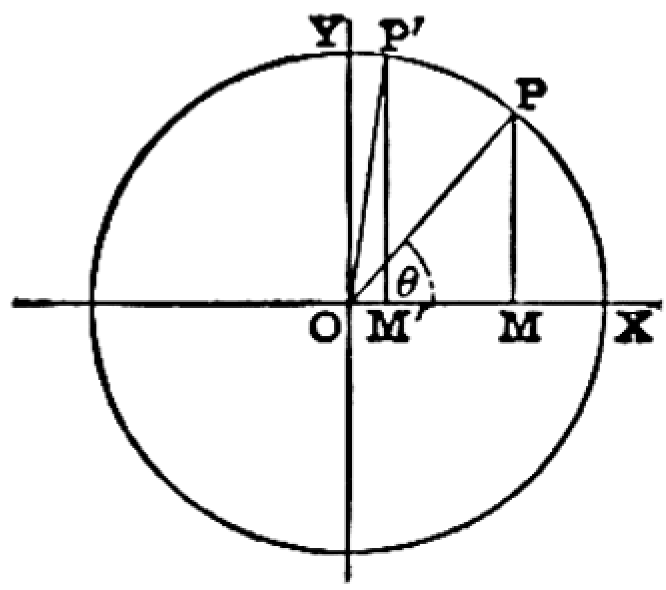


FIG. 2*.—Angular Measure.

of the circle; and generally, if θ is the circular measure of an angle, $\theta = \text{arc}/\text{radius}$.

If r is the length of the radius of a circle and C the circumference, it can be proved that C/r is the same for all circles. This ratio is denoted by 2π; hence $C/r = 2\pi$, or $C = 2\pi r$.[1]

The value of π is 3·1416 or 22/7 very nearly. In Fig. 2* arc XPY $= C/4 = \pi r/2$,

$$\therefore \angle XOY = \frac{\text{arc XPY}}{r} = \frac{\pi r}{2}\Big/ r = \frac{\pi}{2} \text{ radians}$$

$$\therefore \frac{\pi}{2} \text{ radians} = 1 \text{ rt. } \angle$$

and $$1 \text{ radian} = \frac{2}{\pi} \text{ rt. } \angle\text{s} = 57°{\cdot}2958.$$

If an angle contains c radians and d degrees, then, expressing each as a fraction of 2 rt. $\angle$s,

$$\frac{c}{\pi} = \frac{d}{180}$$

This is a convenient equation to convert from one system to the other.

[1] The following results should be remembered:—area of a circle $= \pi r^2$; surface of a sphere $= 4\pi r^2$; volume of a sphere $= \frac{4}{3}\pi r^3$.

Example.—Find the number of radians in 120°.

$\frac{c}{\pi}=\frac{120}{180}$, whence, putting in the value of π, $c=2{\cdot}09$ radians.

Trigonometrical ratios.—A few results from trigonometry are inserted here for future reference. Let a revolving line OP start from OX (Fig. 3*) and sweep out the $\angle$ XOP. Draw PM $\perp$ OX, produced if necessary as in (*b*). Let $\angle$ XOP $=\theta$; OP is the hypotenuse of the $\triangle$ POM. Then we have the following definitions:

$$\text{sine }\theta=\frac{\text{PM}}{\text{OP}}=\frac{\text{side opposite the angle}}{\text{hyp.}},\ (\text{written }\sin\theta),$$

$$\text{cosine }\theta=\frac{\text{OM}}{\text{OP}}=\frac{\text{side adjacent to angle}}{\text{hyp.}},\ (\quad ,, \quad \cos\theta),$$

$$\text{tangent }\theta=\frac{\text{PM}}{\text{OM}}=\frac{\text{side opposite to angle}}{\text{side adjacent to angle}},\ (\quad ,, \quad \tan\theta).$$

In Fig. 3* (*b*) if OM′ is taken positive OM must be negative, and

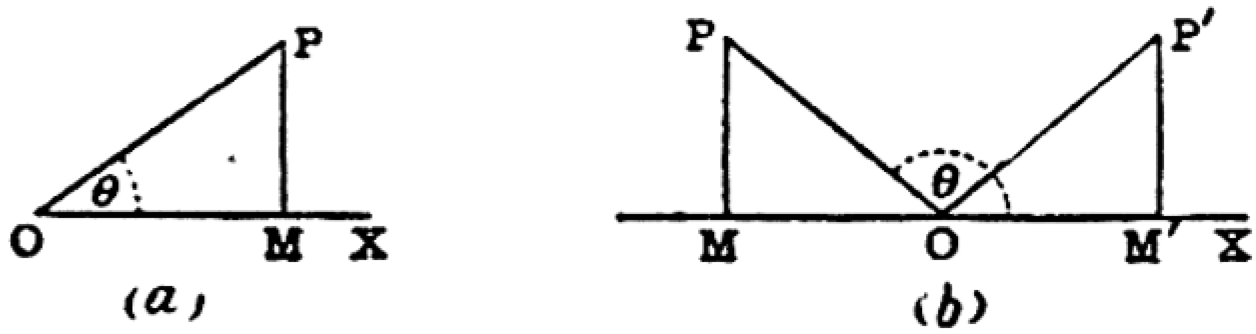


Fig. 3*.

vice versa. The values of these ratios for all angles up to 90° have been tabulated.

Note that $\frac{\sin\theta}{\cos\theta}=\frac{\text{PM}}{\text{OP}}\div\frac{\text{OM}}{\text{OP}}=\frac{\text{PM}}{\text{OM}}=\tan\theta.$

Hence when $\sin\theta$ and $\cos\theta$ are known, $\tan\theta$ can be calculated.

Also $\sin^2\theta+\cos^2\theta=\frac{\text{PM}^2}{\text{OP}^2}+\frac{\text{OM}^2}{\text{OP}^2}=\frac{\text{PM}^2+\text{OM}^2}{\text{OP}^2}=1.$

(Euclid I. 47.)

The three interior angles of a triangle $=2$ rt. $\angle$s.

$$\therefore \text{(Fig. 3* }(a))\ \angle\,\theta+\angle\,\text{OPM}=1\text{ rt. }\angle=\frac{\pi}{2}$$

and

$$\angle\,\text{OPM}=\frac{\pi}{2}-\theta,$$

$$\therefore \sin\left(\frac{\pi}{2}-\theta\right)=\sin\text{OPM}=\text{OM}/\text{OP}=\cos\theta.$$

Similarly $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta.$

In Fig. 3* (*b*) let $\angle$ P′OM′ = θ; make $\angle$ POM $= \theta$, OP′ = OP, and draw PM, P′M′ $\perp$ OX, Then P′M′ = PM,

and $$\angle \text{POM}' = 2 \text{ rt.} \angle \text{s} - \theta = \pi - \theta,$$

also $$\sin(\pi - \theta) = \frac{\text{PM}}{\text{OP}} = \frac{\text{P}'\text{M}'}{\text{OP}'} = \sin\theta,$$

$$\cos(\pi - \theta) = \frac{\text{OM}}{\text{OP}} = \frac{-\text{OM}'}{\text{OP}'} = -\cos\theta$$

(As OM is taken positive, OM′, drawn in the opposite direction, is negative.) For example, $\sin 120° = \sin(180° - 60°) = \sin 60°$.

In Fig. 2* let θ be a very small angle; then both P and M are near X, and arc PX = semi-chord PM very nearly. Hence for small values of θ we may put θ = PM/OP which is frequently useful. In like circumstances $\sin\theta$ = PM/OP = θ, and $\tan\theta$=PM/OM = PM/OX (nearly) = θ.

When P coincides with X, $\theta = 0$; also PM = 0, and M coincides with X. Hence

$$\sin 0° = \text{PM/OP} = 0/\text{OP} = 0,$$
$$\cos 0° = \text{OM/OP} = \text{OX/OP} = 1.$$

Similarly (Fig. 2*) when $\angle$ P′OX = 90°, P′ coincides with Y, P′M′ = OP′ and OM′ = 0.

$$\therefore \sin 90° = \frac{\text{P}'\text{M}'}{\text{OP}'} = \frac{\text{OP}'}{\text{OP}'} = 1,$$

and $$\cos 90° = \frac{\text{OM}'}{\text{OP}'} = 0.$$

In Fig. 4* (*a*) let $\angle$C = 45°, then $\angle$A is 45° if B is a right angle, and BC = AB. Also $\text{AC}^2 = \text{AB}^2 + \text{BC}^2$; hence if AB = BC = 1,

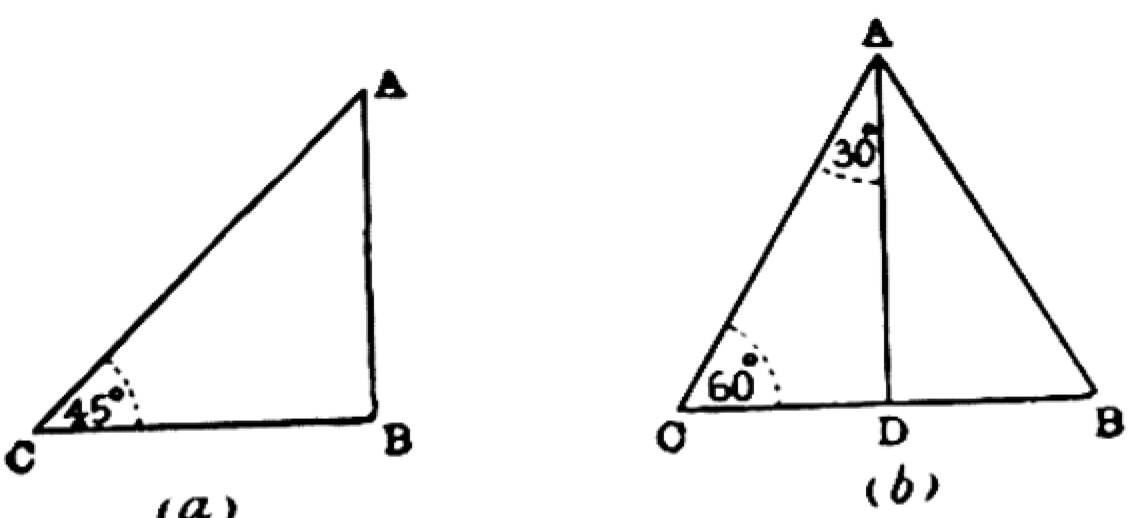


FIG. 4*.

AC = $\sqrt{2}$, and from the $\triangle$ABC the trigonometrical ratios for 45° can be found at once. In (*b*) let ABC be an equilateral triangle.

and AD be $\perp$ BC. Then $\angle C = 60°$, and $\angle CAD = 30°$. Also if $CD = 1$, $AC = 2$; and, as $AC^2 = CD^2 + AD^2$, $AD = \sqrt{3}$. Hence all the sides of $\triangle ACD$ are known, and therefore the trigonometrical ratios for 30° and 60°. The student should calculate the ratios for 120°, 135° and 150° from the formulæ $\sin(\pi - \theta) = \sin(180° - \theta) = \sin\theta$, &c., by putting $\theta = 60°$, 45° and 30° in succession. The results for $\sin\theta$ and $\cos\theta$ are here tabulated.

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

The Vernier.—It scarcely ever happens in practice that a length to be measured is equal to an exact number of divisions on the measuring scale. The vernier is a device which enables a fraction of a division to be estimated with accuracy, and it applies equally well to circular scales. Fig. 5* represents a rough model of a calliper gauge; Q is the scale, and the part V attached to the movable jaw B is the vernier. The zeros of the two scales coincide when the jaws A, B are in contact, so that the position of the vernier zero gives the length of the body P between the jaws. In the Fig. 10 vernier divisions are equal to 9 scale divisions, hence 1 v. div. $= \frac{9}{10}$ scale div., and the difference between a vernier and scale division is $\frac{1}{10}$th of a scale division. It is seen that 7 on the vernier coincides with a scale division; hence 6 on the vernier is $\frac{1}{10}$th division to the right of its corresponding scale division, number 5 is $\frac{2}{10}$th to the right of the next scale division, and so on, until we reach the vernier zero, which is found to be ·7 of a scale division to the right of number 7 on the scale. The length of P is, therefore, 7·7 scale divisions. If 8 on the vernier had coincided with a scale division the required length would have been 7·8. And generally, if n divisions on the vernier are equal to $(n - 1)$ on the scale, it will be seen, by similar

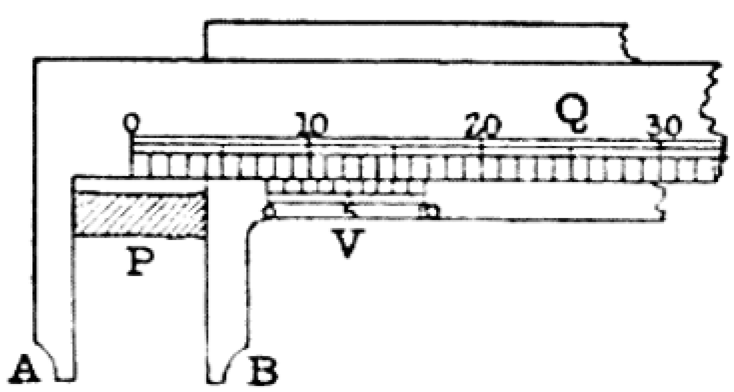


Fig. 5*.—Calliper Gauge.

reasoning, that the difference between two divisions is $\frac{1}{n}$th of a scale division. This is the "least count" of the vernier, and is the fraction to which it enables us to read.

The Micrometer Screw.—An accurate screw is frequently used in one form or other for length measurements. As an example, suppose the pitch of the screw is 1 mm., *i.e.* for each revolution the screw advances 1 mm. Let also the screw have a large circular head, divided into 100 equal parts. Then it is evident that the screw can be turned through $\frac{1}{100}$th of a revolution, and its point advanced or drawn back by 0·01 mm. Fig. 6* shows the application of this principle to the screw gauge. The screw Q is advanced through the nut N by turning the milled head H, which is divided into 50 equal parts by the scale S. Suppose the pitch is ·5 mm. A $\frac{1}{2}$ mm. scale R is engraved on the nut, and when the jaws P and Q are in contact

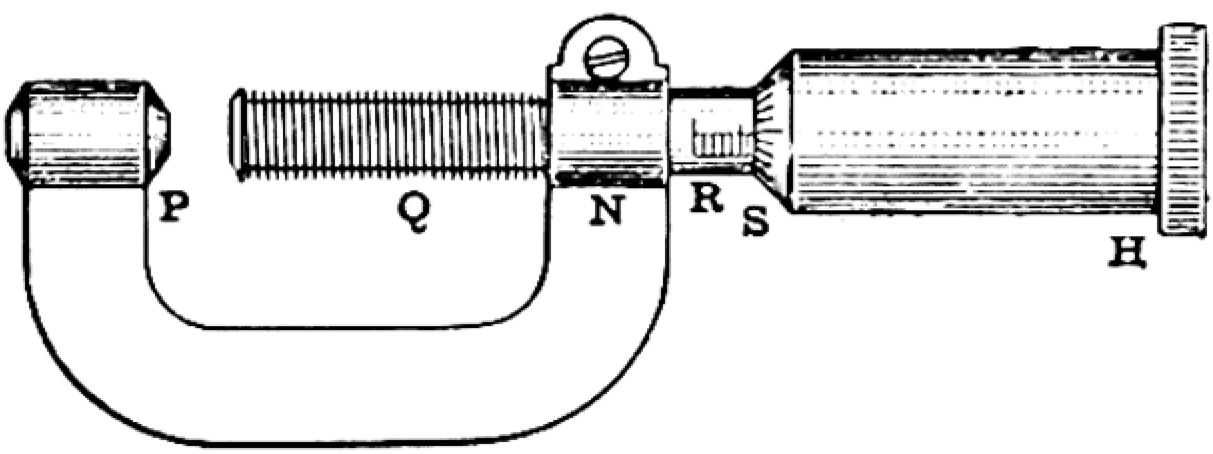


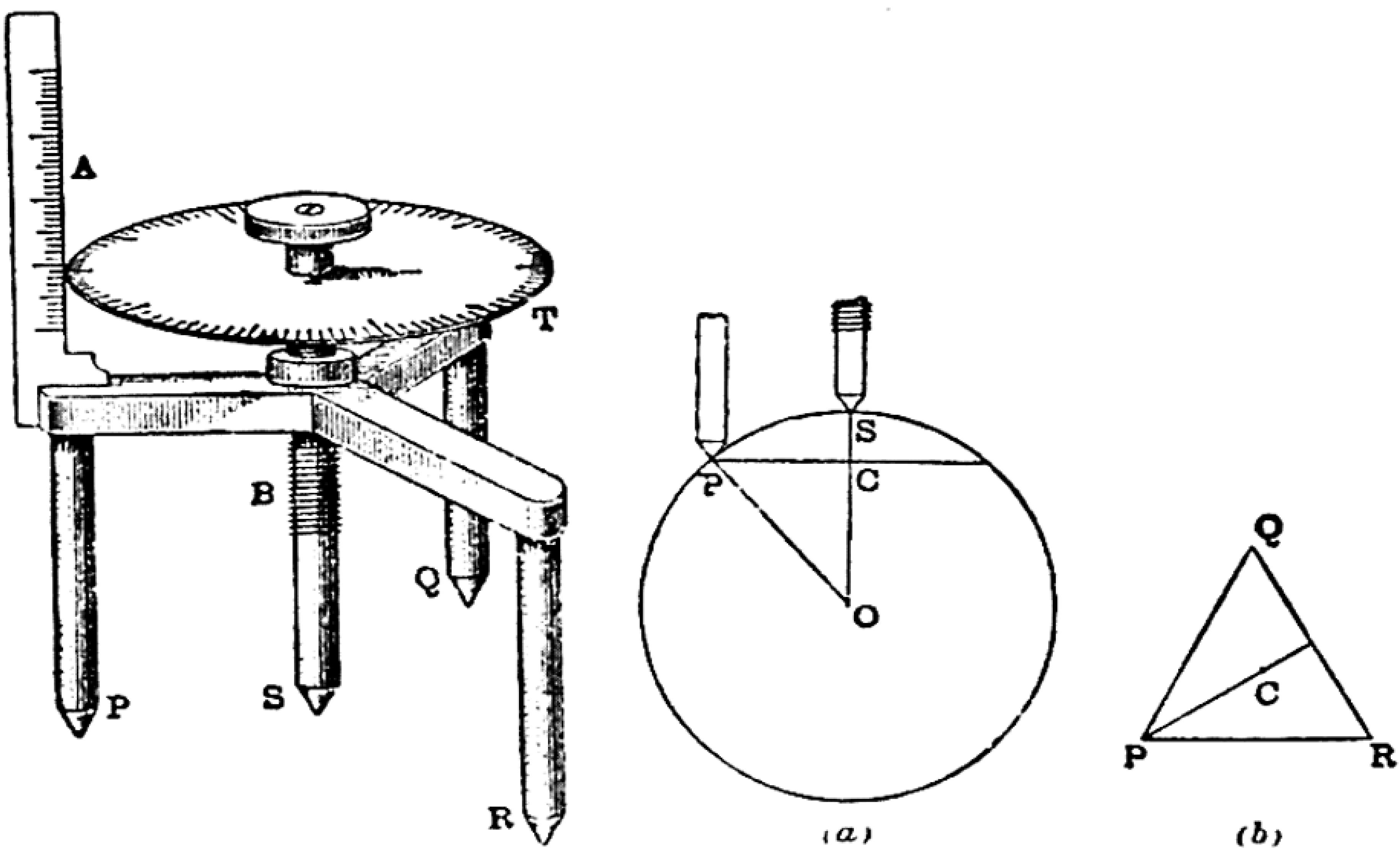
FIG. 6*.—Micrometer Screw Gauge.

the zeros of S and R should coincide. For each complete turn of the screw the bevelled edge at S moves over one division of scale R; fractions of a turn are given by the circular scale S. One division on S corresponds to a movement on the part of Q of $\frac{1}{50}$ of $\frac{1}{2}$ a mm., *i.e.* to ·01 mm. Hence the diameter of a piece of wire in contact with the jaws P and Q is given to the nearest ·5 mm. by the scale R, while the scale S gives the amount to be added to this, in $\frac{1}{100}$ths of a mm., to get the diameter accurately.

In the spherometer (Fig. 7*) T is the circular head, divided into 100 divisions, B is the screw of known pitch, say ·5 mm., and the $\frac{1}{2}$ mm. scale A gives the number of complete revolutions, as in the preceding case. One division of T, therefore, corresponds to $\frac{1}{100}$ of $\frac{1}{2}$ mm. $= \frac{1}{200}$ mm. $=$ ·005 mm. The points of the legs P, Q, R are all in one plane and the lines joining them form an equilateral triangle (Fig. 8* (*b*)). The spherometer is placed on a flat sheet of glass and the screw is turned until S is a little lower than P, Q, R; if it be pushed at A the whole instrument then revolves about S. The centre

leg is screwed upwards until this just ceases ; P, Q, R and S are then in the same plane, and the zeros of scales T and A should be opposite each other. If a thin piece of glass is now placed under the centre leg, the spherometer again revolves round S. The adjustment is made as before, when the thickness of the glass can be read. *E.g.* if A reads 2 divs. (each ·5 mm.), and T reads 45, the thickness is 1 mm. + (45 × ·005) = 1·225 mm. Applications of the vernier or screw are shown in Figs. 1, 2, 21, 142 and 206.

The spherometer is also used to measure the radii of spherical



FIGS. 7* and 8*.—The Spherometer.

surfaces, of which only a portion need be given, such as the faces of lenses. The instrument is placed on the surface and the centre leg screwed in or out, according as the surface is concave or convex, until it just ceases to revolve round S, when the reading of each scale is noted. Fig. 8* (*b*) represents a horizontal plane through P, Q, and R, and the point C vertically under S is shown ; (*a*) represents a vertical section through P and S. Evidently the spherometer has measured the distance CS = h. Let the distance SP in Fig. 7* be a and the radius of the sphere required be R. Then in Fig. 8* (*a*),

$$OP^2 = OC^2 + CP^2,$$

$$i.e.\ R^2 = (R - h)^2 + a^2 = R^2 - 2Rh + h^2 + a^2,$$

whence

$$R = \frac{a^2}{2h} + \frac{h}{2}$$

EXAMPLES ON CHAPTER I*

1. Find the number of radians in 30°, 50°, 130°.

2. Calculate the tangents of 120°, 150°, 30°.

3. A barometer scale is graduated in mms., and 20 vernier divisions equal 19 scale divisions. To what fraction of a mm. can it be read ?

4. The circular scale of a spectrometer is graduated in half degrees, and 30 vernier divisions correspond to 29 scale divisions. What is the least angular distance that can be read on the instrument ?

5. A microscope scale is divided into half mms. ; what kind of vernier will enable one to read to ·01 mm. ?

6. The circular scale divisions of a polarimeter are each equal to one-quarter of a degree, and 25 divisions on the vernier are equal to 24 on the scale. What fraction of a degree can be read on this instrument ?

CHAPTER II*

KINEMATICS AND KINETICS

In order to simplify matters as much as possible it will often be supposed that the bodies dealt with in this chapter are concentrated into such small volumes that any effects arising from their dimensions can be neglected. Such concentrated masses are called material particles. A rigid body is one whose particles remain at fixed distances from each other when it is acted upon by forces. Actually no body is perfectly rigid.

Velocity.—When a particle undergoes a displacement from one position to another, both the distance it has moved and the direction of its displacement must be given in order to define completely its new position. Such quantities which involve direction as well as magnitude are called vector quantities. Those which have magnitude only are called scalars. **The velocity of a body is its rate of displacement**; it is therefore a vector quantity, since it involves direction. A particle has a uniform, or constant, velocity when it moves over equal distances in equal intervals of time, however small these intervals are taken. If an aeroplane has a constant velocity of 90 miles/hour it must travel 132 ft. every second, ·132 ft. every thousandth of a second, and so on. A particle is said to have unit velocity when it passes over unit distance in unit time. In C.G.S. units this is 1 cm./sec. When the velocity of a body is variable, it is still possible to define its velocity at any point. Take a short length s of its path including the point, and let t be the time taken to traverse it. Then the average velocity over this distance is s/t. The velocity at the point is defined as the value of s/t when s, and therefore t, are made very small. This really means that we find the average velocity over a smaller and smaller length, until finally any change of velocity in the small distance s becomes inappreciable.

Acceleration.—When the velocity of a body is changing it is said to be accelerated. **The acceleration is the rate of change of**

velocity. It may be positive or negative; in the latter case it is sometimes called a retardation. When the velocity changes by the same amount every second the acceleration is said to be constant, and it is measured by the increase in velocity per second. A body has unit acceleration, in the C.G.S. system, when its velocity increases 1 cm. per sec. every sec. Students should note that a velocity is measured in cms. per sec., but an acceleration in cms. per sec. per sec.; this is often written cms./sec.^{2}.

Kinematical equations.—If a body moves for t secs. with a uniform velocity u cms./sec., the distance it travels $s = ut$. Let a body move in a straight line, starting with velocity u; let its constant acceleration be a, the velocity after time t be v, and the distance from the starting point after time t be s cms. There are three important relations between these quantities which will now be derived.

The increase in velocity each second $= a$

$$\therefore \text{ the increase in } t \text{ secs.} = at$$

$$\therefore \quad v = u + at \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As the increase in velocity is uniform the average velocity $= (u + v)/2$.

$$\therefore \text{ space passed over, } s = \frac{(u + v)t}{2}.$$

Substitute the value of v from (1) and

$$s = \frac{(2u + at).t}{2}$$

$$i.e. \quad s = ut + \tfrac{1}{2}at^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These equations give v and s in terms of t; if the velocity after passing over a distance s is required the quantity t must be eliminated. Square both sides, of (1), then

$$\begin{aligned} v^2 &= u^2 + 2uat + a^2t^2 \\ &= u^2 + 2a(ut + \tfrac{1}{2}at^2). \end{aligned}$$

The quantity in brackets is s,

$$\therefore \quad v^2 = u^2 + 2as \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If the body starts from rest $u = 0$; also a may be negative in these equations. In solving problems, when the student may not be certain which equation to use, it should be noted that (1) gives the final velocity v after a given time, while (3) gives it when the body is

a distance s from its starting point. Experiments show that if the resistance of the air can be neglected, a body falling freely to earth has a constant downward acceleration of 981 cms./per sec. per sec. approximately, or 32 ft./per sec. per sec., in England. This is called the acceleration due to gravity and is denoted by the letter g. Its value varies slightly from place to place; at the poles $g = 983{\cdot}21$ cms./sec.^{2}, and at the equator $g = 979{\cdot}99$ cms./sec.^{2}. The most accurate method of finding g is from observations of the time of swing of a pendulum (p. 255). The equations above suffice to solve problems on falling bodies if g be substituted for a.

EXAMPLE.—A body is thrown vertically upwards with a velocity of 100 ft./sec. How high will it rise? How long will it be before it returns to earth and what will then be its velocity?

At its highest point $v = 0$ momentarily and s is required; hence use equation (3), putting $u = 100$, $a = g = -32$, since u is upwards and g directed downwards.

$$\therefore \quad 0 = 100^2 - 2 \,.\, 32 \,.\, s,$$

and $$s = 156{\cdot}2 \text{ ft.}$$

When it returns to earth s, the distance from the starting point, is zero, and in the 2nd part of the question t is required. Hence use (2). To find its velocity on return note that $s = 0$ and v is required. Hence use (3). $v^2 = 100^2 - 2 \,.\, 32 \,.\, 0$ and $v = 100$. It therefore returns with the velocity of projection.

Parallelogram of Velocities and of Accelerations.—Since vector quantities include both magnitude and direction, they can be represented by straight lines drawn in the proper direction, and to scale, so that a unit of length represents one or more units of the quantity in question. A velocity 10 miles/hour due E. can be represented by drawing from O (Fig. 9*) a line OA due E., and 10 units in length. A body may have several velocities at the same time. A man walking across a ship in motion is an instance of two velocities (actually of several if the motions of the earth be taken into account). It is found possible to replace two or more velocities by one single velocity which produces the same effect as the others acting simultaneously. This is called the **resultant velocity.** Let a particle have two velocities u and v, represented in magnitude and direction by OA, OB (Fig. 9* (a)); what is the resultant? Complete the $\square^m$ OACB. Imagine a ring being pulled along a stick with a velocity represented by OA, and let the stick move parallel to itself from OA to BC with a velocity represented by OB. Then in 1 sec., if the stick were at rest, the ring would move from O to A, but during this time O has moved to B and A to C. Hence in 1 sec. the ring moves from O to C, and OC is the resultant velocity. This

proposition is called the parallelogram of velocities :—**If a body has two velocities represented by the adjacent sides of a parallelogram, the resultant is represented by the diagonal drawn through the same point.**

Similarly OA and OB might represent changes in velocities per sec. and a similar proposition would hold. But change in velocity per sec. is an acceleration : hence the parallelogram law holds also for accelerations.

Conversely, the velocity OC can be replaced by the two velocities OA, OB ; it is then said to be resolved into its two components.

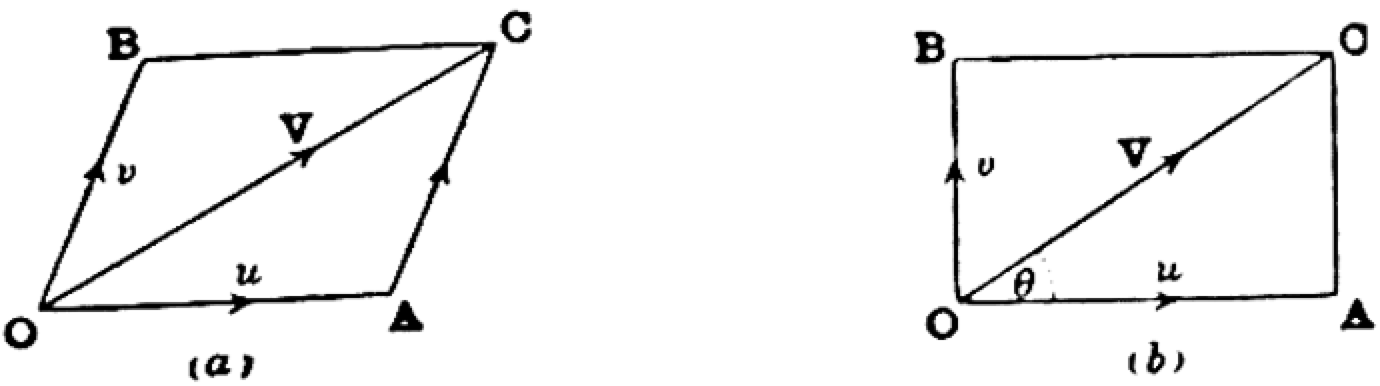


FIG. 9* (a) and (b).—Parallelogram of Velocities.

Any number of parallelograms can be formed on OC as diagonal, but the most important case is that in which the two components are at right angles to each other, (Fig. 9* (b)), when they are called the rectangular components. Let one of them, say OA, make an angle θ with the resultant, and let $OC = V$. Then $OA/OC = \cos\theta$, and $OA = OC \, . \cos\theta = V\cos\theta$. Similarly $OB = AC = V\sin\theta$. Hence the rectangular components can be written down at once if the

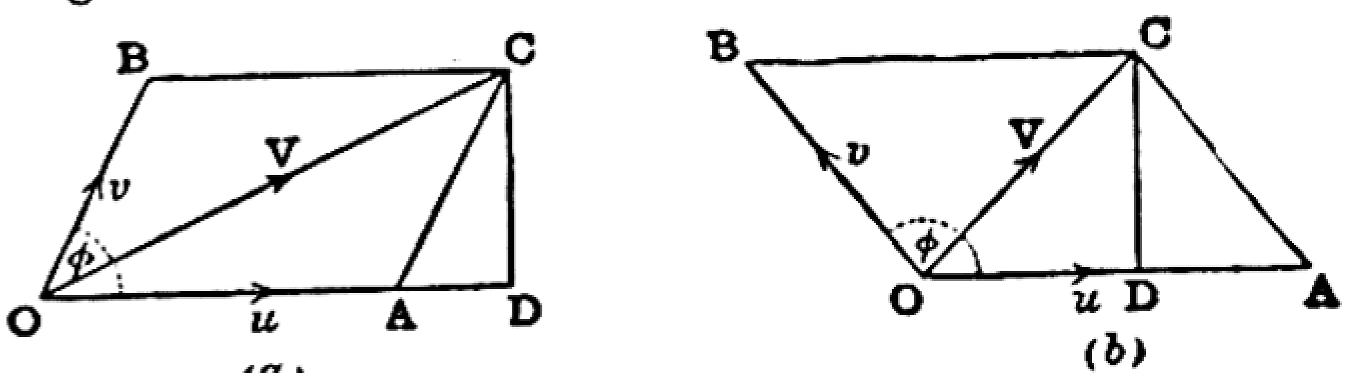


FIG. 10*.—Resultant Velocity.

direction of one of them is known. Also from the Fig., $V^2 = u^2 + v^2$; showing how to calculate V, being given the rectangular components. If the components are inclined to each other at an angle ϕ the resultant can be found as follows :—In Fig. 10* (a) and (b) draw $CD \perp OA$. Then in (a) $AD = AC \, . \cos CAD = v . \cos\phi$,

and

$$
\begin{aligned}
OC^2 &= OD^2 + CD^2 \\
&= (OA + AD)^2 + CD^2 \\
&= OA^2 + 2OA \, . \, AD + AD^2 + CD^2.
\end{aligned}
$$

Noticing that the sum of the last two terms equals AC^2, and substituting for $AD = v \cos \phi$, we have

$$V^2 = u^2 + 2uv \cdot \cos \phi + v^2.$$

In (*b*) $AD = v \cos CAD = v \cos (\pi - \phi) = - v \cos \phi$,
and $$OD = OA - AD.$$

Making these changes, the proof proceeds as before and the same result is obtained.

Triangle and polygon of velocities.—In Fig. 9* the velocity OB can be replaced by AC, when it is seen that the resultant of two velocities, represented by the two sides OA, AC, of a triangle taken in cyclic order, is the third side OC taken in the opposite direction. Hence also the resultant of three velocities represented by the sides OA, AC, CO, of a triangle taken in cyclic order is zero. This is called the triangle of velocities. Similar propositions hold for a polygon. For instance, the resultant of velocities represented by OA, AB, BC, CD (Fig. 11*), the sides of a polygon taken in cyclic order, is the remaining side OD taken in the opposite direction. For the resultant of OA and AB is OB, of OB and BC is OC, and of OC and CD is OD. Similarly, velocities represented by the sides of a closed polygon taken in cyclic order have a resultant equal to zero; *e.g.* the velocities represented by the sides of OABCDO. This is called the polygon of velocities.

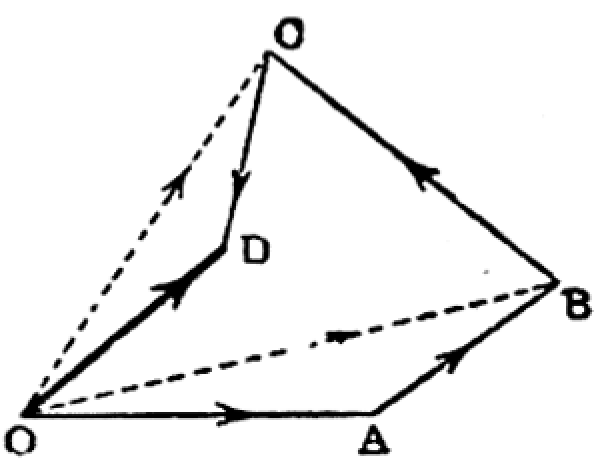


FIG. 11*.—Polygon of Velocities.

Kinetics.—The beginnings of kinetics are largely due to Galileo, but the first general statement of its laws is due to Newton in his laws of motion. Its growth is a good illustration of the methods by which science gradually extends its bounds. First come observations, of perhaps the roughest nature, but sufficient to indicate that all the results can be simply described by a few short statements called scientific laws. Once formulated, these laws are applied to more complicated cases, and their truth is proved not so much by the original observations as by the fact that the consequences deduced from them agree with experience. In the present instance, the best proof of Newton's laws is the fact that the motions of the moon and the planets can be predicted years in advance by calculations based upon those laws.

Mass and Weight.—The mass of a body is frequently defined as the quantity of matter it contains, but this definition has no meaning unless a method is given of comparing masses. The weight of a body is not necessarily a correct measure of its mass, as is shown by the following facts. Weighed on a spring balance, a body which weighs 1 gm. in England would weigh between 2 and 3 mgms. more at the N. pole, and approximately the same amount less at the equator. Even on a sensitive balance of ordinary type it appears to weigh more when a large lump of metal is placed just below the pan. It also weighs less at the top of a high building than at the bottom (p. 2), and less in a deep mine than at the earth's surface. Later on it will be shown that if weighed under exactly the same conditions the weights of bodies are correct measures of their masses; at present we shall take another test for the equality of two masses which is independent of place. Let two bodies move along the same straight line in opposite directions with equal speeds, and let them collide and stick together. If they are reduced to rest by the collision we shall take their masses as equal. An apparatus for performing such an experiment is described on p. 22*. Experiment shows that the result does not depend on their actual velocities so long as these are equal. Being given a standard gram, other equal masses could be fashioned by this test and so masses of 2, 3, 4 gms., &c., built up. Hence the mass of a body could be found, theoretically at any rate, by building up another mass equal to it and testing by collisions. The student can easily see how a gm. could be split up into known fractions. The definition above can now be completed. **The mass of a body is the quantity of matter it contains, and two masses are equal when, colliding with equal and opposite velocities and sticking together, they are reduced to rest.**

The case where the velocities are unequal is dealt with later.

If a mass m is moving with a velocity v the product mv is called its momentum. It is evidently a vector quantity.

Newton's Laws of Motion.—The following three laws are called Newton's laws of motion, although the first and part of the second were discovered by Galileo.

(1) Every body continues in its state of rest or of uniform motion in a straight line unless acted upon by some impressed force.

(2) Rate of change of momentum is proportional to the impressed force and takes place in the direction in which the force acts.

(3) To every action there is an equal and opposite action, or action and reaction are equal and opposite.

Newton's 1st Law.—This really consists of two parts. The first states that no body changes its state of rest or of uniform motion in a straight line unless compelled to do so. This is called the principle of inertia. Evidently such a law cannot be proved by direct experiment, nevertheless, as far as experience goes, the more resisting forces are removed the longer does the motion of a body continue. It is difficult to slide on a rough pavement, easier on a polished floor, and easier still on smooth ice. One can free-wheel on a bicycle with ball bearings much further than on a child's "scooter" which has not got them, and so on. The second part says that if a change of motion does occur it is the result of force. This is really a definition of what force is. Since a change in either the magnitude or the direction of the velocity involves a change in momentum—considered as a vector—the definition can be restated in the following form:—**Force is that which produces or tends to produce change of momentum.** "Tends to produce" is introduced to cover cases where the action of one force is neutralized by other forces.

Newton's 2nd Law.—This law tells us how forces are to be compared, viz. by the rate at which they can generate or destroy momentum. For the sake of simplicity between the units (p. 1), forces are taken not as merely proportional to the momentum they can generate in 1 sec., but as *equal* to this momentum. **Unit force** is then defined as **that force which in 1 sec. can generate unit momentum.** Let a body of mass m change its velocity uniformly from u to v in t secs. under the action of a force F.

Then change in momentum per sec. $= \dfrac{mv - mu}{t} = \dfrac{m(v-u)}{t}$.

But force $=$ change of momentum per sec.

$$\therefore \mathrm{F} = m(v-u)/t$$

Also if a is the acceleration

$$v = u + at$$

or

$$(v-u)/t = a$$

$$\therefore \quad \mathrm{F} = ma.$$

This is the most important equation in dynamics, connecting as it does force and mass with acceleration, and hence, through the formulæ of p. 11*, with the kinematical quantities. If m is 1 gm. and $a = 1$ cm. per sec. per sec., F is the unit force in the C.G.S. system, and is called the dyne. **The dyne is that force which can give to a mass of 1 gm. an acceleration of 1 cm. per sec. per sec.** Replace the

gm. by the lb. and the cm. by the ft. and we have the unit force in the F.P.S. system, called the poundal.

According to this law, if several forces act on a body simultaneously, each produces its appropriate change of momentum, in its own direction, independently of all the others. In Fig. 9* let OA and OB represent the accelerations that two forces P and Q can produce in unit mass; then these lengths are proportional to P and Q. Similarly OC represents the resultant acceleration, and hence the force producing it, thus the parallelogram law applies to forces also. The resultant and the components of forces can therefore be obtained by the formulæ on pp. 13*, 14*. Similarly the propositions called triangle and polygon of velocities are true if "velocities" is replaced by "forces."

Experimental proof of the ▭m law.—Three strings are knotted together at O (Fig. .2*), and weights of P, Q, and R grams are hung from the other ends. Two strings pass over pulleys as in the Fig. The whole is just in front of a vertical drawing board. The point O is in equilibrium under the action of three forces, P, Q, and R gms., acting in the directions OA, OB, and OD; hence one of them, say R, must be equal and opposite to the resultant of the other two, *i.e.* the resultant of P and Q must be a force R gms. acting vertically upwards. Draw on the board lines OA, OB to scale to represent the forces P and Q, and complete the ▭m. It will be found that OC is vertical and that, on the same scale as OA and OB, it represents R gms. Hence the proposition is proved. Also the three forces in equilibrium are represented by OB, BC, CO—the sides of a triangle taken in cyclic order—hence the triangle of forces is established.

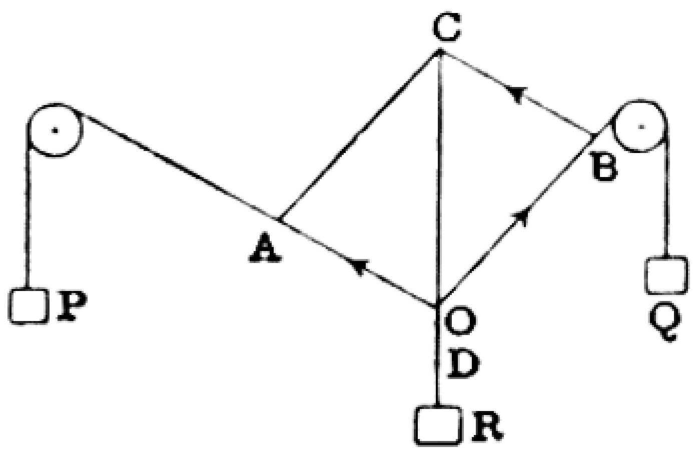


Fig. 12*.—Experimental Proof of the Parallelogram of Forces.

If a constant force F acts for a time t secs. the product Ft is called the impulse of the force, or, more briefly, the impulse. From above,

$$F = m(v - u)/t.$$
$$\therefore Ft = mv - mu.$$

or the impulse is measured by the change of momentum it produces. F may be a large force acting for a very short time, *i.e.* of the nature of a blow, but in every case the impulse is measured by the change in momentum.

Mass and Weight.—The relation between mass and weight can now be cleared up. According to Newton the weight of a body represents the force with which the earth attracts it in accordance

with the law of gravitation (p. 12*). Weight is therefore a *force*. Let a body of mass m gms. fall freely to earth, with acceleration g (p. 12*). The force producing motion is its weight. Substituting in the equation $F = ma$, we have, $F =$ weight w in dynes, and $a = g$,

$$\therefore \quad w = mg \text{ dynes}.$$

In England $g = 981$, hence a force of 1 gram's-weight $= 981$ dynes, and 1 dyne$=1/981$ of a gm.

In the F.P.S. system m is 1 lb. and $g = 32$; whence 1 lb's.-weight $= 32$ poundals.

The fact that g is the same at a given place for both light and heavy bodies shows that the ratio w/m is constant, or the weight of a body is proportional to its mass, if weights are always taken under the same conditions. In that case we may dispense with the collision method of comparing masses and take their weights instead. Engineers frequently express forces in grams or lbs. weight: such units are called gravitational units, and will vary from place to place; the dyne, on the other hand, is an absolute unit, and is the same everywhere, since both mass and the unit of acceleration in the equation $F = ma$ are invariable.

Work and Power.—When a force, acting on a body, causes a displacement of its point of application it is said to do work. The amount done is measured by the product F*s* of the magnitude of the force and of the displacement **measured parallel to the direction in which the force acts.** In Fig. 13* let a force F act along OA and move its point of application from O to M. Draw $MN \perp OA$. ON is called the projection of OM on OA. Then the work done is, by definition, F . ON. This work can be calculated in another way; for $ON/OM = \cos\theta$, and $ON = OM \cdot \cos\theta$.

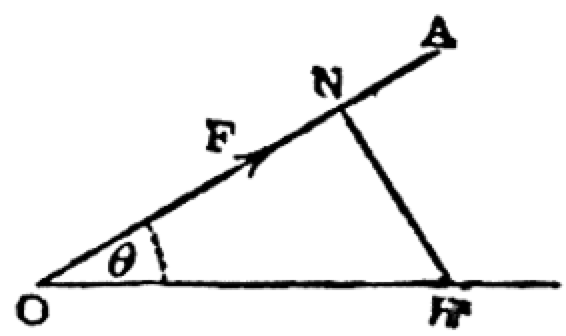


FIG. 13*.—Work done by a Force.

$$\therefore \text{ work} = F \cdot OM \cos\theta = F\cos\theta \cdot OM.$$

But $F\cos\theta$ is the component of F along OM (p. 13*), hence the work is obtained by multiplying the total displacement OM by the component of the force in this direction. If the displacement is in the opposite direction to a force, that force is said to do negative work, or work is done against the force. Thus when a miller raises a sack of corn from a lower to a higher floor he does work against the force of gravity.

Notice that the work does not depend on the time taken to do

it. **The unit of work is that done by unit force when its point of application is moved over unit distance parallel to the direction of the force.**

In the C.G.S. system this unit is called **the erg.** It is the work done by 1 dyne in moving its point of application over 1 cm. The corresponding unit in F.P.S. units is called the foot-poundal. If forces are measured in gravitational units—gms. or lbs.—the units are called gms.-cms. or foot-pounds respectively. Evidently a gravitational unit is g times an absolute unit. One ft.-lb. of work is done when 1 lb. is raised vertically through 1 ft. The erg is too small for convenience in electrical engineering, so another unit, called **the joule,** is frequently used. One joule $= 10^7$ ergs of work.

The power, or activity, of an agent is its rate of doing work. An activity of one joule per second is called a watt. This and its multiple the kilowatt (=1000 watts) are largely used in engineering. Another unit of activity is the horse power, equal to 550 ft.-lbs./sec.

Example.—To find the relation between horse power and watts, being given the relations 1 in. $= 2{\cdot}54$ cms., 1 lb. $= 453{\cdot}6$ gms.

$$\begin{aligned} \therefore\ 550 \text{ ft. lbs./sec.} &= 550 \times (2{\cdot}54 \times 12) \times 453{\cdot}6 \text{ gm. cms./sec.} \\ &= 550 \times 2{\cdot}54 \times 12 \times 453{\cdot}6 \times 981 \text{ ergs/sec.} \\ &= \frac{550 \times 2{\cdot}54 \times 12 \times 453{\cdot}6 \times 981}{10^7} \text{ joules/per sec.} \end{aligned}$$

$$\therefore\ 1 \text{ H.P.} = 746 \text{ joules/sec.} = 746 \text{ watts.}$$

Kinetic and Potential Energy.—Bodies may be capable of doing work on account of their motion or their position or state. The capacity of a body for doing work is called its energy. If it possesses this capacity in virtue of its motion its energy is called **kinetic energy.** Thus a train can move some distance after steam is shut off against the frictional resistances it experiences: it does work against such forces. To find the kinetic energy of a mass m gms. moving with a velocity u cms./sec., let us suppose its motion is opposed by a force F dynes, which brings it to rest after it has moved over s cms. From definition, its kinetic energy $= \mathrm{F}s$ ergs, since this is the work it can do on account of its velocity. From equation (3) (p. 11*) the final velocity v is given by

$$v^2 = u^2 - 2as,$$

where a is the retardation produced by F. Also final velocity $= 0$.

$$\therefore\ u^2 - 2as = 0,$$
$$as = \tfrac{1}{2}u^2.$$

Multiply by m.

$$\therefore\ mas = \tfrac{1}{2}mu^2.$$

Also
$$F = ma.$$
$$\therefore\quad Fs = \tfrac{1}{2}mu^2.$$

Hence the kinetic energy is $\frac{1}{2}mu^2$ ergs.

If the force acts so as to increase the body's velocity from u to v, while it traverses a distance s, the work done by the force is equal to the change in kinetic energy. For

$$v^2 = u^2 + 2as.$$
$$\therefore\quad as = \tfrac{1}{2}(v^2 - u^2),$$

and work done $= Fs = mas = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ . . . . . . (4)

This is a most important result of the 2nd law of motion.

A body may be devoid of kinetic energy and may yet possess the capacity for doing work on account of its position or condition. It is then said to have **potential energy.** As instances :—a compressed spring can overcome resistance while resuming its normal length ; a volume of compressed gas can be made to move a piston during expansion ; a raised weight can, by the intervention of pulleys, be made to raise other weights while it falls to the floor. In the last instance more work could be done by allowing the weight to pass through a hole in the floor to the room below, showing that the potential energy of a body is to be measured by the work it can do in coming to some standard position or state. The earth's surface is usually taken as the zero position. When m gms. are raised vertically h cms. the force opposing motion is $w = mg$ dynes, and the work done is mgh ergs ; this is the potential energy. If the mass falls freely this is converted into kinetic energy ; for its velocity at earth is given by $v^2 = 2gh$ (equation (3), p. 11*), and its kinetic energy $\frac{1}{2}mv^2 = mgh$. This is a simple illustration of conservation of energy (p. 113).

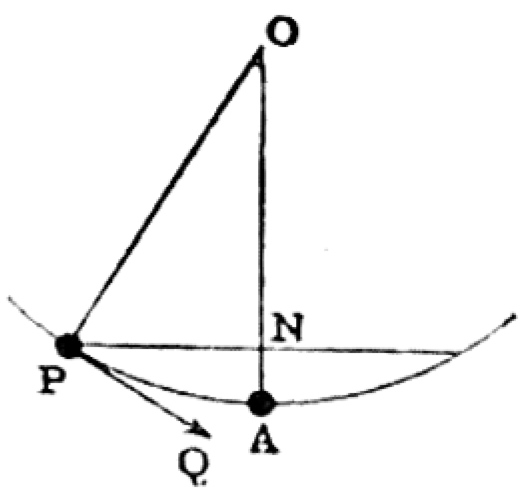


FIG. 14*.—Theory of the Ballistic Balance.

The following example will be made use of in the next paragraph. A (Fig. 14*) represents a mass m suspended from O by a long string of length R, the whole constituting a simple pendulum. The weight is pulled aside a short distance to P and released ; it is required to find its velocity at A, its lowest point. When it is at P the particle is moving in the direction of the tangent PQ, and therefore in a direction at right angles to OP, so that during a small movement

the tension in the string does no work, for "the displacement parallel to the direction of the force" is zero (p. 18*). This is true at every point, hence the tension does no work, and the increase in kinetic energy, from equation (4) above, arises from the work done by the only other force acting, viz. the weight mg. If the vertical displacement is $NA = h$, the work done in moving from P to A is mgh.

$$\therefore \quad \text{increase in K.E.} = \tfrac{1}{2}mv^2 = mgh \text{ (eqn. (4), p. 20*)}$$

or

$$v^2 = 2gh,$$

just as if the particle had fallen freely. Exactly as on p. 8* we have

$$PN^2 = 2Rh - h^2,$$

and if h is small compared with R the term h^2 is small and can be neglected.

$$\therefore \quad h = PN^2/2R,$$

and

$$v^2 = 2gh = \frac{g}{R} \cdot PN^2.$$

$$\therefore \quad v = \sqrt{(g/R)} \cdot PN.$$

This shows that when the string is long and h is small the velocity at A is proportional to PN, the distance the particle is pulled aside horizontally before being released.

Newton's 3rd Law.—Newton gave various meanings to the terms "action" and "reaction." When the hand is pressed on a table the table presses back with equal force; the attraction of the earth on the moon is equal and opposite to the attraction of the moon on the earth; the pull of the coupling on the railway carriages is equal and opposite to the pull it exerts on the engine which moves them. In the last instance students frequently wonder why, if the 3rd law be true, there should be any motion. The difficulty generally arises from the fact that they are not clear which body's motion it is they are considering. The horizontal forces acting on the carriages are the pull at the coupling and the friction at the rails, and the carriages move forward because the first is greater than the second. As the engine forces its wheels round, the friction at the rails tends to move both the rails and the earth backward, but the reaction corresponding acts on the engine and is forward. The horizontal forces acting on the engine *from outside* are the pull of the coupling backward and the reaction at the rails forward, and it is because the latter is the greater that the engine moves. If the train is considered as a whole the only horizontal impressed forces acting, *i.e.* those from outside, are the reaction at the rails and the frictional resistances,

and motion takes place because the reaction is the greater. In these instances action and reaction refer to forces, but if the forces are equal, the momenta they generate are also equal. For example, the forward momentum of a bullet is equal to the backward momentum of the rifle ; pull the rifle tightly to your shoulder and you become part of the backward moving mass, and for equal momentum the velocity of "kick" is reduced. When a man springs from an unmoored boat the effects of the action and reaction are seen—the

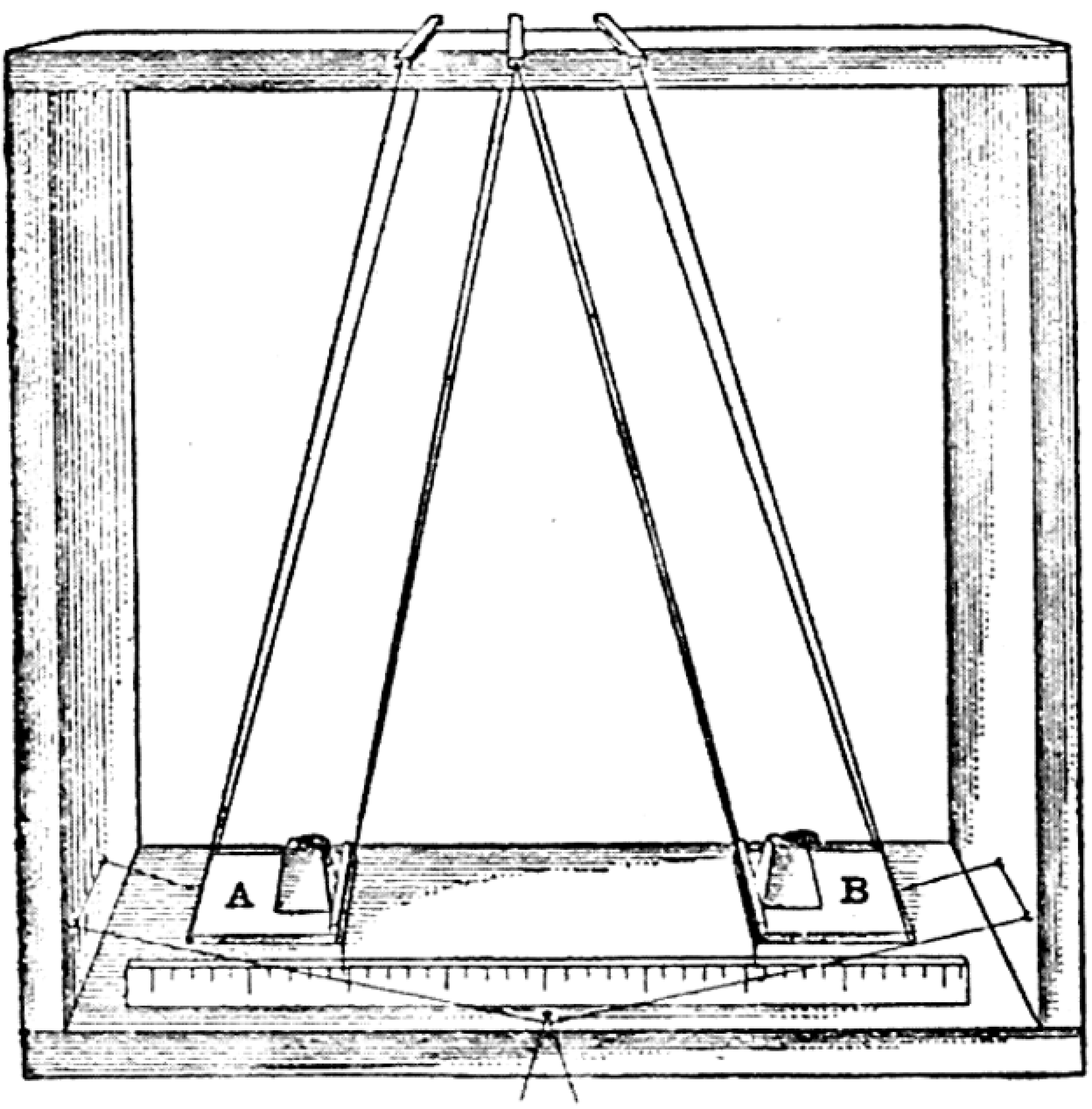


Fig. 15*.—Hicks' Ballistic Balance.

man goes in one direction and the boat in the other with equal momenta—but when the boat is moored it becomes part of a much larger body—the earth, and its motion then is not apparent. Action and reaction, then, can be interpreted to mean momentum. Fig. 15* shows an apparatus, called Hicks' ballistic balance, by means of which the 3rd law can be verified. It consists of two carriages, A and B, hung by strings so that no rotation can take place, and so arranged that they are in contact at their lowest points of swing. Various masses can be placed on the carriages, which are then pulled apart and allowed to collide. Springs, not shown in the fig., prevent

them separating after collision. A horizontal scale shows how far the carriages have been displaced horizontally, and hence, from the last paragraph, what are their velocities at collision. If OA (Fig. 14*) is 109 cms., $v = \sqrt{\left(\frac{981}{109}\right)}$. PN = 3PN, so that one cm. on the scale corresponds to a velocity at collision of 3 cms./sec. If after collision a body swings through x cms. its velocity immediately after collision was $3x$ cms. sec.

EXPERIMENT.—Put unequal masses on the carriages; let m_1 and m_2 be the total masses, u_1 and u_2 their velocities, with proper signs, at collision, v their common velocity after collision. Prove $m_1u_1 + m_2u_2 = (m_1 + m_2)v$. This shows that the algebraic sum of the momenta is unchanged by collision; what one loses the other gains, proving the 3rd law. The fact that the momentum is unchanged in amount is called the principle of conservation of momentum.

Motion in a Circle.—Let a particle of mass m be revolving with uniform speed v in a circle of radius r, and let P_1, P_2, etc. (Fig. 16*), represent successive positions at intervals of t secs. From Newton's 1st law, as the direction of motion is changing there must be a force acting on the particle, otherwise it would move along the tangent P_1A_1. From centre O draw OQ_1, OQ_2, etc., to represent the velocity of the particle when it is at P_1, P_2, etc.; the points Q_1, Q_2, etc., all lie on a circle of radius $OQ_1 = v$. We shall prove that the acceleration of the particle at P_1 is represented in magnitude and direction by the velocity of the corresponding point Q_1 on the second circle. While it passes from P_1 to P_2 the particle changes its velocity from OQ_1 to OQ_2: but OQ_2 is the resultant of velocities represented by OQ_1 and Q_1Q_2 (p. 14*); hence the change in velocity in time t is represented by Q_1Q_2, and the average rate of change is Q_1Q_2/t. Now take t very small; when P_1P_2 and Q_1Q_2 also become small, and Q_1Q_2/t becomes the rate of change of velocity of the particle very near and finally *at* P_1, if t is taken small enough. Or, as rate of change of velocity is acceleration, Q_1Q_2/t represents the acceleration of the particle at P_1. But, when t is small, Q_1Q_2/t also represents the velocity of Q_1; hence the acceleration of P_1 is represented in magnitude and direction by the velocity of Q_1. Now the instantaneous velocity of Q_1 is along the tangent Q_1B_1,

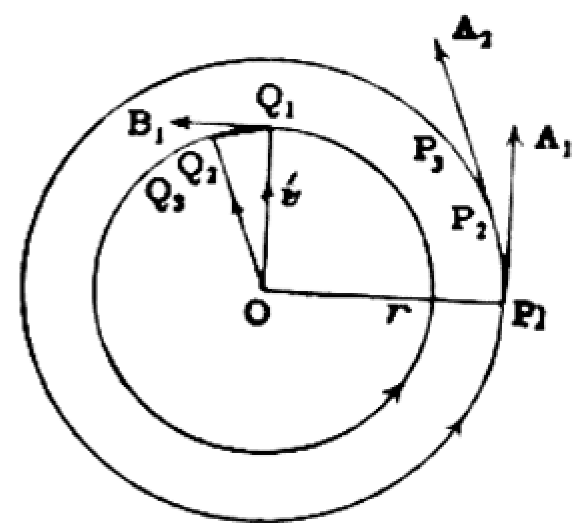


FIG. 16*.—Motion in a Circle.

which is parallel to P_1O; showing that the acceleration of the particle is towards the centre along P_1O.

Also
$$\frac{\text{vel. of } Q_1}{\text{vel. of } P_1}=\frac{\text{circumference of circle } Q_1Q_2}{\text{circumference of circle } P_1P_2}$$

$$\therefore \quad \frac{\text{vel. of } Q_1}{v}=\frac{2\pi v}{2\pi r}=\frac{v}{r}$$

$$\therefore \text{ accel. of the particle} = \text{vel. of } Q_1 = v^2/r,$$

and the force urging it towards the centre $= ma = mv^2/r$ dynes, in C.G.S. units. This is the tension in the string when a particle is whirled in a horizontal circle. Similarly when a fly-wheel revolves a call is made on the tensile strength of the material to keep it from flying to pieces. In a cream separator the new milk is put in a closed vessel which revolves at great speed, the heavy milk particles fly off to the circumference, thereby forcing the lighter cream particles to the centre, whence they can be drawn off.

EXAMPLES ON CHAPTER II*

1. After moving over 625 ft. from rest, a body has a velocity of 125 ft./sec.; find its acceleration. (L. '88.)

2. A jet of water is projected against a wall so as to strike it at right angles. If the velocity of the jet be 80 ft./sec. and 100 lbs. of water strike the wall every second, what pressure will be exerted against the wall, (1) when the water does not rebound, (2) when it rebounds with a velocity of 10 ft./sec.? (L. '83.)

3. A bullet weighing 25 gms., and moving with a velocity of 300 metres/sec., is stopped by impact against a bone, being brought to rest in a distance of 3 cms. from first striking. Calculate the average force exerted by the bullet on the bone. (L. 1900.)

4. A cage weighing 240 lbs. is lowered with uniform acceleration down the vertical shaft of a pit, the velocity changing from 100 to 200 ft. per sec. while the cage descends 600 ft. Determine the force exerted by the cage on the rope by which it is lowered. (L. '84.)

5. A mass of 1 gm. hangs over the edge of a smooth horizontal table, and is attached by a string to a mass of 980 gms., which slides without friction on the table. Find the potential energy, in gms.-cms., lost by the system in 10 secs., starting from rest. ($g=981$.) (L. '93.)

6. A bullet of mass 20 gms. is shot horizontally from a rifle, the barrel of which is 1 metre long, with velocity 400 metres/sec. into a mass of 50 kgms. of wood floating on water. If the bullet buries itself in the wood, find the velocity of the latter directly after it is struck. Also find the average force in gms. weight exerted on the bullet by the powder. (L. '85.)

7. What is the horse power of an engine which can pump 1000 gallons of water per minute from a well and project it with a velocity of 80 ft./sec. through a nozzle which is at a height of 40 ft. above the surface of the water in the well? [A gallon of water weighs 10 lbs.] (L. '82.)

8. A reservoir of water of area 330,000 sq. ft. is initially at a depth of 10 ft. How many ft.-lbs. can it supply to a turbine on a level with the bottom, and what horse power can it maintain on the average if it is emptied in 10 hours? [1 cu. ft. of water weighs 62·4 lbs.] (L. '94.)

CHAPTER III*

STATICS

Three Forces at a Point.—In order to specify a force completely there must be known:—(1) its magnitude, (2) its direction, (3) its point of application. These can be represented on a diagram by drawing from the given point a line to the proper scale in the given direction.

For the purpose of calculating the conditions of equilibrium in certain cases, the following proposition—which is the converse of the triangle of forces—is important:—**If three forces acting at a point are in equilibrium they can be represented by the sides of a triangle taken in cyclic order.** In Fig. 17* one of the forces, say R, is equal and opposite to the resultant of the other two—for this reason it is called their equilibriant. The three forces are therefore in the same plane and R is represented by CO. Hence the forces P, Q, R are represented by the sides OB, BC, CO of the triangle OBC taken in order. If another triangle be drawn with its sides parallel to those of triangle OBC, its sides will be in the same ratio as those of the original triangle, and can still represent the forces. But when there are more than three forces their ratio cannot be fixed by drawing a polygon with its sides parallel to them; for having made such a polygon, draw a line inside the figure parallel to one side, and another polygon is formed whose sides are in different ratios.

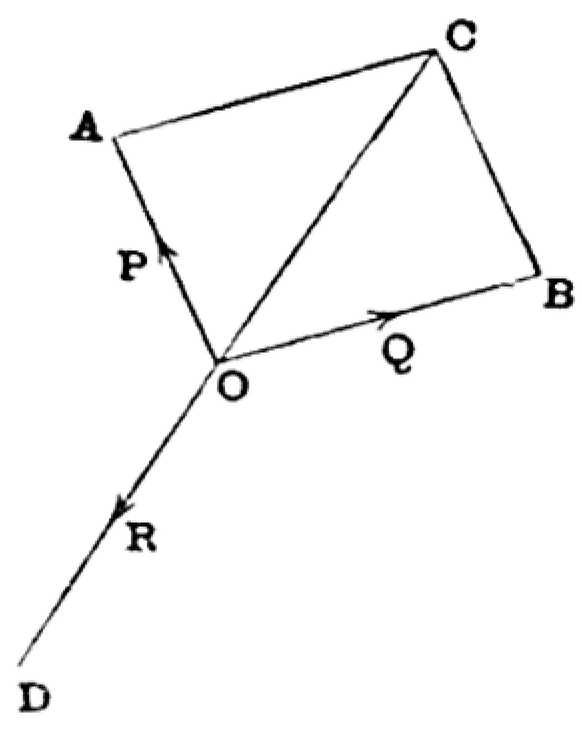


FIG. 17*.—Converse of Triangle of Forces.

Several Forces at a Point.—It will be supposed that the forces are all in one plane. Take two axes at right angles, OX and OY, and resolve each force into its rectangular components along them.

Let the algebraic sum in the two directions be X and Y respectively; these can replace the original forces, and the resultant is given by $R^2 = X^2 + Y^2$ (p. 13*). For equilibrium R must be zero, and therefore both $X = 0$ and $Y = 0$. Hence the condition for equilibrium is that the sum of the components in any two directions at right angles shall vanish.

Inclined Plane.—As an application of the results of the last two paragraphs, let us find the conditions for the equilibrium of a particle resting on a smooth inclined plane (Fig. 18*). By a smooth plane is meant one in which there is no frictional force parallel to the plane; the particle of weight W presses on the plane with a force R, and the reaction, also R, is normal to the plane. Let the height $AB = h$, the base $BC = b$, the length $AC = l$, the inclination $= \theta$, and let the force P parallel to the plane just keep the particle at rest. The forces acting are P, R, and W as shown. As there is equilibrium they can be represented by the sides of a triangle taken in order. From any point L in OL draw LM parallel to CA, and produce the line representing R to M. Then the ΔOLM has its sides, taken in order, parallel and in the same direction as the forces; so that OL represents W, LM represents P, and MO in like manner R. Also LO and MO are perpendicular to BC and AC respectively, therefore $\angle$ LOM $= \theta$,

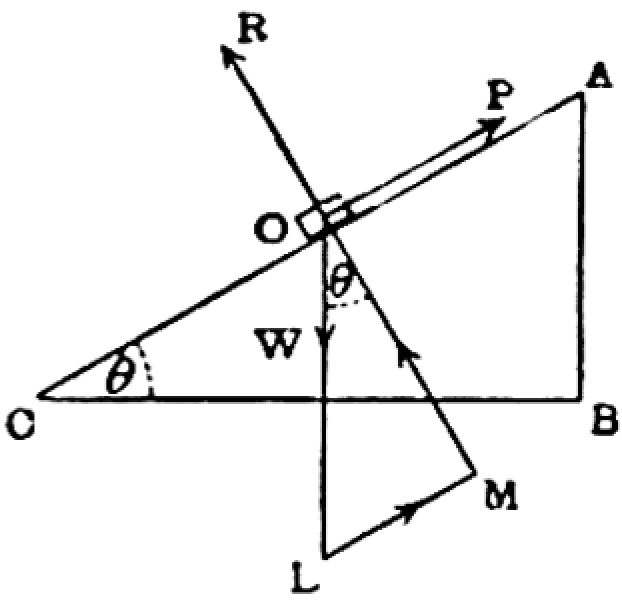


FIG. 18*.—Inclined Plane.

and $$P/W = LM/OL = \sin\theta.$$

$$\therefore\quad P = W\sin\theta = W\cdot\frac{h}{l}.$$

Also $$R/W = OM/OL = \cos\theta.$$

$$\therefore\quad R = W\,.\cos\theta = W\cdot\frac{b}{l}.$$

Now apply the method of the preceding paragraph. Resolve $\parallel$ and $\perp$ to AC. (The advantage of choosing these directions is that R does not appear in the first nor P in the second result.)

$\parallel$ to plane, $$P - W\,.\cos COL = 0.$$

$$\therefore\ P = W\,.\cos COL = W\cos(90 - \theta) = W\sin\theta \text{ (p. 4*)}.$$

$\perp$ to plane, $$R - W\sin COL = 0.$$

$$\therefore\quad R = W\,.\sin COL = W\,.\cos\theta.$$

Let P be parallel to the base as in Fig. 19*. Each side of $\triangle ABC$ is $\perp$ to a force, so that if it were turned through 90°, as in the small fig. to the right, each side would be parallel to and in the same direction as a force. Hence $\triangle ABC$ can be taken as the triangle of forces, the side perpendicular to a force being taken to represent that force. (This principle should be remembered.)

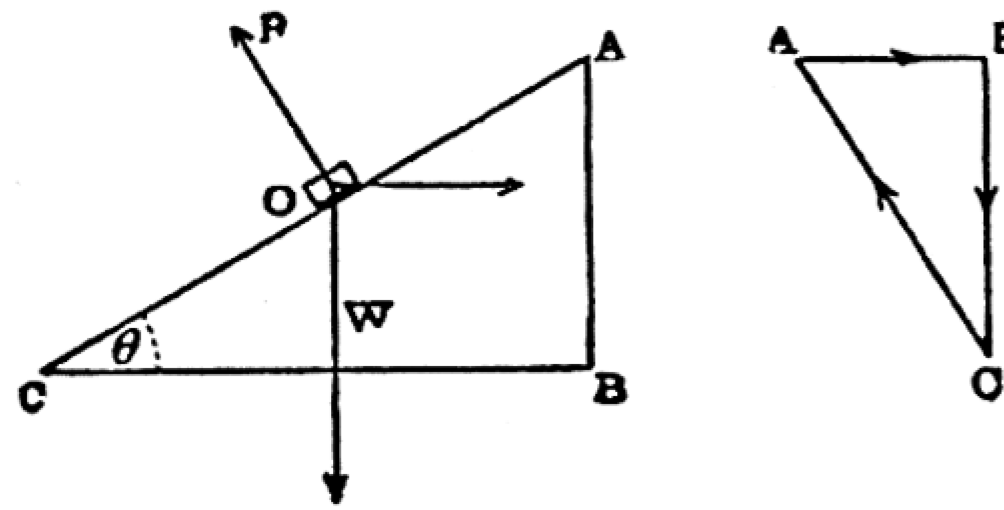


FIG. 19*.—Inclined Plane.

$$\therefore \quad P/W = h/b = \tan\theta,$$

and $$R/W = l/b = 1/\cos\theta.$$

In either case, if a force slightly greater than P were applied, the weight would move up the plane. The formulæ show the advantage of such an arrangement in raising a weight, as, by proper inclination, P can be made much less than W. It is much easier to push a barrow up an inclined plank than to raise it vertically, but the work done against gravity is the same in each case, viz. mgh ergs, where m is the mass raised in gms. (p. 20*). This suggests a still simpler method of obtaining the results; for the gain in potential energy must be equal to the work done. Suppose the mass is dragged from C to A: the gain in energy, in gravitational units, is Wh; the work done by R is zero, since the displacement is perpendicular to it; and in Fig. 18* the work done by $P = Pl$.

$$\therefore \quad Pl = Wh \quad \text{or} \quad P = W \,.\, h/l \text{ as before.}$$

In the second case the work done by $P = Pb$, because b is the displacement parallel to P's direction.

$$\therefore \quad Pb = Wh \quad \text{or} \quad P = W \,.\, h/b.$$

Moment of a Force.—When it is a question of the equilibrium of particles attention can be confined to the possibility of motions of translation, but with rigid *bodies*, whose dimensions affect the result, it is necessary to take into account rotations also. Let AC (Fig. 20*) be the line of action of a force whose magnitude is P, and OL the perpendicular from O to AC; then P . OL is called the torque or the moment of the force round O. To see its physical significance let a weight W gms. be hung from a lever supported at O (Fig. 21*), and let another weight P be moved until a balance is obtained. Alter

P in amount and get a balance as before. It will be found in every case that W . AO = P . OB, *i.e.* the moments of the forces P and W round O are equal and tend to turn the lever in opposite directions. But the lever balances when the turning effects neutralise each other; the moment of P round O is therefore a measure of its turning effect. Moments are taken of opposite sign according as they tend to turn a body in a clockwise or anti-clockwise direction. If a force

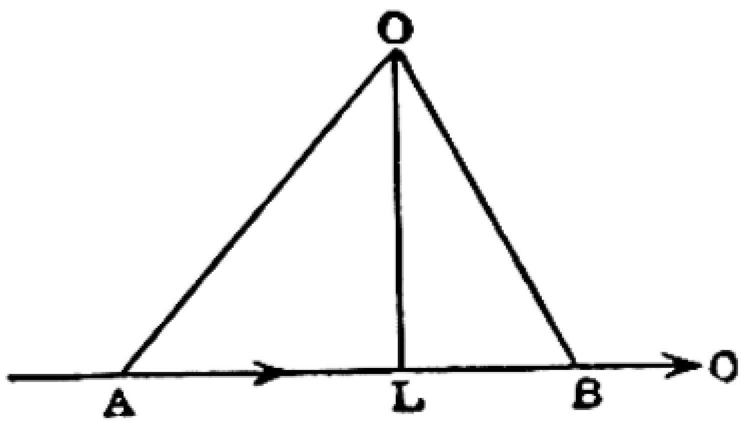


Fig. 20*.—Moment of a Force.

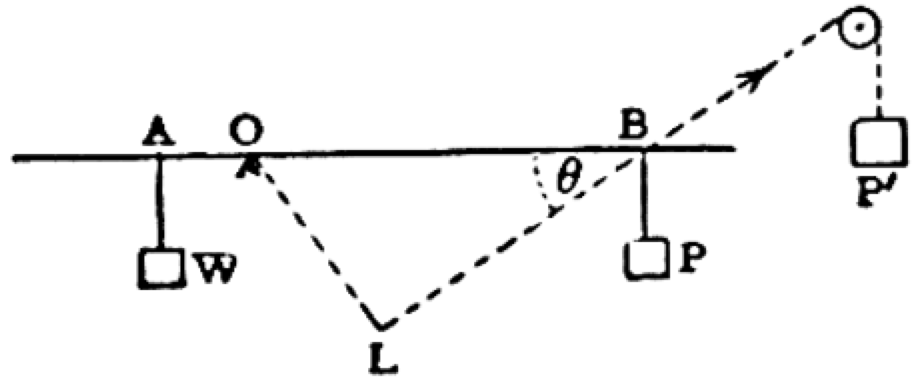


Fig. 21*.—Illustration of a Moment.

is not perpendicular to the lever, as P′ in Fig. 21*, it may be resolved into its components along and perpendicular to OB: the former passes through O and has no moment round that point; the latter is equal to P′ . sin θ, and its moment round O is P′ sin θ . OB. Instead of this we might have drawn OL ⊥ P′, when the moment is P′ . OL, but as OL = OB . sin θ, the result is the same as before. In Fig. 20* let AB represent P in size; then the moment P . OL = AB . OL = twice area of △OAB, showing that a moment can be represented graphically by an area.

Moment of the Resultant of Two Forces.—Let OA, OB, be the lines of action of two forces and OC that of their resultant (Fig. 22*). We will prove that the sum of the moments of the two forces round any point M is equal to the moment of their resultant round the same point. Draw FME ∥ OB and EL ∥ OA. The three forces are now represented by OF, OL and OE, and their moments by 2△OFM, 2△OLM, and 2△OEM. The last two moments are opposite in sign to the first. We have to prove that

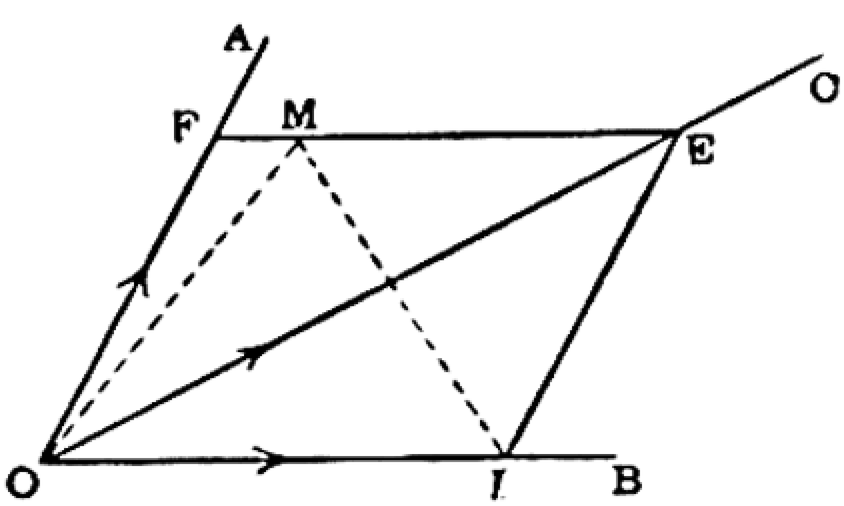


Fig. 22*.—Resultant Moment.

$$2\triangle OLM - 2\triangle OFM = 2OEM.$$

As $\triangle OLM$ is on the same base and between the same parallels as $\square$ FL,

$$\therefore 2\triangle OLM = \square FL = 2\triangle OFE.$$

$$\therefore 2\triangle OLM - 2\triangle OFM = 2\triangle OFE - 2\triangle OFM$$
$$= 2\triangle OEM,$$

so proving the proposition. Being true for two forces, it is true for their resultant and a third force, and so on for any number of forces. It follows from this proposition that the sum of the moments of any number of forces about any point in the line of action of their resultant is zero.

Parallel Forces.—In the last fig. let O be supposed to move away to a very great distance along OC, then OF and OL become parallel, and their resultant OE becomes equal to their sum or difference, according as they are in the same or opposite directions. In the first case they are called like and in the second unlike parallel forces. The position of the resultant is found by the theorem of the last paragraph. Let two parallel forces P and Q act at A and B (Fig. 23*), and let their resultant R act at C. The forces are like in (*a*) and unlike in (*b*). In (*a*) $R = P + Q$, and in (*b*) $R = Q - P$ (if $Q > P$).

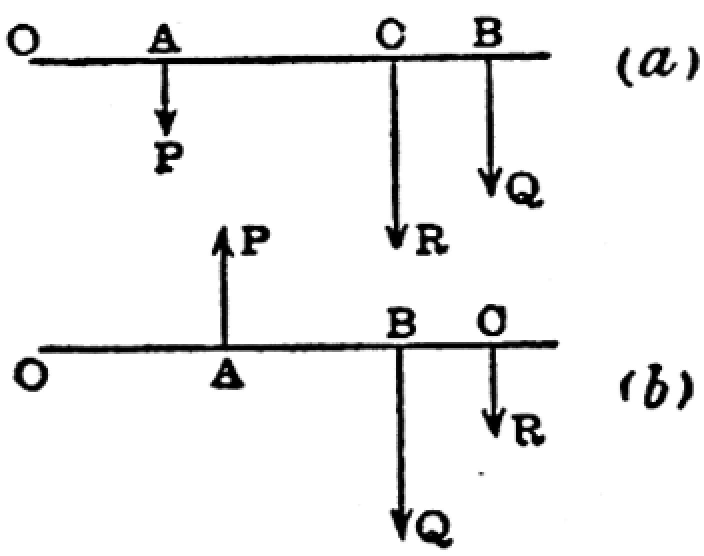


Fig. 23*.—Parallel Forces.

To find the position of C, remember that the moments of P and Q round this point are equal and opposite,

$$\therefore \quad P \, . \, AC = Q \, . \, BC \quad \text{or} \quad AC/BC = Q/P$$

in both cases. Evidently in (*b*) the point C must fall outside AB if the forces are to have oppositely directed moments, and the result just obtained shows it is nearer the larger force. It is often more convenient to take moments round some other point O, and to express that the sum of the moments of P and Q is equal to the moment of R.

Then in (*a*) $\qquad R \, . \, OC = (P + Q)OC = P \, . \, OA + Q \, . \, OB,$
and in (*b*) $\qquad (Q - P)OC = Q \, . \, OB - P \, . \, OA,$

and again the position of C is found. Next suppose the directions of the forces are changed so that they all make an angle θ with OC. To get their moment round O each force is resolved perpendicular

to OC (p. 29*), and this component is multiplied by its distance from O. The components are P sin θ, etc., and the last equation becomes

$$(Q - P)OC \,.\, \sin\theta = Q \,.\, OB \,.\, \sin\theta - P \,.\, OA \,.\, \sin\theta.$$

Dividing out by sin θ it is seen that the result is the same as before, and the position of C is unchanged. C is called **the centre of the parallel forces.** The centre of parallel forces acting at fixed points is the point at which their resultant acts however their direction is changed.

Centre of Gravity.—On p. 318 an example is given of a number of parallel forces acting on a magnet. Another system is of special importance. Any body can be regarded as being built up of innumerable small particles, the weight of each being equivalent to a force acting vertically downwards. The resultant of these is the weight of the body, and the point where it acts—the centre of parallel forces—is called the **centre of gravity** of the body. If it be supported at this point the body will be in equilibrium in all positions. For many purposes the whole mass of a body can be supposed to be concentrated at its centre of gravity.

Couples.—A system of two equal but unlike parallel forces is called a couple. The resultant of such a system is zero and it can produce no motion of translation; there may, however, be rotation. In Fig. 24* draw AB perpendicular to the direction of the two forces P, and take moments about any point O in this line. The resultant moment is

in (*a*) $\quad P \,.\, AO + P \,.\, OB = P \,.\, AB,$
and in (*b*) $\quad P \,.\, AO - P \,.\, OB = P \,.\, AB.$

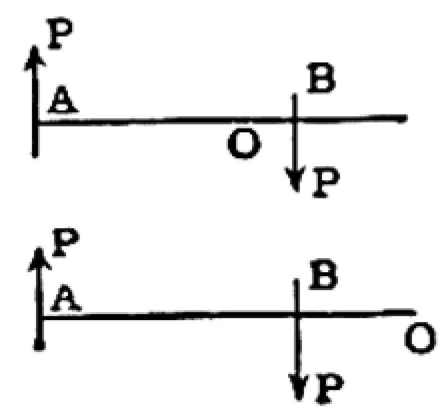


FIG. 24*.—A Couple.

This constant moment is called the moment of the couple, and AB is its arm. If a couple produces rotation work will be done. To find its amount suppose AB (Fig. 24* (*a*)) to make one complete revolution round O, P remaining $\perp$ AB. The angle turned through $= 4$ rt. $\angle$s $= 2\pi$ radians (p. 3*); also B describes a circle of radius OB and circumference $2\pi \,.\, OB$, and A one whose circumference is $2\pi \,.\, OA$.

$$\text{Work done} = P(2\pi \,.\, OA + 2\pi \,.\, OB) = P \,.\, AB \,.\, 2\pi$$

$\therefore$ Work = (mom. of couple) $\times$ angle in radians described by arm.

This will be in ergs if P is in dynes and AB in cms.

Conditions of Equilibrium.—For purposes of reference the conditions for equilibrium are collected here, it being assumed that the forces are all in one plane.

(1) **For a Particle.**—If there are two forces they must be equal and opposite. If there are three forces they must either (*a*) be parallel and their resultant zero, or (*b*) if not parallel, they must be capable of being represented by the sides of a triangle taken in cyclic order. No matter how many forces there are, if there be equilibrium, the components in two directions at right angles must vanish.

(2) **For a Rigid Body.**—(Both translation and rotation to be taken into account.) For no translation the resultant must be zero; for no rotation the sum of the moments round *any* point must be zero. If all the forces pass through one point they cannot produce rotation, and the conditions are the same as for a particle. If they do not all pass through the same point, the simplest condition is that the sum of the components in two directions at right angles should vanish, and the moments round any point should vanish.

Machines.—A machine is a contrivance for overcoming a resistance at one point by the application of a force, usually at another point. The resistance W to be overcome is called the weight or the load, and the applied force P the power (not to be confused with power defined on p. 19*). If P is the force that just balances W, the ratio W/P is called the mechanical advantage of the machine. The inclined plane has already been dealt with.

Levers are divided into three classes according to the position of the point about which they turn—called the fulcrum. The different types are indicated in Fig. 25*, where C is the fulcrum, P the power, and W the load. In addition to P and W the only other force acting on the lever is the reaction R at the fulcrum. Hence the resultant of P and W must act at C and be equal and opposite to R (no translation). Also the sum of the moments of P and W about any point on the line of action of their resultant is zero, that is, in all three cases

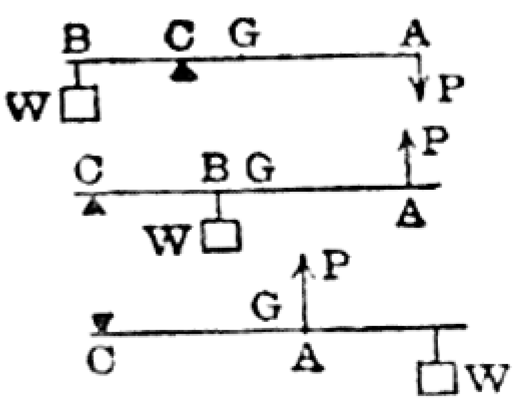


FIG. 25*.—Levers.

$$P \,.\, AC = W \,.\, BC,$$

and R is the algebraic sum of P and W. Examples of the lever of the first class are a pump handle, the lever used by tramwaymen to move a tram over dead points, the pole and cradle used to raise

sheep into the tub holding sheep dip, and the common balance. Double levers are a pair of scissors or pincers. Examples of the second class are a pair of nut-crackers and a wheelbarrow. In the third class $W<P$, and there is always mechanical disadvantage, but in many cases space is saved. The forearm is an example; the fulcrum is the elbow joint, the power is applied obliquely by the biceps muscle at a point below the elbow, and the weight is held by the hand. Double levers are the forceps in a box of weights and sugar tongs. If the weight of the lever, W_1, has to be taken into account, there is an additional force W_1 acting at the centre of gravity G of the lever, but the sum of the moments round C is still zero.

The wheel and axle is shown in Fig. 26*. Examples are the capstan used in ships to raise the anchor, and the windlass used to raise water from a well. (The wheel is represented by the radius of the handle.) Suppose P just balances W. Let radius of axle $= b$, and radius of wheel $= a$. Then in

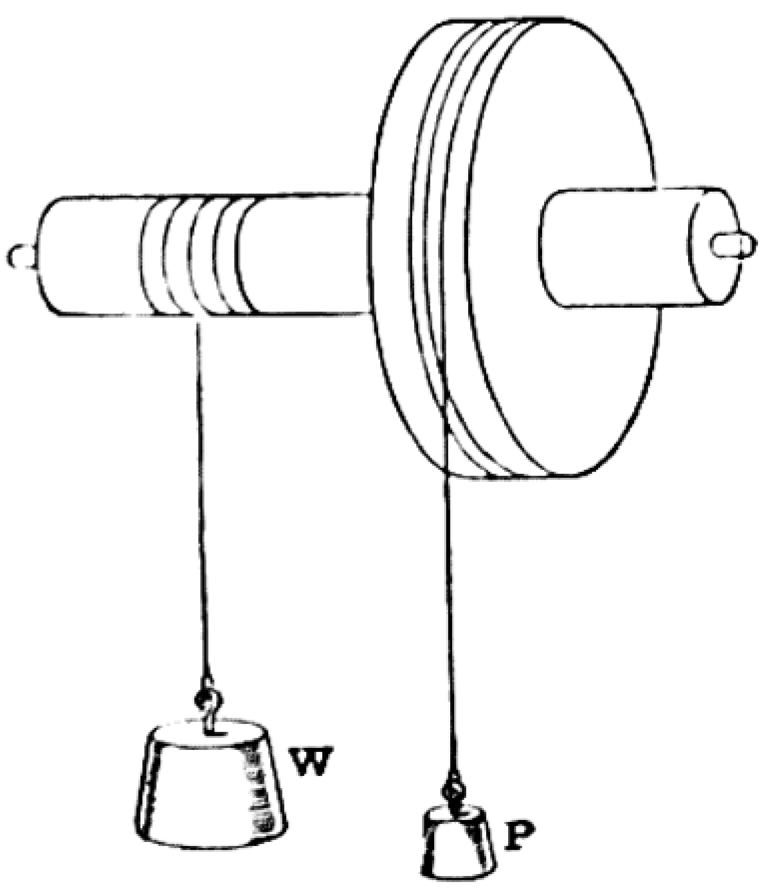


FIG. 26*.—Wheel and Axle.

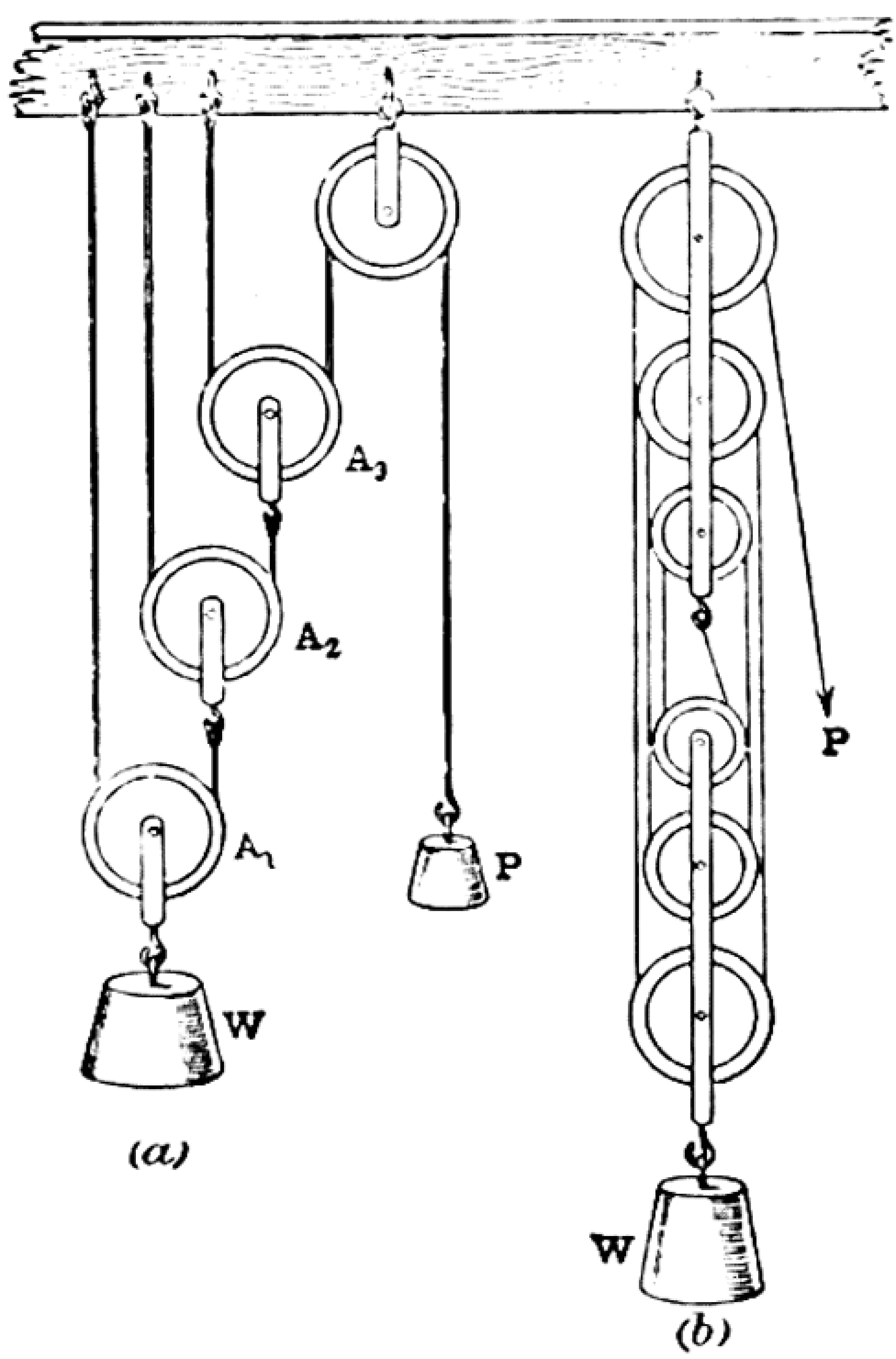


FIG. 27*.—Pulleys.

one complete revolution when W is raised, the work done by P = work done on W (p. 28*), *i.e.* $P \cdot 2\pi a = W \cdot 2\pi b$, or $Pa = Wb$.

Of the various systems of pulleys we will deal with two only, and for the sake of illustration two methods of calculating the

mechanical advantage will be used. In the Archimedes system (Fig. 27* (*a*)) a separate string passes round each pulley. Let W ascend x cms.; then A_2 ascends $2x$ cms.; A_3 goes $2^2 . x$ cms., and P descends $2^3 . x$ cms. Also work by P = work done on W,

$$\therefore \quad P . 2^3 . x = W . x \quad \text{or} \quad W/P = 2^3.$$

In the common system (Fig. 27* (*b*)), the pulleys of the top block are usually all mounted on the same axis, and similarly with those of the bottom block. The same string passes round all the pulleys, and, since it supports the power, its tension is P. As there are six strings going to the lower block the upward pull is 6P. If there are n pulleys $W = n . P$. The weights of the pulleys have been neglected in each case.

The Balance.—In principle the common balance is simply a lever of the first class with equal arms. The beam AB (Fig. 28*) turns round a fulcrum C, which, to diminish friction, is made of an agate or steel knife-edge resting on a smooth agate plane. Let W be the weight of the beam and pointer, G be their centre of gravity, and suppose that nearly equal weights P and Q hang from the arms of length a. Let $CG = b$. Suppose the beam is tilted through an $\angle \theta$. Taking moments round C,

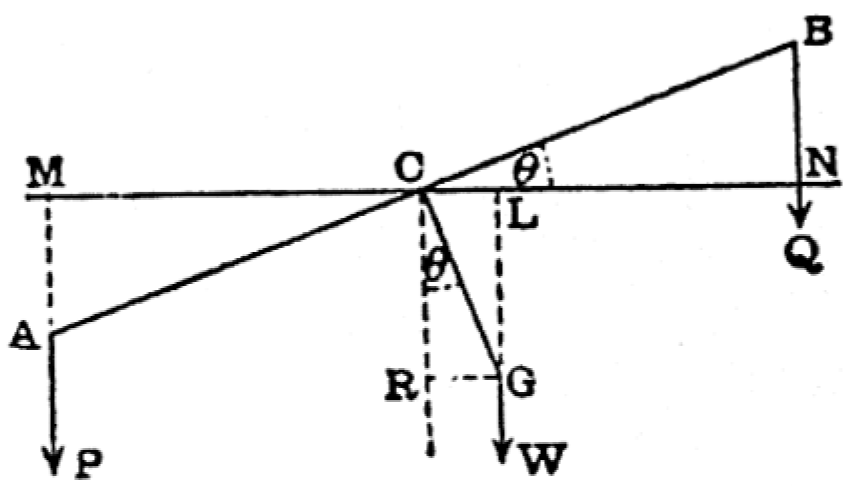


FIG. 28*.—The Balance.

$$P . CM = W . CL + Q . CN.$$

Also $$CM = CN = a \cos \theta \text{ and } CL = GR = b . \sin \theta.$$

$$\therefore \quad Pa \cos \theta = Wb \sin \theta + Qa \cos \theta \quad . \quad . \quad . \quad (1)$$

$$\therefore \quad \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{(P - Q)a}{W . b} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

A good balance should have the following characteristics:—(1) It should be true; *i.e.* the beam must be horizontal when equal weights, or no weights, are in the pans. This is secured by making the arms exactly equal in length and weight, and the pans equal in weight. (2) It must be sensitive. This means that for a small difference between P and Q the angle of tilt, θ, must be large. For a given difference between P and Q the equation shows that $\tan \theta$, and therefore θ, is large when a is large, and W and b are small. Hence the beam must be long and light, and its centre of gravity near C.

(3) It must be stable, *i.e.* it must return quickly to its position of rest when deflected, with equal weights in the pans. Equation (1) shows that when P and Q are equal the only restoring couple arises from the weight of the beam. Hence for stability Wb must be large. (Of course the C.G. must be below C.) It can also be shown that for a quick swing a light beam is required. Evidently the conditions for (2) and (3) are at variance and in practice a compromise must be effected. The scientist requires sensitivity even at the sacrifice of some speed in weighing, while for a grocer a less accuracy combined with speed is sufficient.

Suspended Bodies.—When a body is suspended so that it can turn freely round its point of suspension O (Fig. 29*), it will come to rest with its centre of gravity G vertically under O. For if its C.G. were at G_1, the weight would have a moment round O, causing the body to turn.

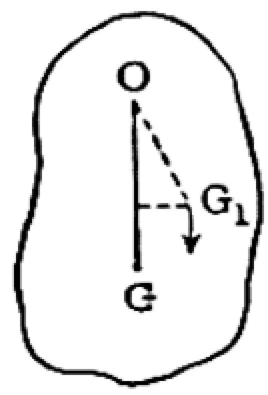


FIG. 29*.—Suspended Body.

EXPERIMENT.—Hang up a sheet of cardboard by a string at one point O and draw the vertical. Repeat for another point of support. The intersection of the two verticals is the C.G.

Similarly when a body rests on a plane, horizontal or inclined, there are two forces acting on it, its weight at the C.G. and the resultant reaction of the plane, and for equilibrium these must act in the same straight line. The vertical through the C.G. must therefore pass through the area of contact of the base of the body with the plane. If the C.G. of the body shown in Fig. 30* were at G_1, the body would topple over, but there is equilibrium when it is at G_2. When a body returns to its rest position after a slight displacement, it is said to be in stable equilibrium; if it moves still further away after the displacement its equilibrium is called unstable. When, like a sphere on a table, it rests indifferently in any position it is said to be in neutral equilibrium.

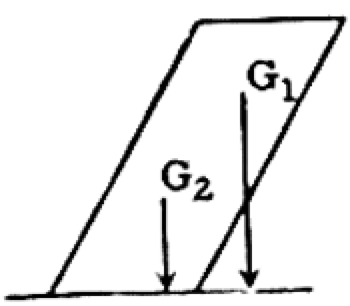


FIG. 30*.

EXAMPLES ON CHAPTER III*

1. A particle slides down a smooth inclined plane inclined to the horizontal at an angle θ. Show tnat its acceleration is $g \cdot \sin\theta$.

2. Find graphically and by calculation the resultant of forces of 10 and 16 gms., acting at an angle of (a) 30°, (b) 60°.

3. Find the resultant of forces of 4, 5, and 6 gms. making angles of $120°$ with each other, (*a*) graphically, (*b*) by resolving them along two directions at right angles.

4. The arms of a balance are not quite equal in length. When a body is placed in one pan it appears to weigh W_1, but when placed in the other its apparent weight is W_2. Show that its true weight is $\sqrt{(W_1W_2)}$, and that the lengths of the arms are in the ratio $\sqrt{W_1} : \sqrt{W_2}$.

5. A weight of 240 gms. is supported from O by two strings, OA, OB, 30 and 40 cms. long respectively. The other ends of the strings are attached to points A and B 50 cms. apart horizontally. Find the tension in each string.

6. Prove that the work done on a particle by a system of forces during any displacement, such that the forces do not alter, is equal to the work done by their resultant. If the forces are in equilibrium their resultant is zero; hence deduce the work principle (p. 28) for machines.

7. A uniform plank, 5 ft. long and weighing 20 lbs., lies symmetrically on the top of a cube of 1 ft. face. What force must be applied at one end in order to tip up the plank? What force must be thus applied to tip it up after a weight of 5 lbs. has been hung on the other end of the plank? (L. '98.)

CHAPTER IV*

HYDROSTATICS

Fluids.—Solids are substances which can oppose a big resistance to any change in their size or shape. Fluids, on the other hand, are substances, like water and air, which can offer no resistance to shape changes when they take place slowly. For rapid changes internal friction—called also viscosity—comes into play. This accounts for the large horse power required in fast steamers and in aeroplanes. For many purposes, as in the following pages, these frictional effects can be neglected. The term fluids includes both liquids and gases. Liquids differ from gases in being very difficult to compress. A gas will expand until it fills the containing vessel, however large it may be. On account of the absence of friction it is clear, (1) that the surface of a liquid at rest must be horizontal: if it were inclined, the particles would slide down the inclined plane. (2) The pressure on any solid surface in contact with a fluid at rest is normal to that surface. For if it were not so, it could be resolved into components perpendicular and parallel to the surface, and the latter component would cause motion to take place.

Pressure in a Fluid.—If a fluid exerts the same pressure on every cm.^{2} of a surface its pressure is said to be uniform, and is expressed in dynes or gms. per cm.^{2} Strictly this should be called the intensity of the pressure. When it is not uniform, a point is taken and a plane area S is described around it so small in extent that the pressure on it can be regarded as uniform. If p is the total pressure on this area, then p/S is called the pressure *at* the point. It is evidently the pressure that would be exerted on unit area if it were everywhere of the same intensity as it is over S. The pressures at two points in the same horizontal plane and connected by fluid at rest are equal. For imagine the points A and B (Fig. 31*) to be connected by a smooth tube; if the pressures on the ends were different the fluid would flow to the left or right. Also the pressure at any point is the

same in all directions. For imagine the end A to be turned in any directi n keeping at the same depth; the liquid can be at rest in the tube only if the pressures along the axis at A and B are equal, and altering the direction of A will not cause motion. On account of the freedom with which liquids move, an increase of pressure at one point is transmitted equally in all directions. For example, when the piston of the pump is pushed in the pressure in the bicycle tyre is increased everywhere to the same extent.

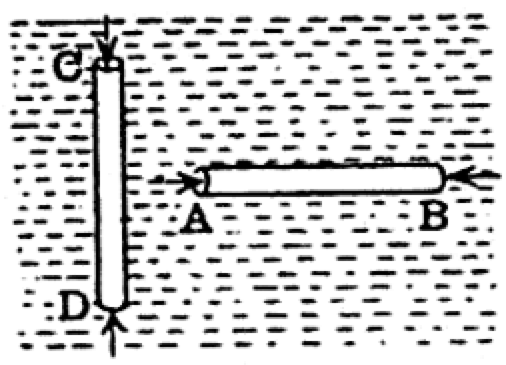


FIG. 31*.—Pressure in a Fluid.

Density and Specific Gravity.—It is frequently necessary to know the relative weights, bulk for bulk, of different materials. Tables of densities or specific gravities are used for this purpose. **The density of a substance is its mass per unit volume.** It is expressed in gms. per c.cm., or lbs. per cub. ft., etc. The units used should be stated in every case. The density of water is 1 gm./cm.^{3}, or 62·5 lbs./cu. ft. approximately. If d is the density of a substance whose volume is v, then its mass

$$m = vd.$$

The specific gravity is the ratio of the weight of a certain volume of that substance to the weight of an equal volume of some standard substance.—The standard substance is usually taken to be water at 4° Cent. As at this temperature the density of water is 1 gm./cm.^{3}, the numbers representing densities and specific gravities are the same in the C.G.S. system. It should be noticed that the specific gravity is the ratio of two weights and is therefore a *number*, which will be the same in all systems of units.

Variation of Pressure with Depth.—Let C and D (Fig. 31*) be two points in a fluid in the same vertical line. Suppose they are connected by a narrow, smooth cylinder. Then, in order that there shall be no downward flow, the pressure on the end D must be greater than that on C by an amount equal to the weight of the liquid in the cylinder. Suppose the section of the cylinder to be 1 sq. cm., then if d is the density of the liquid and CD $= h$, it is seen that the difference in pressure at the points C and D is $p = hd$ gms./cm.^{2} The density of gases is so small that this difference of pressure can be neglected in ordinary closed vessels, though it must be taken into account in large scale operations, such as mountain ascents or balloon voyages (p. 44*). Neglecting the weight, the pressure of a

gas is the same at all points in a vessel. When a U-tube contains liquid (Fig. 32* (*a*)) the pressure at B = pressure at C, and therefore the height AB = height CD, although the vertical limbs may have different sectional areas. If the U-tube contains two liquids which do not mix, as in Fig. 32* (*b*), let B be the interface where the liquids meet, d_1 and d_2 the densities of the liquids on the left and right respectively, and h_1 and h_2 the lengths of the columns AB and CD. Then the pressure at B = the pressure at C (in the same horizontal plane).

$$\therefore\ h_1 d_1 = h_2 d_2.$$

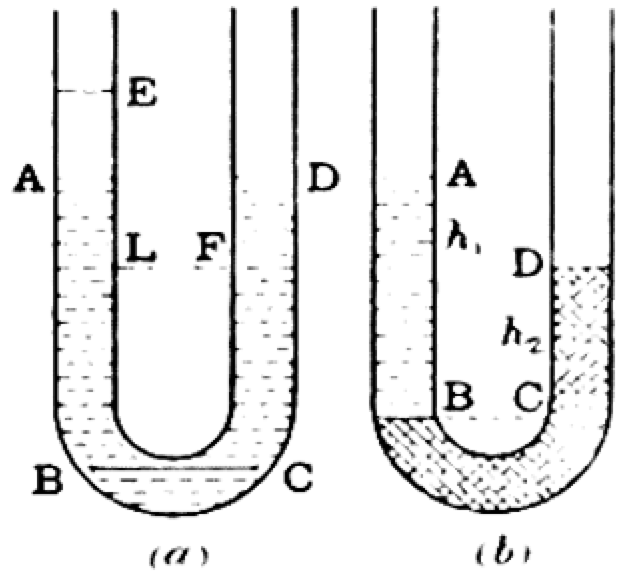


FIG. 32*.—Pressure in a U-Tube.

This result can be used to compare the densities. A U-tube containing liquid can also be used as a pressure gauge. For example, suppose the gas mains are connected to the right limb of the U-tube shown in Fig. 32* (*a*): the liquid is forced downwards in this limb, and upwards in the other. Let it come to rest at E and F; then the gas just balances the column of liquid of height EL, and this accordingly measures its pressure.

Principle of Archimedes.—When a body is partly or wholly immersed in a fluid the pressure on its surface gives rise to an upward thrust and the body apparently loses weight, as, *e.g.*, in swimming. This apparent loss can easily be found both theoretically and experimentally. Let A (Fig. 33*) represent the body. Imagine it to be removed and the space it occupies filled with the fluid. The forces arising from the surrounding fluid are unaltered, but now it is seen that the resultant upward thrust just supports the weight of the volume A. Hence the loss of weight of the original body is equal to the weight of the fluid that would fill the space A. This is known as the principle of Archimedes, after its discoverer. **When a body is immersed in a fluid there is an upward thrust on it of an amount equal to the weight of fluid it displaces,** and the body apparently loses weight by this amount.

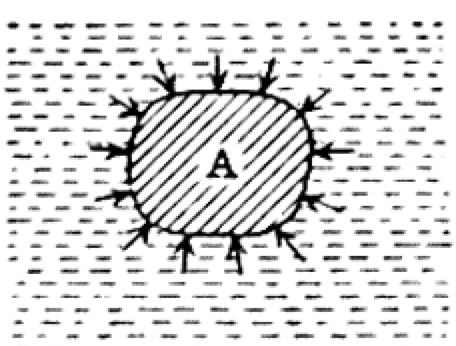


FIG. 33*.—Principle of Archimedes.

EXPERIMENT.—In Fig. 34* B is a solid metal cylinder which just fits in the hollow cylinder A. Weigh the two together and then let B be totally immersed in water, as in the fig. Balance is destroyed, but it may be restored by filling

A with water, showing that the loss of weight of B is equal to the weight of water it displaces. Corresponding to the upward thrust there is a downward "reaction," and the weight of the water in the beaker is apparently increased. The following experiment shows how much this increase is.

EXPERIMENT.—Find B's loss of weight in gms.; then remove it and place the beaker C on the balance. Suspend B from a stand with thread so that it is again immersed. The beaker gains in weight and its gain will be found equal to B's apparent loss.

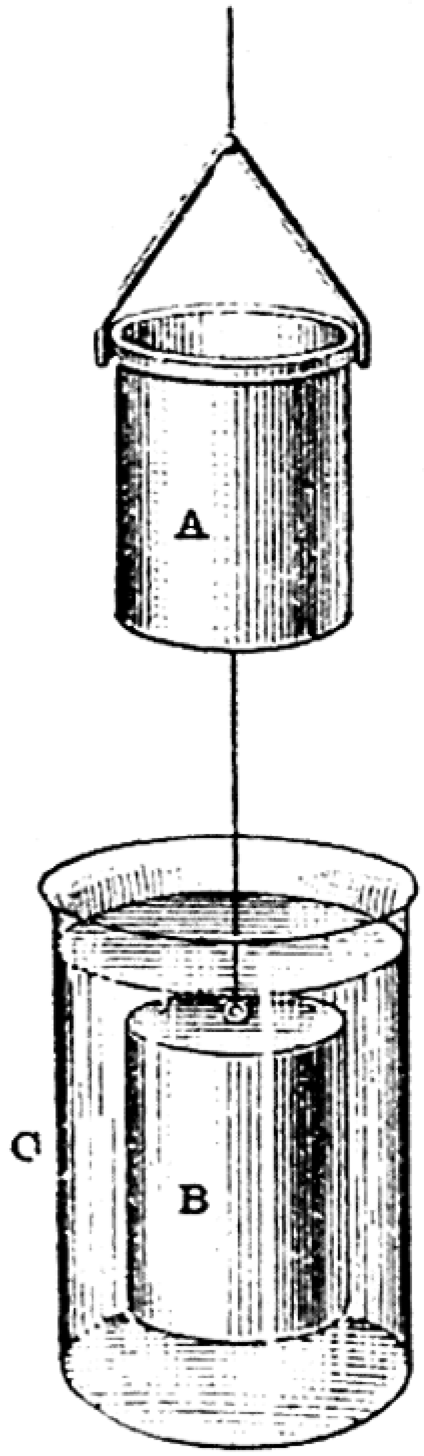


FIG. 34*.—Proof of Archimedes' Principle.

In accurate work the weight of a body and the balance weights must both be corrected for the air they displace.

Determination of Specific Gravity. — The strengths of solutions and the purity of substances are frequently tested by finding their specific gravities. If the specific gravity of a ring, said to be gold, is determined and is found to be less than that of pure gold, then it is certain that either the ring is not solid throughout or it is made of some lighter alloy. The specific gravity of a solid is found from Archimedes' principle by weighing the body first in air and then when hanging completely immersed in water. Let these weights be W and W_1 respectively. Then

$$\text{loss of wt. in water} = \text{wt of water displaced} = W - W_1 \text{ gms.}$$

and

$$\text{S.G.} = \frac{\text{wt. of body}}{\text{wt. of an equal vol. of water}} = \frac{W}{W - W_1}.$$

As 1 gm. of water occupies 1 c.cm., it is evident that the volume of the body is $(W - W_1)$ c.cms. This is the most accurate method of finding volumes.

Suppose the same body is now weighed in another liquid, and that its apparent weight is W_2. Then the weight of liquid it displaces is $(W - W_2)$ and, as the weight of the same volume of water is $(W - W_1)$ gms.,

$$\text{the S.G. of the liquid} = \frac{W - W_2}{W - W_1}.$$

If in the first case the solid is soluble in water, it must be weighed in air and in some other liquid, of known specific gravity s, in which

it is insoluble. If the weights are W and W_1, as before, the weight of liquid displaced is $(W - W_1)$ gms.; and, as 1 c.cm. of the liquid weighs s gms., the volume of liquid displaced is $(W - W_1)/s$ c.cms. But the weight of this volume of water is $(W - W_1)/s$ gms.,

$$\therefore \text{S.G. of the solid} = \frac{W}{(W - W_1)/s} = \frac{W}{W - W_1} \cdot s.$$

That is, to find the specific gravity of a body soluble in water we first find the specific gravity referred to some other liquid, then multiply this by the specific gravity of the liquid.

The best method of finding the specific gravity of a liquid is to find in succession the weights of liquid and of water required to fill a small narrow-necked bottle; the S.G. of the liquid is the ratio of these weights. The specific gravity bottle method is also used when the body is in the form of a powder. W gms. of the powder are weighed in the bottle, which is then filled up with water. Let the contents—powder and water—weigh W_1 gms. Next the weight of the water alone required to fill the bottle is found; let this be W_2. If it were possible to add the powder to the bottleful of water without any liquid escaping, the combined weight of the contents would be $W + W_2$; actually water escapes and the weight is less than this, viz. W_1. Hence water displaced by powder $= W + W_2 - W_1$,

and $$\text{S.G. of powder} = \frac{W}{W + W_2 - W_1}.$$

The methods described above are all used, with additional precautions, in accurate work. The following are much less accurate and, with the exception of the common hydrometer, are used only to illustrate and verify the principles already developed in earlier pages. On account of the accuracy with which weighings can be effected balance methods are usually to be preferred to all others.

Nicholson's Hydrometer consists of a hollow cylinder A (Fig. 35* (*a*)), carrying upper and lower pans C and B. It is suitably weighted to float upright in a liquid. To find the specific gravity of a solid, the hydrometer is floated in water and weights are added to C until a mark P is just in the surface. The solid is then placed in the upper pan and weights W_1 are removed to bring P again in the surface; the weight of the solid is W_1. The body is then removed to the lower pan when, on account of the up-thrust, further weights W_2 must be removed from C to keep the mark in the surface. The weight of water displaced by the solid is W_2, and the specific gravity

required is W_1/W_2. To find the specific gravity of a liquid, the hydrometer is floated first in the liquid and secondly in water, weights being added to C in each case to bring P in the surface. Let W_1 and W_2 be the weights required, and W the weight of the hydrometer alone. From Archimedes' principle the weights of liquid and water displaced are $(W + W_1)$ and $(W + W_2)$, and the volume is the same for each,

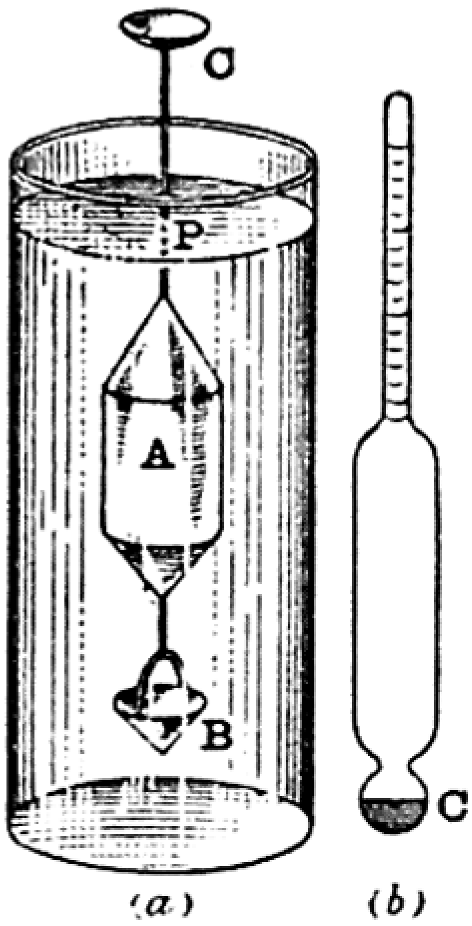


Fig. 35*.—Hydrometers.

$$\text{hence} \qquad \text{the S.G. of the liquid} = \frac{W + W_1}{W + W_2}$$

The common hydrometer shown in Fig. 35* (*b*) is frequently used in works' practice. It consists of a hollow glass bulb surmounted by a slender cylindrical stem, on which a scale is engraved; a lower bulb C is weighted so as to make the instrument float in an upright position when placed in a suitable liquid. When it is placed in a liquid to be tested, the hydrometer sinks until it displaces its own weight of the liquid; the specific gravity of the liquid is then given directly by the scale reading in the surface.

The U-tube method (Fig. 32* (*b*)) can be used to find the specific gravity of a liquid which does not mix with water. When the liquids are miscible the U-tube is inverted and the liquids are sucked up the limbs (Fig. 36*). The pressure on the liquid in the beakers is the same in each case, viz. atmospheric pressure, and this just balances the weights of the liquid columns. Hence, as before, $h_1d_1 = h_2d_2$, whence, if d_1 refers to water, d_2 can be found. Used in this manner the U-tube is called Hare's apparatus.

Pressure of the Atmosphere.—Gases differ from liquids in being less dense, more compressible, and less viscous. If a litre flask closed by a stopcock be exhausted of air and weighed, it can easily be shown that the readmission of the gas increases its weight. A litre of dry air at a temperature of 0° Centigrade, and a pressure of 76 cms. of mercury (see below), weighs 1·293 gms. Any one who uses a bicycle pump knows that air is compressible, and the slow rate at which fog particles fall to earth is a result of the viscosity of the air acting in combination with a relatively large surface. (A number of particles have a much larger surface than if they are all collected

into one liquid drop.) As air has weight the atmosphere should produce a pressure, and this, in fact, it can be readily shown to do. If a glass tube 5 mms. in diameter and 80 cms. or more in length be closed at one end, filled with mercury, and then inverted with its open end under mercury, it is found that a column of the liquid about 76 cms. long remains in the tube (Fig. 37* (*a*)). As the pressure at the level of B must be the same inside and outside the tube, the pressure of the atmosphere just balances that of a column of mercury

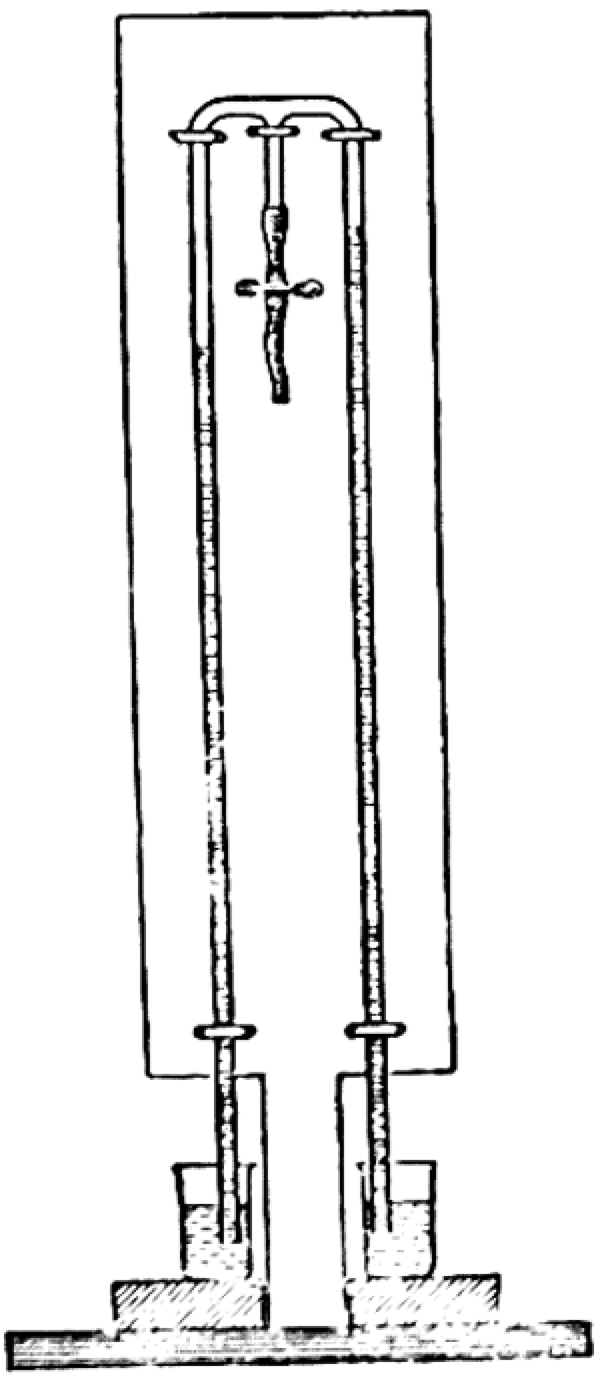


Fig. 36*.—Hare's Apparatus.

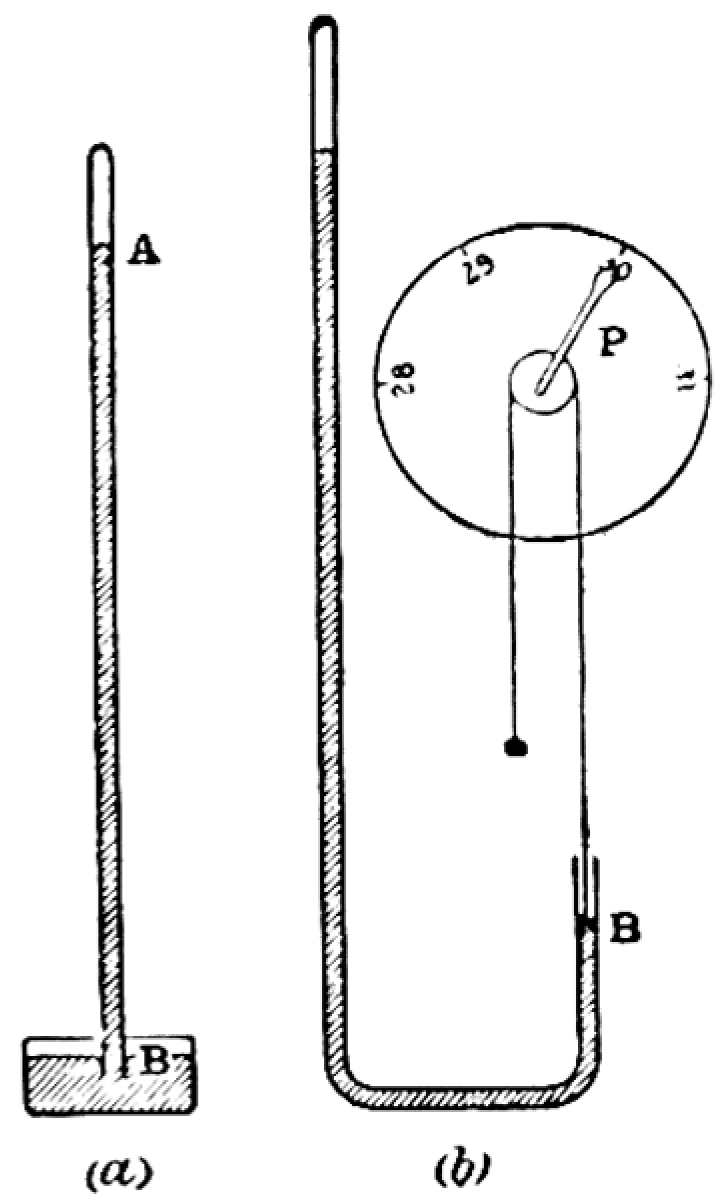


Fig. 37*.—Barometers.

of vertical height AB. If the tube be inclined the mercury runs up it until its vertical height is the same as before, since the pressure at B depends only on the *vertical* depth of this point below A 38*). This height AB is called the height of the barometer. The space above A is called the Torricellian vacuum; actually it is filled with mercury vapour, but this exerts a very small pressure. The normal barometric height is 76 cms., so that the pressure of the atmosphere per sq. cm. is that due to 76 c.cms. of mercury. Taking the density of mercury as 13·6 gms./cm.^{3}, this is $(76 \times 13{\cdot}6)$ gms. weight =

1033·6 gms./cm.². To bring it to dynes multiply by g (= 981). For many purposes gas pressures are expressed in cms. of mercury, *i.e.* in terms of the length of the column they are capable of balancing. For convenience the vessel in (*a*) is frequently replaced by a bend in the tube (Fig. 37* (*b*)); it is then called a siphon barometer. As the height of the barometer is found to be connected with the probable state of the weather, the instrument in this form is frequently used as a weather glass. A thin string, carrying a weight at each end, passes round the axis of a pointer P. One of the weights floats on the surface of the mercury at B and, as this rises or falls, its movements are communicated to the pointer. An accurate form of barometer is described on p. 5. If water were used instead of mercury in a barometer its height would have to be 13·6 times as large to produce the same pressure; this gives the height of the water barometer as approximately 34 ft. Just as the pressure in a liquid varies with the depth, so does the pressure of the atmosphere decrease as we ascend (see p. 91). The height of a balloon, indeed, is calculated from this decreased pressure as measured by a portable barometer. For this and similar purposes an aneroid barometer is used. This consists of an airtight metal box closed on one side by a thin metal sheet. Variations in pressure produce movements in the sheet which, by suitable gearing, are magnified and communicated to a pointer moving over a dial on which the values of the pressures or heights are marked.

Boyle's Law.—The law governing the compressibility of a gas is called Boyle's law, after its discoverer Robert Boyle. In the form of an equation it states that $p_1v_1 = p_2v_2$, where p_1 and p_2 are the pressures of a given mass of gas and v_1 and v_2 are the corresponding volumes. For the method of proving the law see p. 7.

Lift and Force Pumps and Brahmah Press.—The common pump depends for its action on atmospheric pressure. The piston B (Fig. 38*) carries a valve C which opens upwards; lower down in the barrel is another valve D opening in the same direction, and below this a narrower pipe goes to the water reservoir. Normally C and D are closed by their own weight. Let us consider the action from the start:—When B is raised the pressure of the air between C and D is reduced, D opens on account of the air pressure below, and air from the pipe passes into the space CD. The piston is then lowered, when the increased pressure closes D and opens C, thus allowing air to escape upwards. These operations are repeated until all the

air is removed from the pipe, and the atmospheric pressure on the reservoir forces water up above D to take its place. At the succeeding downward stroke this water closes D and escapes above C. On now raising the piston the water closes C and is carried upwards to escape at S. Evidently DE must be less than the height of the water barometer—about 34 ft.—otherwise the atmospheric pressure could not force the water above D. If it be required to raise water to a greater height, as with a fire engine, a force pump must be used. One form of this is shown in Fig. 39*, where it forms part of a Brahmah press. The only additional part is the valve B opening upwards. Suppose the cylinder L to be full of water. When the solid plunger P is raised, atmospheric pressure forces water from the reservoir,

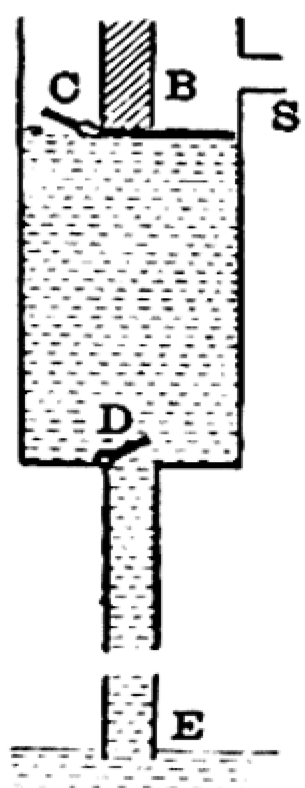


FIG. 38*.—Lift Pump.

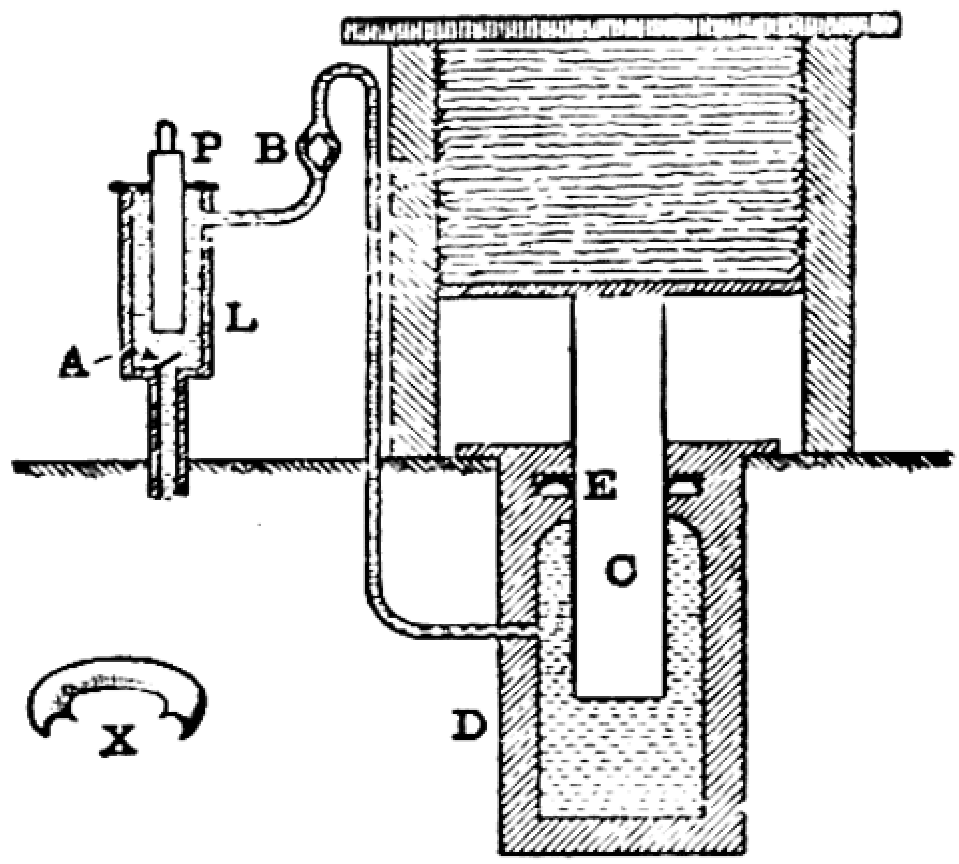


FIG. 39*.—Brahmah's Press.

past the valve A, into L. When P descends A is closed; at the same time B is opened by the increased pressure and water is forced upwards, whence it may be led away in any direction. In the fig. it is shown as being led into a second strong iron cylinder D, forming part of the Brahmah press. A large cylinder C, called the ram, moves up and down in this chamber, suitable packing at E preventing leakage. The packing takes the form of a well-oiled leather collar, placed in the groove near E, and of the shape shown at X; the pressure of the water on the concave side then presses it strongly against the metal on either side. When valve B is open the pressure on the plunger P is transmitted to the ram, and as the sectional area of this is n times that of the plunger, where n is a large number, the total upward pressure on the ram is n times that on P. By this means very large pressures can be produced.

Air Pumps.—The simplest form of air pump is the ordinary glass filter pump shown in Fig. 40*. The side tube C leads to the vessel to be exhausted, while A is connected with the water mains. When a strong jet of water is forced down A the air surrounding the narrow nozzle becomes entangled and is carried away down B; thus a partial vacuum can be produced in a vessel joined to C. Even when a current of air is sent down A a suction effect is produced, as can readily be shown by blowing down it while a tube from C leads into a vessel of water. The liquid immediately rises and passes down B. If steam

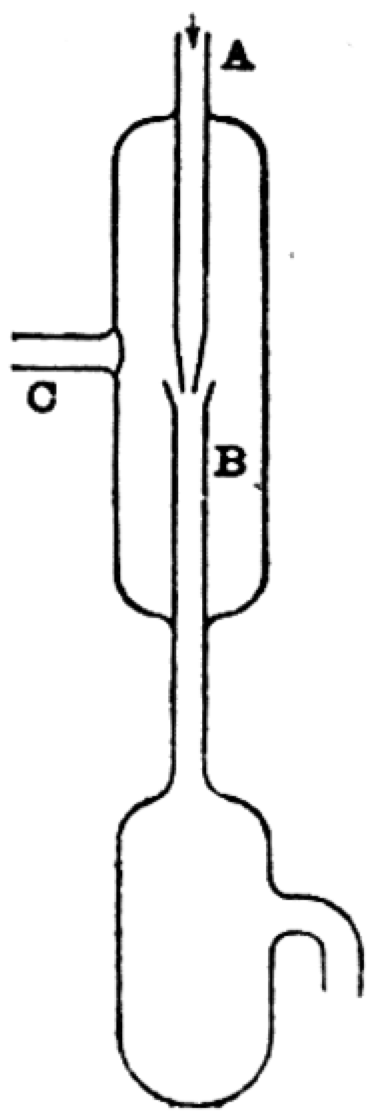


Fig. 40*.—Filter Pump.

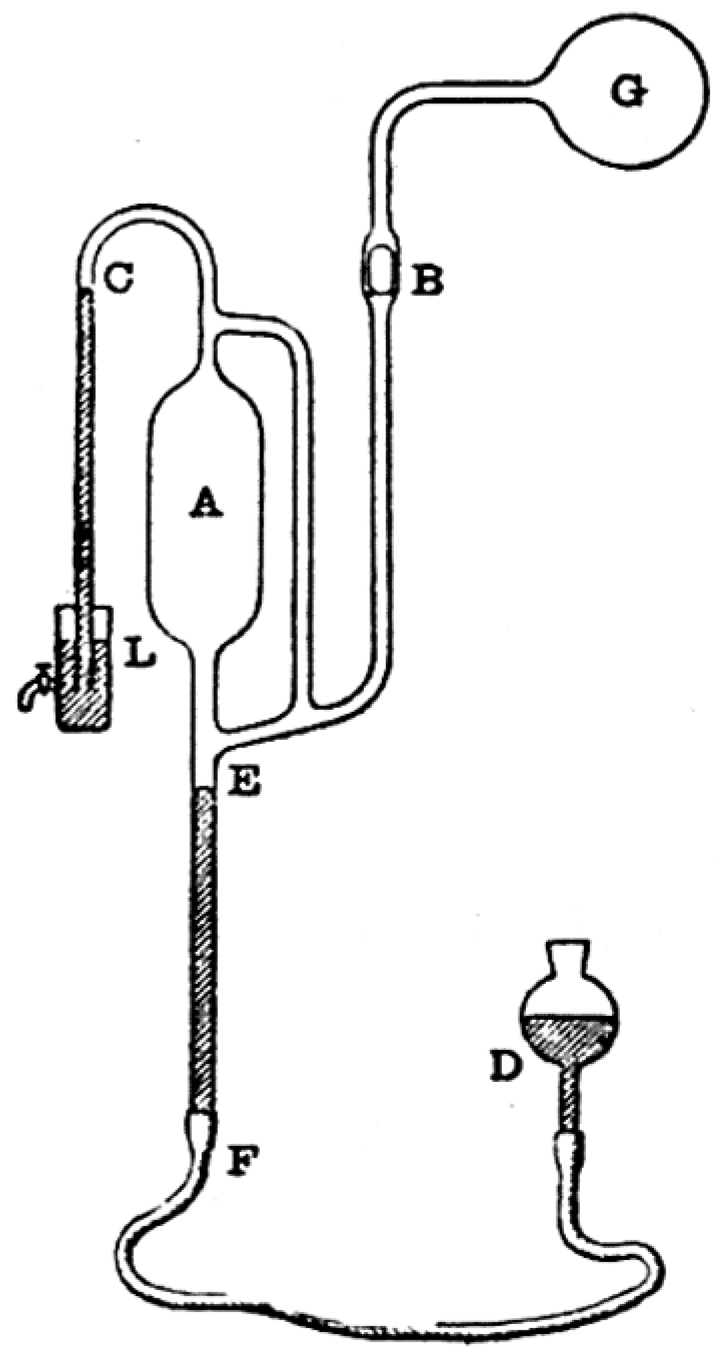


Fig. 41*.—Töpler Air Pump.

be used instead of air we have an illustration of the action of the steam injector used to force water into boilers. There is also a piston and valve air pump, but as this operates in a similar manner to the lift pump of Fig. 38*, no further description is necessary, except to say that the narrow tube below D goes to the vessel to be exhausted, and it is the expansive force of the gas which opens this valve at each stroke of the piston. If each valve in Fig. 38* opened downwards, air could be compressed in any vessel fixed on to the end of the narrow tube DE. This is the action of a bicycle pump, except that

the valve corresponding to D is in the tyre. It is evident that with a piston and valve air pump the exhaustion cannot be pushed beyond the stage where the pressure of the gas becomes too feeble to raise the valve D. For further exhaustion another type must be used; one such, called a Töpler pump, is illustrated in Fig. 41*. The apparatus is made of glass, except for the rubber connection tube F. The reservoir D and the vessel L contain mercury, and the length of each of the tubes FE and LC is slightly greater than the barometric height. The lower end of C is under mercury. The apparatus to be exhausted is sealed on just beyond the valve B, whose purpose is to prevent the passage of mercury upwards. When the reservoir D is raised mercury passes up the side tubes at E and closes the valve; it also flows into A and down the narrow tube C, sweeping the air along and causing it to escape through the mercury in L. The reservoir is then lowered, when the valve falls by its own weight, but on account of its shape does not close the tube leading downward, so that the air in G expands and again fills A. Meanwhile the lower end of C is sealed from the atmosphere by mercury, some of which rises up the tube. The stroke is then repeated, each time removing air of volume A. Pressures as low as ·0001 mm. of mercury can be produced by this pump. Dewar has shown that still lower pressures can be produced by sealing on to the apparatus a small bulb containing coco-nut charcoal; when this is immersed in liquid air it is found that the charcoal rapidly absorbs most gases.

The Siphon.—The siphon (Fig. 42*) is a convenient device for emptying a vessel of liquid when taps are not in use. It consists of a bent tube with a long and a short limb. Suppose the tube is filled with some of the liquid in the vessel; then the pressure at B, being equal to that at A in the same horizontal plane, is that due to the atmosphere, plus the weight of a column of liquid of height DA, and the downward pressure at C is greater than the atmospheric pressure owing to the liquid column BC. Hence liquid escapes and atmospheric pressure forces more up the short limb to take its place. It is clear that the siphon will not work if the vertical distance between D and E is greater than the height of a barometer made of the liquid in question.

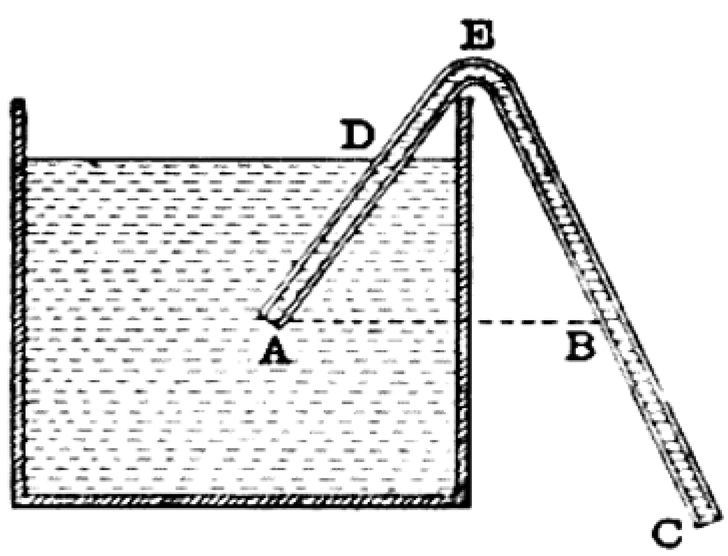


FIG. 42*.—The Siphon.

EXAMPLES ON CHAPTER IV*

1. The specific gravity of gold is 19·3; that of silver is 10·4. What is the composition of an alloy of gold and silver whose specific gravity is 17·6? (L. '80.)

2. A cu. ft. of water weighs 1000 oz. A man weighing 160 lbs. floats with 4 cu. in. of his body above the surface. What is his volume? (L. '81.)

3. A piece of lead weighing 17 gms. and a piece of sulphur have equal apparent weights when suspended from the arms of a balance and immersed in water. When the water is replaced by alcohol of density ·9, 1·4 gms. must be added to the pan from which the lead is suspended to restore equilibrium. Determine the weight of the sulphur, the density of lead being 11·333. (L. '86.)

4. A bottle whose volume is 500 c.cs. is sunk mouth downwards below the surface of a pond. How far must it be sunk for 100 c.cs. of water to run into the bottle? The height of the barometer at the surface is 76 cms., and the specific gravity of mercury is 13·6. (L. '93.)

5. A flask, which when filled with water weighs altogether 410 gms., has 80 gms. of a solid introduced, and being then filled up with water weighs 470 gms. Find the s.g. of the solid and the volume of a kgm. of it. (L. '90.)

6. Some air is in the space above the mercury in a barometer. When the mercury stands at 29 in., the space above the mercury is 4 in. long. The tube is then pushed down into the cistern so that the space above the mercury is only 2 in. long, and now the mercury stands at 28 in. At what height would it stand in a perfect barometer? (L. '94.)

7. Show that if a piston is moved along a cylinder against a constant pressure the work done in a stroke is equal to the product of the pressure into the volume swept out by the piston. (L. '97.)

8. A closed cylindrical vessel 3 ft. in diameter and 1 ft. high is connected with a vertical tube of 1 sq. in. and 10 ft. high from the top of the vessel. Calculate (*a*) the weight of water that will fill the vessel and tube; (*b*) the force tending to burst off the bottom of the cylinder; and (*c*) the pressure on the bottom. (1 cu. ft. of water weighs 62·5 lbs.) (L. '98.)

9. A man 1·7 metre high changes from the vertical to the horizontal position. If the density of the blood be 1·03, calculate the change in blood pressure in his head, assuming that it stays constant in his feet. (L. 1900.)

PHYSICS

CHAPTER I

GENERAL PROPERTIES OF MATTER

THE study of Mechanics has shown that a frequent result of the action of force on matter is the generation of energy, either kinetic or potential. It is found, however, that energy may manifest itself in other and more complicated forms; although these would probably be reducible to the simpler forms if our knowledge were more complete. The study of these various forms of energy, their modes of propagation from place to place, and the conversion of one form into another is the province of Physics. Properly to understand these, it is found necessary to investigate the properties of the smallest particles of which all matter is built up; consequently the structure of matter is one of the main problems of Physics. The types of energy dealt with in the following pages are those associated with Heat, Light, Sound, Magnetism and Electricity; but before beginning their study it will be convenient to deal with some of the general properties of matter, which do not fall strictly under any of the above heads, as these properties are frequently met with in experimental work.

Newton's Law of Gravitation.—Matter may be defined as that which occupies space. This definition does not make any hypothesis as to the structure of matter; in fact, this question is one of the main problems of Physics. In addition to occupying space, all matter possesses mass and is subject to the law of gravitation. This law, discovered by Sir Isaac Newton, states that **every particle of matter attracts every other particle with a force which is proportional to the product of the masses and inversely as the square of the distance between them.** Thus if m and m' are the masses and R the distance between them, the force of attraction

$$F \propto \frac{mm'}{R^2} \quad \text{or} \quad F = k \cdot \frac{mm'}{R^2}$$

where k is a constant. Thus the earth attracts the moon, and *vice*

versâ, and each attracts and is attracted by the sun and other astronomical bodies. As illustrations of laboratory experiments that have been made to detect and measure the attractive force between bodies, the two following may be quoted. Prof. Krigar-Menzel weighed a body on the top of a high building, then he attached it to the balance by a long vertical wire, which passed through the rooms below, and weighed it again. In the second position it was nearer the earth and should be more strongly attracted, *i.e.* its weight should be greater; this was found to be the case. Similarly Prof. Poynting attached equal masses to the arms of a balance and found that one was attracted downwards when a large block of lead was placed immediately beneath it. From experiments of this type the value of the constant k in the above equation can be found, for all the quantities except k can be measured. Once this value is known the mass of the earth and of the planets can be calculated.

Elasticity.—When force is applied to a body it may move it as a whole or it may merely alter the relative positions of the particles composing it. In the latter case the size or shape of the body is changed and it is said to be **strained.** If it tends to recover its original size or shape after the forces are removed the body is said to be **elastic.** Consider any small plane area in a strained elastic body: there will be attractive or repulsive forces between the particles on opposite sides of the plane tending to move them back to their original positions; the magnitude of this force per unit area is called the **stress.** Bodies which tend to recover their original volume after a deformation are said to possess **volume elasticity,** those which tend to recover their shape, *e.g.* after a twist, are said to possess **simple rigidity.** So long as their volume is unaltered, liquids and gases do not permanently resist change of shape; they have only volume elasticity. Solids, on the other hand, have elasticity of both types. If we consider only alterations of length, the strain is measured by the change in length per unit length; thus if a wire of length L cms. is strained until its length is $(L \pm l)$ cms., the strain is l/L. When the volume varies the strain is measured by the change in volume per unit volume, *i.e.* if the volume V is altered by forces to $(V \pm v)$, the strain is v/V. If the strain exceeds a certain value, which varies with the material, the body is permanently deformed and is incapable of recovering its original configuration; it is said in such cases that the elastic limits have been exceeded.

For smaller strains, within the elastic limits, experiment shows that the strain is proportional to the stress ; hence

$$\text{stress} \propto \text{strain}$$

or
$$\frac{\text{stress}}{\text{strain}} = E$$

where E is a constant. This ratio is called the **modulus of elasticity.** For volume changes E is called the **bulk modulus** of elasticity ; if we are concerned only with variations of length, E is called **Young's modulus.** Let a wire of length L and radius R cms. be stretched by a force of F dynes, and suppose the increase of length is l cms. Across each section of the wire there acts a force F, hence the stress $= \frac{F}{\pi R^2}$. Also the strain is l/L, and Young's modulus, which we shall denote by Y, is

$$Y = \frac{F/\pi R^2}{l/L}$$

The ratio l/L is a mere number independent of the units of length, hence Y is given in dynes/cm.² If the force is applied by hanging a weight of M gms. to the lower end of a vertically suspended wire

$$F = Mg$$

and
$$Y = \frac{Mg \,.\, L}{\pi R^2 \,.\, l}$$

Suppose that a body of volume V cms.³ is subjected at every point of its surface to an increase of pressure P dynes/cm.² at right angles to the surface, and let v be the volume change produced. Then the stress is P, the strain is v/V, and the bulk modulus

$$= \frac{P}{v/V} = \frac{PV}{v}$$

As in the previous case v/V is a number and the bulk modulus is expressed in dynes/cm.²

Hooke's Law.—Young's Modulus for a Wire.—It has already been stated that within certain limits stress and strain are proportional ; for alterations in length this is known as **Hooke's law.** This law states that **the linear extension is proportional to the stretching force.** The spring balance is a common application of this rule, the extension

of the spring is proportional to the weight of the body hung from its lower end. The student should verify in a similar manner that Hooke's law holds for a piece of rubber band. When we come to metal wires the extensions to be measured are very small, and either a sensitive apparatus must be used to determine them or they must be made larger by using a long wire. The latter alternative is generally ruled out, as it is difficult to keep the temperature [1] of a long wire constant, and any variation may cause an alteration in length as large as that produced by stretching.

EXPERIMENT.—**Searle's Apparatus.**—With this apparatus a wire about 2 m. long can be used. Two wires A, B (Fig. 1) are hung from the same support, and each carries at its lower end a brass rectangle to the lower sides of which suitable weights can be hung. Stretching across from one rectangle to the other is a spirit level; this turns freely round a hinge at one end and at the other rests on the point of a vertical screw C. The pitch of the screw, *i.e.* the distance it advances with one complete turn, is 1 mm., and its large circular head is divided into 100 divisions. Thus if it is turned through one division the point advances or retreats 0·01 mm. Each wire is stretched taut by a kgm. weight and the screw reading is taken when the air bubble is at the middle of the level. An additional kgm. is then added to one wire, this stretches it slightly and the bubble is displaced; it is brought back to its standard position by turning the screw. Evidently the amount the latter advances measures the extension. Readings are taken with increasing and decreasing loads and the average extension for 1 kgm. is found. This is l of the last paragraph when $M = 1000$ gms. The diameter of the wire is taken at different points with a screw gauge, the length L can be measured with a metre rule, and hence Y can be found. As the wires are suspended from the same support, any yielding of this affects both wires equally and is without influence on the spirit level. If the extensions are plotted against the load the points will lie very approximately on a straight line, thus proving Hooke's law. The stress at which Hooke's law ceases to be true

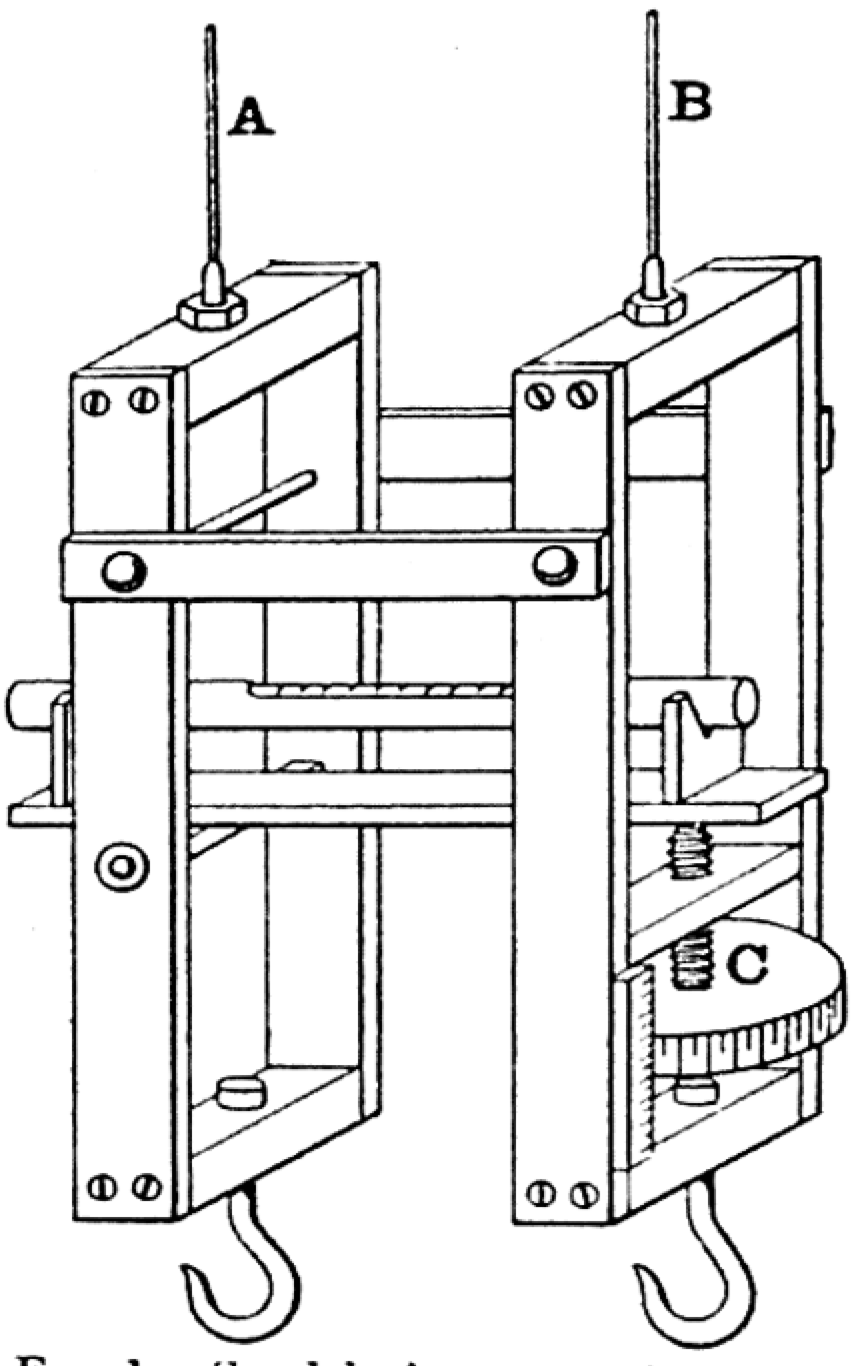


FIG. 1.—Searle's Apparatus for finding Young's Modulus.

[1] Temperature is dealt with in Chap. II.

is called the yield-point; beyond this the elastic limits are exceeded and the wire is permanently stretched.[1]

Bulk Modulus of Solids and Liquids.—The measurement of this quantity is difficult and will not be dealt with, but the volume elasticity of gases can easily be determined. Before giving the necessary experiments it will be convenient if some form of standard barometer is described. It is assumed that the student already knows the principle of the barometer.

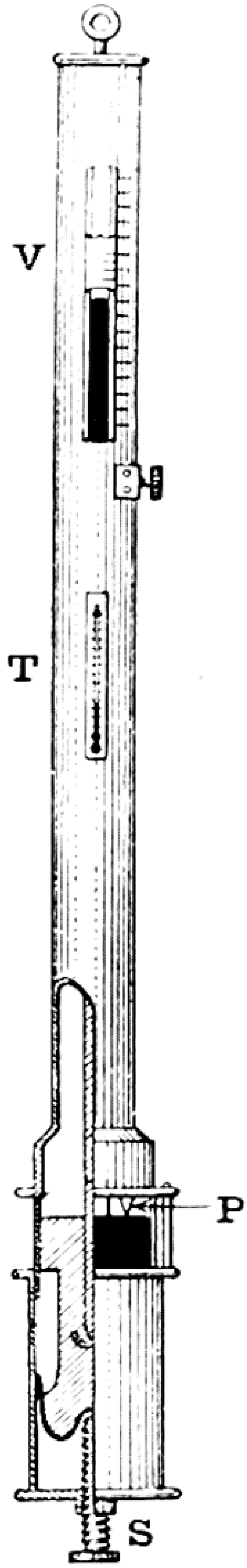


Fig. 2.—Fortin's Barometer.

Fortin's Barometer.—To construct a barometer a glass tube 80 cms. or more in length is closed at one end, filled with mercury, and its lower, open, end is placed in a vessel of the same liquid. The height of the mercury surface in the tube over that outside is the height of the barometer, and the pressure the column produces per cm.^{2} is the atmospheric pressure. It is convenient to read the height by a scale fixed to the column, but as the pressure varies, mercury flows into or out of the cistern and the scale zero must constantly be shifted to coincide with the mercury surface. This difficulty is overcome as follows in the Fortin barometer. The bottom of the cistern is made of wash-leather which rests on the broad end of a vertical screw S, while immediately above the mercury there is a small ivory pointer P (Fig. 2) whose tip coincides with the zero of the scale. When a reading is to be taken the screw is turned until the mercury just touches the bottom of the pointer, the scale zero is then on the liquid surface. The length of the column is read by a vernier V which is moved over the graduated scale by a rack and pinion. Even when the pressure is constant the height varies with the temperature on account of the expansion of the liquid; in giving barometric heights it is therefore usual to reduce the observed height to what it would be at 0° Centigrade. A thermometer T attached to the column gives the necessary temperature; the method of calculating the

[1] For another apparatus, see Barton and Black, "Practical Physics," p. 41.

correction is explained on p. 56. To diminish the effects of surface tension (p. 9), the diameter of the glass tube should be large, 5–10 mm. at least.

Boyle's Law.—To investigate the elasticity of a gas we have to determine how the volume depends on the pressure. This point may be investigated with the apparatus shown in Fig. 3, due to Dr. Baillie.[1] Two wide glass tubes, A and B, containing mercury are connected by flexible rubber tubing. Each can be moved along vertical iron guides by a string and pulley arrangement, and the difference in level of the mercury surfaces is read on a movable scale D. Vessel A is graduated in c.cms. from the tap C. This part of the apparatus must first be filled with well-dried air or other gas. With this object some calcium chloride is placed in the funnel above the tap, the latter is opened, and the reservoir B is raised, thus expelling the air from A. The reservoir is next lowered very slowly and air, dried by the calcium chloride, enters A through the tap. This operation is repeated several times; finally the tap is closed. Suppose mercury stands at the same level in each limb, the pressure of the gas in A is then equal to the pressure at the surface of B, *i.e.* to the atmospheric pressure, which is measured by the height of the barometer. Let this be H cms. at the room temperature. The volume of the enclosed gas is given by the graduations on A. B is next raised and A lowered until the mercury stands at A′, B′. The gas pressure P is now (H + A′B′) cms. of mercury; A′B′ is read on the scale D, and the volume V is found as before. A series of readings is taken in this manner. If B is lower than A, the pressure of the confined gas is (H — difference in level in the two limbs), hence observations can be made with pressures greater and less than that of the atmosphere. Careful experiments of this type have shown that the product (pressure × volume) is constant, provided the temperature does not vary. This relation was first discovered by Robert Boyle in 1662 and

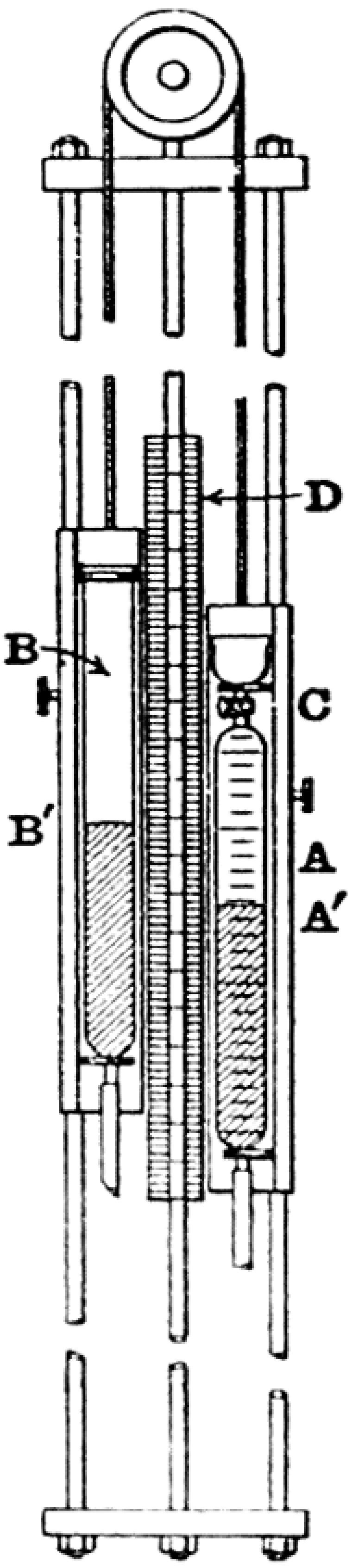


Fig. 3.—Boyle's Law Apparatus.

[1] Barton and Black's "Practical Physics," p. 37.

is called **Boyle's law.** On the Continent it is known as Marriotte's or as the Boyle-Marriotte law. Put in the form of an equation it is

$$PV = \text{constant}$$

or if P_1, V_1 represent a second pressure and volume

$$PV = P_1V_1$$

The equation shows that if the pressure on a gas is doubled its volume is halved. More extended and elaborate experiments have shown that the law is not strictly true for any gas, but the deviations are so small in the case of gases like air, hydrogen, oxygen, nitrogen and helium that we shall assume it is obeyed accurately. Other gases such as sulphur dioxide, carbon dioxide, and ammonia are more compressible than the law requires; their volume is reduced to less than half when the pressure is doubled. When we state the volume of a gas it is clear we must give also the temperature and pressure at which this is measured; the normal temperature and pressure (N.T.P.) are taken as the melting-point of ice, and a pressure of 76 cms. of mercury at 0° Centigrade. The above equation may be written in a slightly different form. Let ρ be the density of the gas when its pressure is P and volume V. Since density is the mass of unit volume the mass of the gas

$$m = V\rho$$

or

$$V = m/\rho$$

Hence

$$PV = \frac{Pm}{\rho}$$

or

$$\frac{P}{\rho} = \frac{PV}{m} = \text{const.}$$

since m is constant while we deal with the same mass of gas.

Any change which takes place at constant temperature is called an **isothermal** change. Thus Boyle's law gives the isothermal relation between the pressure and volume of a gas.

Isothermal Elasticity of a Gas which obeys Boyle's Law.—Let a mass of gas occupy a volume V at a pressure P, and suppose when the pressure is altered by a small amount to $(P + p)$ that the volume becomes $(V - v)$, the temperature remaining constant. The increase in stress is p and the strain it produces is v/V, hence the isothermal bulk elasticity is

$$E = \frac{p}{v/V} = \frac{pV}{v}$$

But $\quad (P + p)(V - v) = PV$ from Boyle's law

or $\quad - Pv + pV - pv = 0$

Now p and v can be made as small as we please, hence their product pv can be made so small as to be negligible compared with the other terms of this equation (see p. 40). We may therefore neglect the third term, and

$$pV = Pv$$

or
$$\frac{pV}{v} = P$$

Thus the isothermal elasticity of the gas is equal to its pressure. E is usually given in dynes/cm.^{2}, while the pressure P per cm.^{2} is measured in cms. of mercury, P must therefore be expressed in dynes/cm.^{2} Taking the normal pressure we have to find the weight in dynes of a column of mercury 76 cms. high and 1 cm.^{2} in section. Since the density of mercury is 13·6 and $g = 980$,

$$\begin{aligned} P &= 76 \times 1 \times 13{\cdot}6 \text{ gms./cm.}^2 \\ &= 76 \times 1{\cdot}36 \times 980 \text{ dynes/cm.}^2 \\ &= 1{,}013{,}000 \text{ approximately} \end{aligned}$$

This result will be required later.

Kinetic Theory of Matter.—In order to connect and explain the many facts that have been accumulated by experiment, some hypothesis as to the structure of matter is necessary. The kinetic theory is the one which has proved most fruitful in these respects. According to this theory it is supposed that all substances are built up of very small particles called molecules, just as a handful of sand is composed of fine granules. We may suppose, for simplicity, that the molecules are small spheres; then, even when the spheres are in contact, the substance is not continuous, but there are interspaces between the molecules which are unoccupied by matter. It is further supposed that the molecules are not generally in contact with their neighbours, but that each is moving to and fro in a continuous state of agitation, sometimes moving freely, at other times in collision with surrounding molecules. In gases the average separation of the molecules is large compared with the dimensions of a single molecule, so that considerable freedom of motion is possible. In liquids the molecules are supposed to be closer together; thus, although a molecule may thread its way through the mass like a person in a crowd, collisions are more frequent than in gases. Within

solids the motion is still more restricted ; a molecule now oscillates to and fro round a mean position and is never far removed from it. If the molecules in a solid possessed great freedom of movement the shape of solid bodies would constantly be changing ; gases, as we know, occupy the whole volume of the containing vessel, no matter how this is varied. It is easy to see that the molecular separation is greater in gases than in liquids, for when a gas is converted to liquid there is usually a large decrease in volume. Thus 1700 c.cms. of steam, which is water in gaseous form, condense to form 1 c.cm. of water. The theory supposes also that the hotter a body is, the more violent does the molecular agitation become ; in fact, in the simplest case of gases, it is assumed that the temperature is proportional to the average kinetic energy of the molecules. An increase of temperature, which means an increase of heat within the substance, thus corresponds to an increase in the kinetic energy of the molecules, and we are lead to make a further supposition, viz. that heat is merely energy. This is a view of which the correctness will be established in later chapters. The kinetic theory gives a qualitative, and in some cases a quantitative, explanation of a number of properties of matter. Thus the pressure of a gas is due to the bombardment of the walls of the containing vessel by the rapidly moving molecules. When a body is compressed the molecules are moved closer together. Again, in solution it is supposed that molecules of the solid become detached and wander away through the molecules of the liquid, so that if we could examine a minute quantity of a solution we should find it far from homogeneous. Similarly porous bodies are those in which the particles are so far apart that the molecules of other substances can find their way into the interstices. Or take diffusion : if a few c.cms. of copper sulphate solution are placed in the bottom of an upright tube and the remainder is filled with water, it is found after some days that the salt molecules have gradually wandered throughout the whole mass of liquid, in spite of the fact that copper sulphate is heavier than water. This process is called **diffusion**. Gases diffuse more rapidly than liquids on account of the greater freedom of the molecules. Other applications will appear later.

Properties of Liquids.

Surface Tension.—It has been stated that liquids do not permanently resist change of shape when their volume is unaltered ; this is

only approximately correct and it ceases to be true when most of the liquid is in or near the surface layer, as in the case of a thin film. Thus air has to be forced into a soap-bubble to make it expand, and, if the mouth be removed from the pipe stem, the bubble contracts and forces the gas out again. In this case the volume actually occupied by the liquid is constant, but the extent of its surface is changed. The liquid film behaves, in fact, like a football bladder—it resists an increase in its area and decreases in size directly the external force is removed. Numerous experiments can be given to show that the surface of a liquid acts as if it were a stretched membrane.

EXPERIMENT.—Dip a camel-hair brush in a beaker of water; the single hairs project in all directions. Remove it from the liquid and the hairs are all drawn together as if connected by a stretched membrane. This experiment and many others are given by Prof. Boys in his book on "Soap-Bubbles."

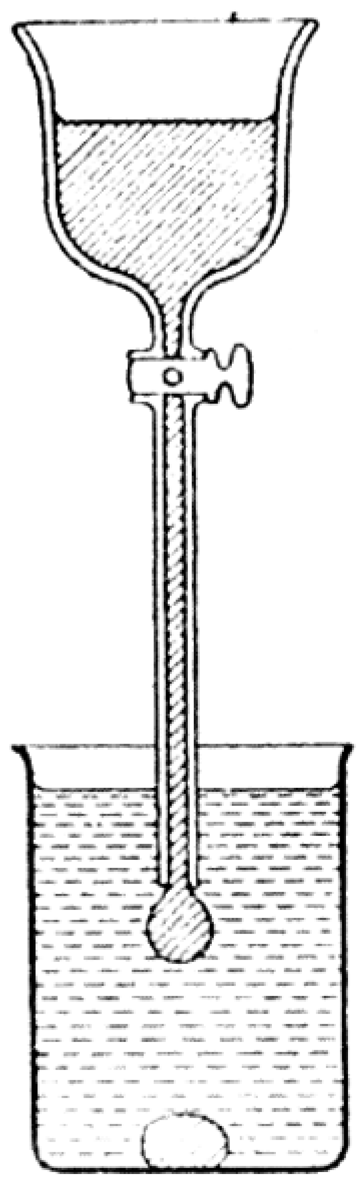


FIG. 4.—Darling's Experiment on the Formation of Drops.

EXPERIMENT.—Place a needle carefully on a water surface; it rests in a small depression just as a heavier body would do if placed on a sheet of stretched rubber. The ability of certain insects to walk on water depends on the same property.

EXPERIMENT.—Make a shallow dish about 2 inches square from fine copper gauze and cover its bottom with a loose piece of paper. Pour water in and then remove the paper; the liquid does not flow out because it must increase its surface before it can escape through the fine holes.

EXPERIMENT.—The formation of a water drop at the end of a vertical tube can be imitated exactly by fastening to a circle of wire a sheet of thin rubber, such as part of a toy balloon. When water is gradually poured on to the rubber it forms a pendant drop very similar to the water drop, and finally contracts like the liquid into a narrow neck before it breaks. When the liquid drops at the end of the tube are large, and if they are caused to form slowly, the similarity becomes more striking. This is done in the next experiment.

Darling's Experiment.—The formation and rupture of a drop can be more easily observed if, by some means, the effective weight of the drop is diminished. A convenient arrangement for doing this is shown in Fig. 4. At 64° aniline has a density equal to that of water, while at temperatures just below this it is slightly the denser of the two; hence if a drop of aniline be formed in water at a temperature near 60° most of its weight will be supported by the surrounding liquid. The funnel shown in Fig. 4 contains aniline, and its lower end, which should have a diameter from 5–10 mms., is immersed in water whose temperature is about 60°.

By opening the tap slightly a large drop of aniline may be formed at the exit; this finally forms a narrow neck and ruptures. The similarity in the behaviour of liquid and rubber surfaces then becomes very striking.

All these experiments show that the surface of a liquid acts like a stretched membrane. Imagine a line 1 cm. long drawn on the surface, the tension tends to pull the liquid apart on opposite sides of this line, and **the magnitude of the force per unit length is called the surface tension of the liquid.** This force is confined to an extremely thin skin of the liquid; hence in the case of a soap-bubble or isolated film there is a fully developed surface tension on *each* side. When the area of such a film is extended more liquid goes from the interior into the surface, but the tension remains constant until the point of rupture is nearly reached. In this respect a liquid surface differs from a stretched membrane. The student will perhaps understand more clearly the definition given above from the following illustration: Suppose we have a wire rectangle (Fig. 5) with a soap film ABCD stretched across it, and suppose the side AB can slide along the other wires as guides. On account of the surface tension the film tends to contract and pull the wire towards CD; if T is the surface tension, the force that must be applied to hold it in position is 2T . AB. The multiplier 2 comes in since both sides of the film have to be taken into account. For water at 0° T is about 75 dynes per cm., for clean mercury it is about 430. The illustration just given shows a typical effect of surface tension, viz. the tendency that a liquid surface has to contract its area unless hindered by other forces. Thus raindrops are spherical because that shape has the least surface for the same volume. This circumstance is turned to account in the manufacture of lead shot. Molten lead is made to fall from the top of a tower into water some distance below; during its descent it takes the form of small spheres which rapidly solidify. The surface tension of mercury is so large that small drops of mercury spilt on the table are spherical in spite of their weight. The surface tension of oil is less than that of water, hence when an oil drop is placed in a beaker of water it is pulled in all directions until it is spread over the entire surface.

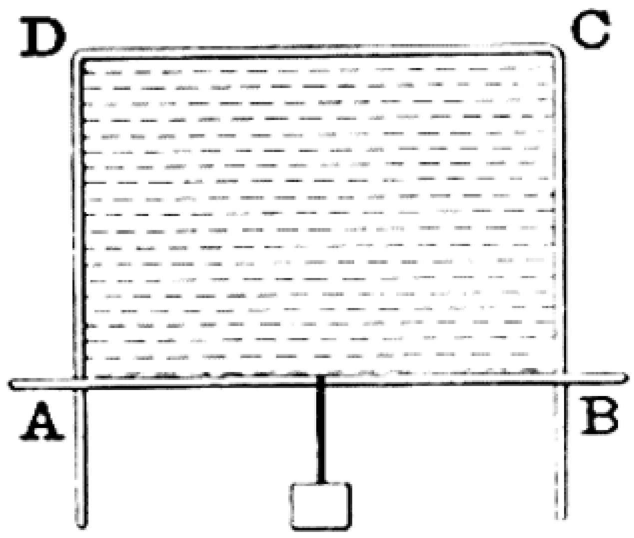
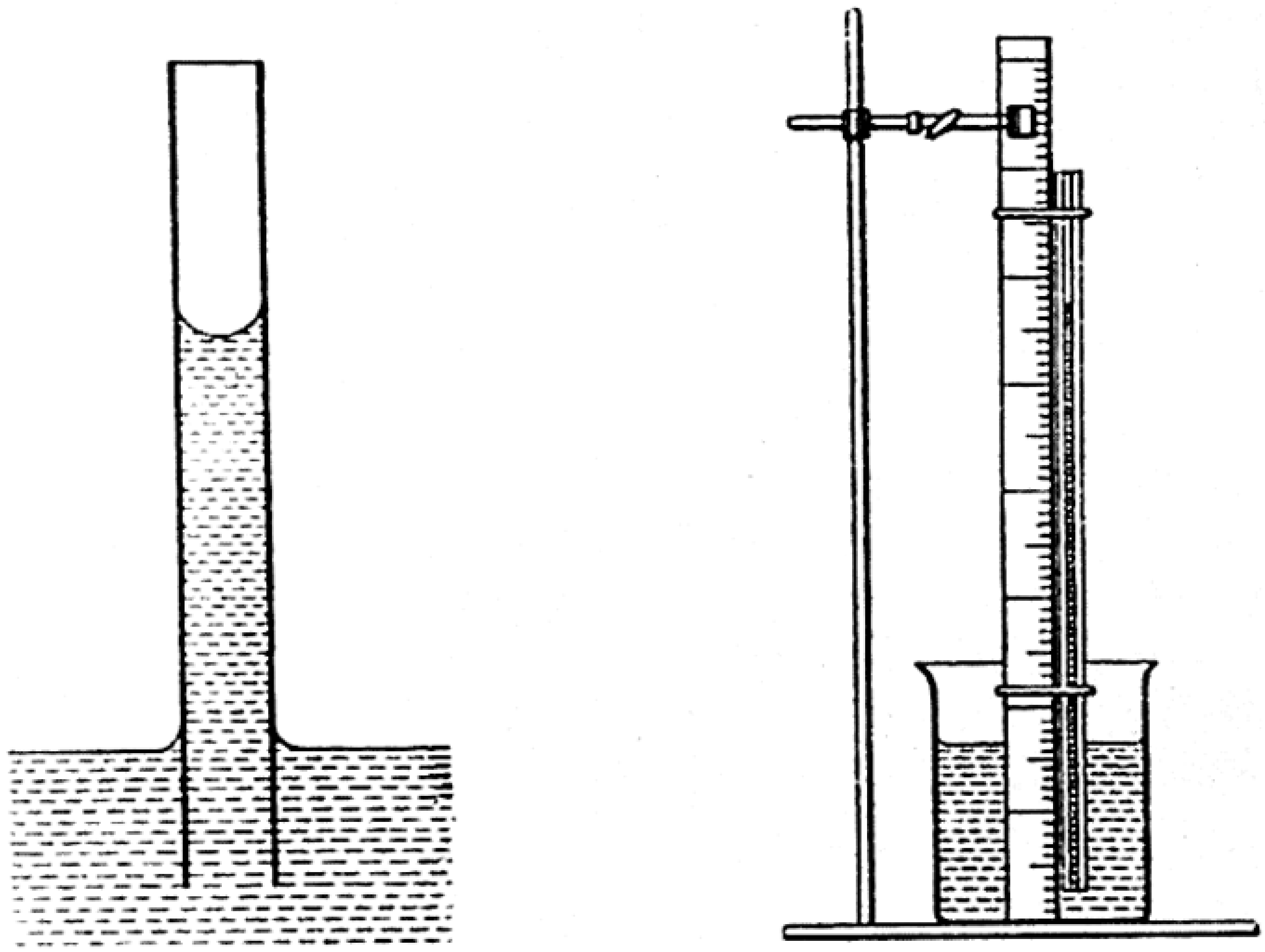


FIG. 5.

EXPERIMENT.—Place a film of water on a glass plate and let a single drop of alcohol fall on it. The surface tension is diminished and the film is pulled away in all directions.

Methods of measuring Surface Tension.—*1st method.*[1] When a glass tube of fine bore is held vertically with its lower end in a liquid which wets it, it is found that the liquid rises in the tube to a definite height which depends on the nature of the liquid and on the internal diameter of the tube. The upper end of the column is hemispherical with its convex face downwards, as in Fig. 6, A. This upper surface



(A) (B)

FIG. 6.—Rise of Water in a Capillary Tube.

clings to the glass and by means of its surface tension it supports the weight of the column below it; this gives a method of finding the surface tension T. Let d be the density of the liquid, h the height which it rises, and R the radius of the tube. Then the surface tension acts along a length $2\pi R$ and the upward force is $2\pi R \cdot T$ dynes; the weight of liquid it supports, in dynes, is $\pi R^2 hdg$,

hence
$$2\pi RT = \pi R^2 hdg$$
$$T = \frac{Rhdg}{2}$$

To carry out the experiment the glass tube is thoroughly cleaned and washed out with the liquid; it is then fixed to a graduated scale

[1] Barton and Black, "Practical Physics," p. 48.

and placed in a vertical position in the liquid, as shown in Fig. 6, B. The quantity h can thus be found. The diameter of the bore is next measured with a microscope or by other means, and the density of the liquid is found with a specific gravity bottle.

Mercury may be taken as typical of those liquids which do not wet a solid placed in contact with them; these liquids do not rise in capillary tubes like the water in the last experiment.

EXPERIMENT.—Push a capillary tube into mercury; it will be found that the liquid is lower inside the tube than outside, exactly the reverse of what happens with water. It will also be noticed that the surface is convex towards the air.

EXPERIMENT.—Pour mercury into a glass U-tube one limb of which is wide while the other consists of a fine capillary. The liquid surface is lower in the narrow tube. If the bore is very fine a considerable pressure will be required to force the mercury along it. An instance of this is given in the next chapter in connection with the filling of a thermometer.

The rise of a liquid up blotting paper is due to surface tension; the interspaces between the fibres form a large number of capillary tubes through which the liquid ascends. The ascent of a liquid through a lump of sugar is due to a similar cause.

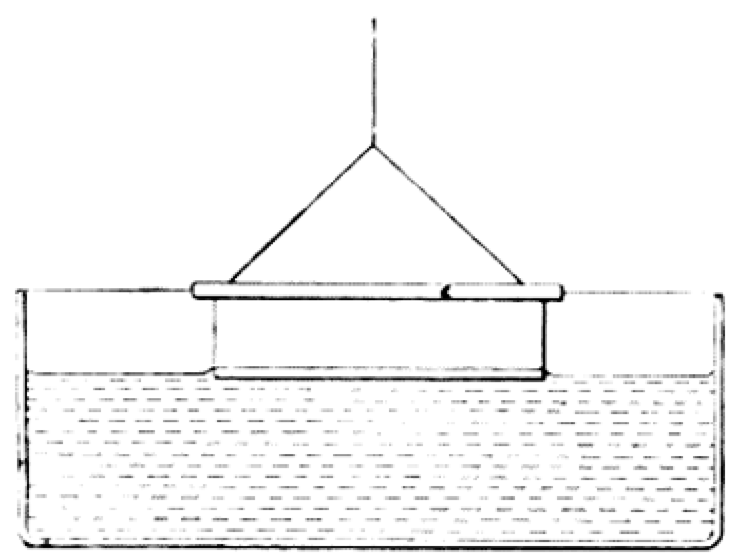


FIG. 7.—Method of finding Surface Tension by Weighing.

2nd method. In this method the pull due to surface tension is determined directly by means of a balance. A plate of glass, such as a microscope slide, is fixed in a strip of wood and hung with its plane vertical from one arm of a balance over a vessel of water, as in Fig. 7. An equal weight is placed in the other pan. The vessel is raised until the water just touches the glass, when the surface tension pulls down the arm. The height of the liquid and the weights in the other pan are altered until the lower edge of the plate is just in the liquid surface when the beam of the balance is horizontal. Let l be the length and c the thickness of the plate, m the additional weight required to balance the downward pull of the liquid. Taking into account each side of the glass, the surface tension acts on a length $2(l+c)$. Hence the pull is

$$2(l+c)\mathrm{T} = mg \text{ dynes}$$

whence T can be found.

EXAMPLE.—To increase the pull six plates were fixed in two wooden strips and the additional weight required was 6·82 gms. Length of plate = 7·6 cms., thickness = 2 mm., hence

$$l + c = 7{\cdot}8 \text{ cms.}$$

and

$$7{\cdot}8 \times 2 \times 6 \times T = 6{\cdot}82 \times 980$$

whence T for water = 71·4 dynes/cms.

When six plates are used a sensitive balance is unnecessary.

Diffusion.—We have already given an example of diffusion and have shown how the process is to be pictured according to the kinetic theory. Let us return to this example (p. 9). Imagine a horizontal plane drawn across the tube just above the copper sulphate some time after the diffusion has begun. As the salt is more concentrated below than above this plane more of its molecules will wander across in an upward than in a downward direction. It can be shown that the excess is proportional to the difference in the concentrations of the salt immediately above and below the plane. The rate at which diffusion proceeds depends on the velocity of the molecules, it accordingly takes place more rapidly as the temperature rises and is quicker for light than for heavy molecules. Owing to their greater freedom of motion gases diffuse much more rapidly than liquids.

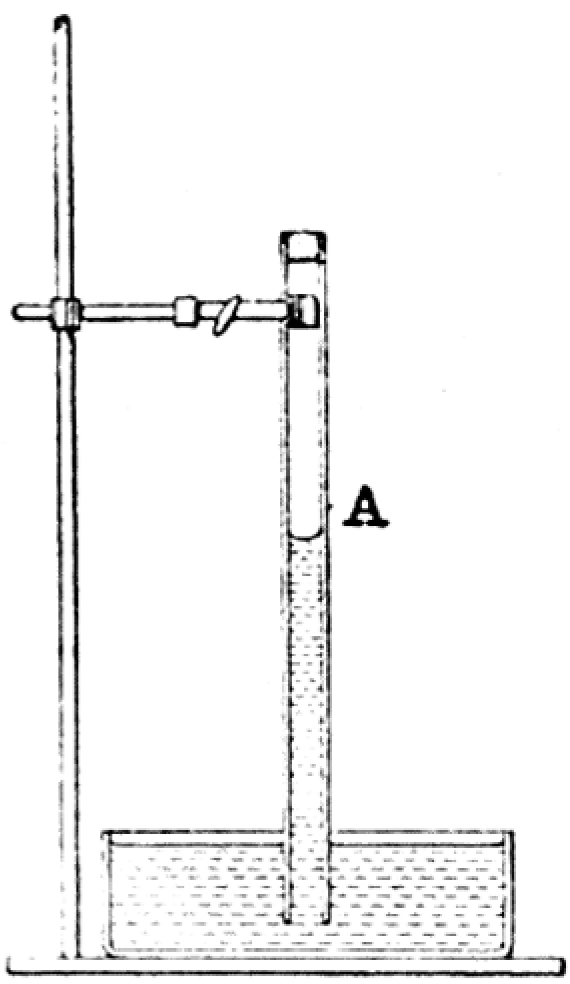


FIG. 8.—Diffusion of Gases.

EXPERIMENT.—The glass tube A (Fig. 8) is closed at its upper end with a plug of plaster of Paris. Fill it with hydrogen and invert it with its open end under the surface of water. Hydrogen is lighter than air and therefore diffuses more rapidly; owing to this the gas escapes through the porous plug more rapidly than air can enter and water rises in the tube. If the tube contains air and is surrounded by an atmosphere of hydrogen the latter gas diffuses inwards and increases the pressure.

The time required for a given volume of a gas to diffuse through the plug, *i.e.* the rate of diffusion, varies inversely as the square root of the density of the gas.[1] When a mixture of two gases is passed through a porous tube, such as the stem of a clay pipe, the lighter of the two diffuses most rapidly through the walls, hence the mixture

[1] This is Graham's law.

which escapes at the exit is richer in the heavier gas. By this means the mixture may, to a great extent, be separated into its two constituents.

Viscosity.—When the hand is placed in a large vessel of water and is moved slowly through the liquid it experiences little resistance, but if it is moved rapidly it is opposed by a considerable force. The different layers of liquid are caused to slide over each other and this motion is opposed by frictional forces between adjacent layers. These forces arise from the **viscosity** or internal friction of the water. The case is very similar to that of a block of wood being dragged across a floor, except that the friction arises between adjacent layers of the same substance. Thus the layer of water on the bed of a river is at rest and the upper layers slide over it, the velocity gradually increasing as the surface is approached.

Osmosis.—The process of diffusion is found to be greatly modified if the two liquids are separated by certain membranes. This is illustrated by the following experiment.

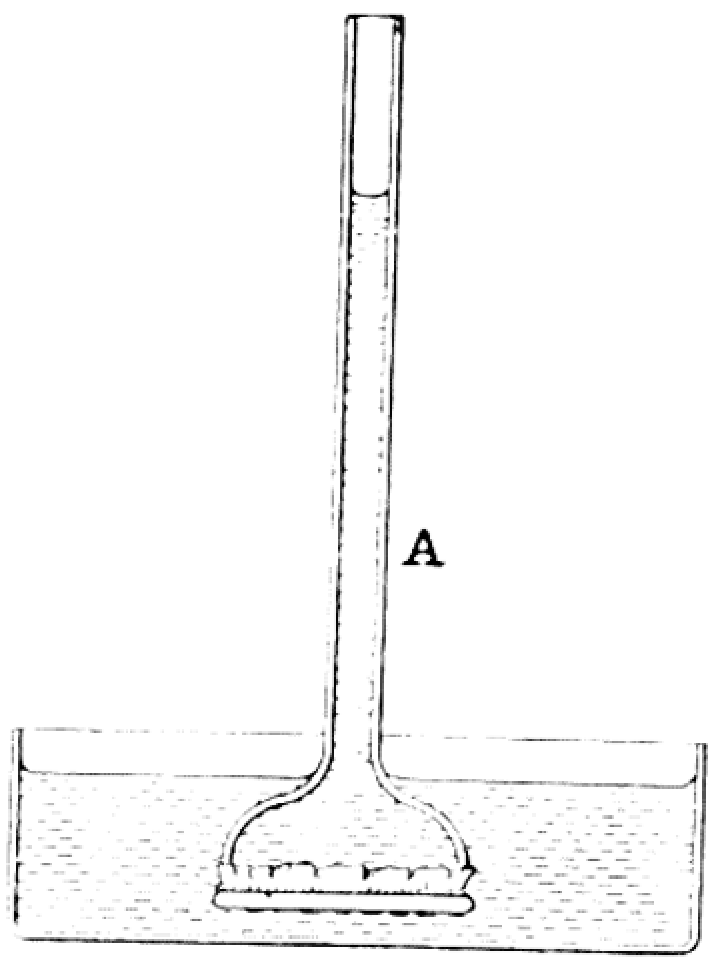


FIG. 9.—Apparatus to show Osmosis.

EXPERIMENT.—The tube A (Fig. 9) is closed at its wide lower end with a piece of parchment and a strong solution of sugar is poured into it. It is then immersed in water, as shown in the figure, with the liquids at the same level inside and outside. After a short time the liquid is found to stand the higher inside the tube.

At first sight this appears to be analogous to the diffusion illustrated in Fig. 8, but there is this important difference, no sugar passes from the interior to the exterior. The membrane allows water to pass through it freely but opposes the passage of the sugar molecules. Membranes which behave in this manner are called semi-permeable. There is doubt as to the mechanism, but we may perhaps picture the process as follows: The water molecules hit the membrane on each side and pass through; but a less number hit the inner side, since some of the molecules of the solution are sugar. Hence, on the whole, water passes to the interior. If the

solution in A is subjected to pressure by means of a piston, the inward diffusion of the water can be stopped. The pressure necessary for this is called the **osmotic pressure** of the solution, and the whole process is called **osmosis.**

EXAMPLES ON CHAPTER I

1. Give an account of the phenomena of diffusion in liquids and gases and describe some experiments to illustrate them. How is diffusion explained theoretically? (L. '08.)

2. How would you show that different substances diffuse at different rates through the same liquid? How would you account for the difference and for the fact that the rate of diffusion depends on the temperature? (L. '10.)

3. Taking 10 lbs. as the unit of mass, one minute as the unit of time, and a yard as the unit of length, compare the unit of force with that belonging to the ft.-lb.-sec. system. (L. '80.)

4. An engine of 1 horse-power is capable of doing 33,000 ft.-lbs. of work per minute. What is the H.P. of an engine that can pump 1000 gallons of water per minute from a well and project it with a velocity of 80 ft. per second through a nozzle which is 40 ft. above the surface of water in the well? (L. '82.)

5. Describe a good form of standard barometer. What is the effect of surface tension or "capillarity" upon the height of the mercury in the barometer tube? On what does the magnitude of this effect depend, and how may a barometer reading be corrected for this error? (L. '83.)

6. Describe how to measure the relation between the pressure and volume of a mass of gas at constant temperature. If 310 c.c. of a gas at a pressure of 230 mm. of mercury are subjected to a pressure of 760 mm. what will be the resulting volume? (L. '88.)

7. A narrow glass tube is closed at one end and contains air which is shut off from the atmosphere by a long thread of mercury. Show how to obtain the height of the barometer from observations of the lengths of the air and mercury columns when the tube is (1) horizontal, and (2) vertical.

8. Some air is in the space above the mercury in a barometer of which the tube is uniform. When the mercury stands at 29 in. in the tube the space above the mercury is 4 in. long. The tube is then pushed down into the cistern so that the space above the mercury is only 2 in. long, and now the mercury stands at 28 in. At what height would it stand in a perfect barometer? (L. '94.)

9. State the laws of diffusion of gases through a plug of porous material. A mixture of hydrogen and oxygen in equal proportions is contained (1) In a vessel in which there is a porous plug, (2) In a vessel in which there is a hole, say, 1 mm. in diameter; the mixture is allowed to escape into a vacuum. How will the proportion of hydrogen and oxygen be affected, if at all, when the escape has been going on for a short time? (L. '96.)

10. Describe an experiment showing how the density of gases affects their rate of diffusion through a porous septum. Do changes of pressure and temperature affect the quantity of gas that disappears in a given time? If so in what ways do they affect it? (L. '98.)

11. If a lump of sugar be held just below the surface of tea in a cup it dissolves much more rapidly than it does if it is allowed to drop to the bottom of the cup, but not so fast as if it is well stirred. How do you account for this? (L. '02.)

12. In measuring the surface tension of a liquid, as on p. 12, it is usual to measure h to the lowest part of the concave surface of the liquid in the tube; hence a correction is necessary for the liquid raised above this point. Show that the correct formula for T is then $T = \frac{1}{2}(h + \frac{1}{3}R)Rdg$.

CHAPTER II

THERMOMETRY

We are all familiar with the sensations of hotness and coldness; the physical agent which produces these sensations is called *heat.* If one body is hotter than another, as indicated by our sensations, we say that the **temperature** of the one is higher than that of the other. Temperature is defined as **a number denoting the hotness of a body measured according to some arbitrarily chosen scale.** It is easy to show that our sensations do not enable us to compare temperatures accurately.

Experiment.—Take three bowls of water, the first cold, the second tepid, the third hot. Place the left hand in No. 1 and the right hand in No. 3; after half a minute transfer both hands to No. 2. It appears hot to the left hand but cold to the right. Our sensations then are greatly influenced by contrast, and other means must be used to measure temperature.

An instrument used for measuring temperatures is called a thermometer. An important part of the study of heat is concerned with temperature measurement or thermometry. When the temperature of a body changes it is assumed that it has lost or gained heat. Thus if a block of hot copper is placed in a beaker of cold water the copper becomes cooler and the water hotter; these changes are ascribed to a transfer of heat from the hot to the cold body, and generally, as later experiments will show, heat tends to flow from places where the temperature is higher to those where it is lower.

A system of thermometry in order to be scientifically useful must enable two observers at different places to measure temperatures that shall be comparable with one another. Thus if observer A finds that a substance melts at a temperature of 50°, as recorded by his thermometer, observer B should know what temperature this corresponds to on his thermometer, although the two instruments

may not have been compared directly, and may be altogether different in construction. To construct such a temperature scale there must be some zero of temperature that can be easily reproduced, and also some standard temperature difference in terms of which any other temperature interval can be expressed. Before showing how this is arranged let us examine first some of the effects produced by a temperature change; we shall then be in a position to apply one or more of them to temperature measurement.

General Effects produced by Heat.—When bodies are heated they generally increase in length, area, and volume. The well-known experiment with Gravesande's ring illustrates this. A metal ball is made of such a size that it just passes through a round hole in a sheet of metal when both are at the same temperature; if the ball is heated it can no longer pass through, showing that it has expanded.

Liquids and gases also expand when their temperature is raised, but the observations are here complicated by the expansion of the vessel in which they are contained. This is shown in the following experiment.

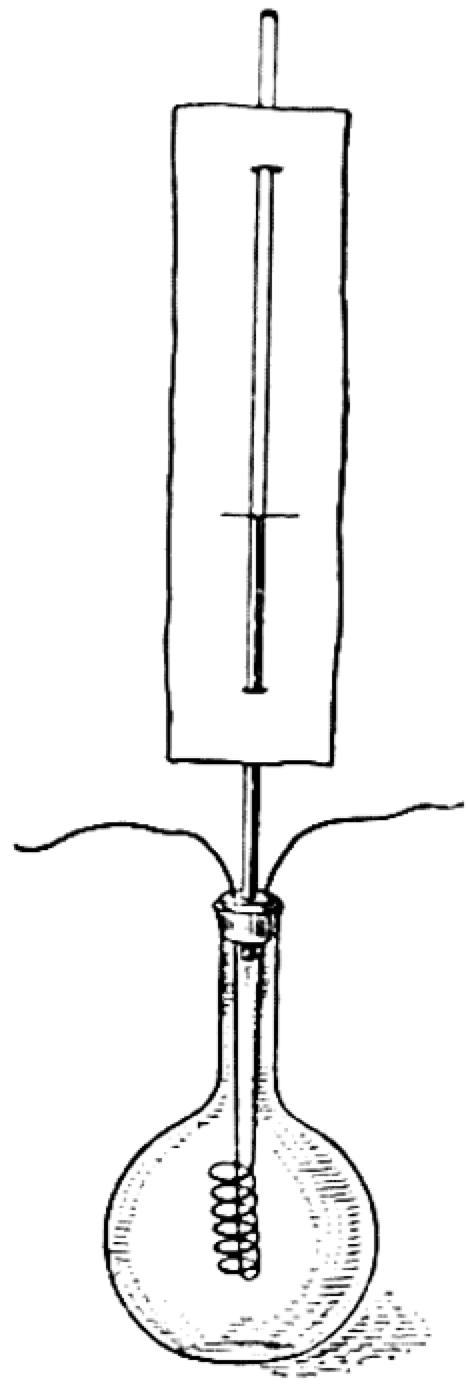


FIG. 10.—Apparatus to show Expansion of a Liquid and of the Flask containing it.

EXPERIMENT.—Fill a small glass flask with cold water which has been coloured and pass through the cork a narrow bore glass tube about 30 cms. in length; press in the cork until the liquid rises halfway up the tube and mark the position of the surface (Fig. 10). If the flask is plunged into warm water it will be noticed that the liquid in the tube falls momentarily, then comes to rest, and finally rises above the mark. The reason for this is apparent. As the flask is heated from the outside the glass expands first, and if this alone took place the index would fall, but after a short time heat reaches the liquid also through the glass walls and its expansion more than counterbalances that of the flask. If this is the correct explanation we ought to get rid of the initial fall by raising the temperature of the liquid before allowing heat to reach the flask. The expansion of the latter will then be masked at every stage by the larger expansion of the water. This can be done by immersing a coil of iron wire in the water, as shown in the figure, and passing an electric current through it. As will be seen later, this heats the wire, and the liquid consequently expands before the flask. No fall of the index is then observed.

EXPERIMENT.—Fill the flask of the last experiment with air or other gas,

but leave a drop of liquid in the tube to act as an index. A very slight warming now produces a big expansion, showing that gases are more expansible than liquids.

When a gas is heated while its volume is kept constant, it is found that its pressure increases. This can be demonstrated conveniently by the apparatus shown in Fig. 34, p. 65. The bulb A, which contains air, communicates by rubber tubing with the mercury reservoir B. When the air is heated it tends to expand, but its volume can be kept constant by raising the reservoir until the mercury in the left-hand limb returns to its original position. The mercury in B then stands higher than in A, and the difference in levels shows by how much the pressure of the gas exceeds the atmospheric pressure.

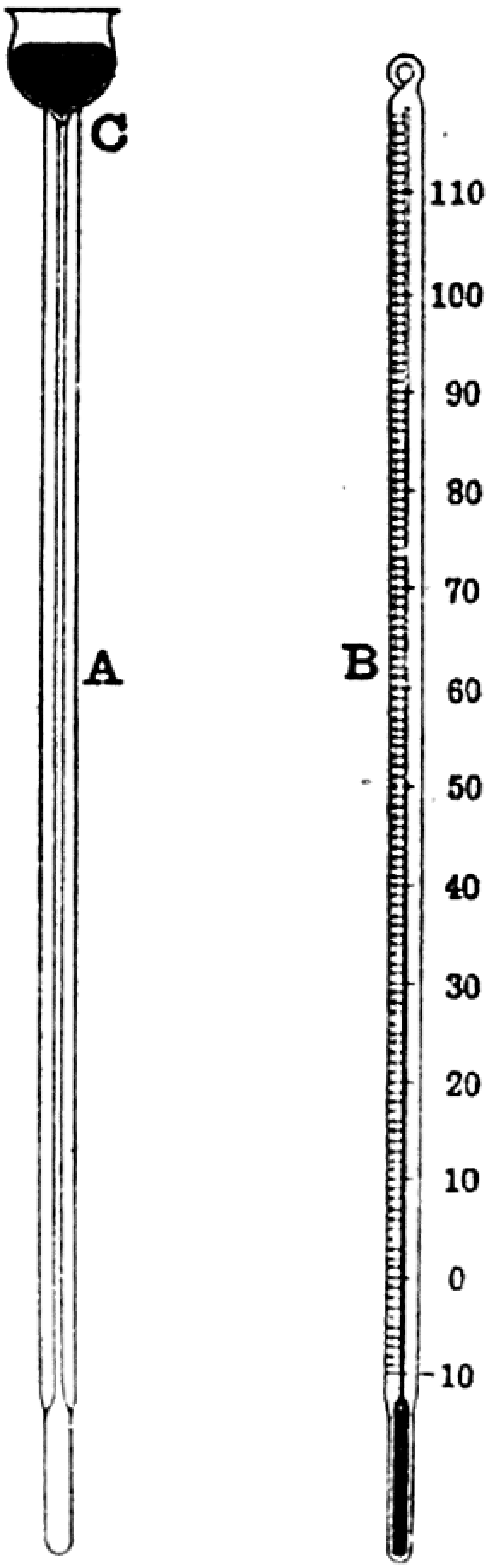


FIG. 11.—Mercury Thermometer.

The electrical properties of bodies are also altered by heat, but the discussion of these is deferred for the present.

Any of the above changes produced by heat might be used as a means of measuring temperature; at present we will consider the expansion of a liquid contained in a glass bulb. The liquid most generally chosen is mercury.

Mercury Thermometers.—Our object, in the first place, is to construct an apparatus which will enable us to observe readily the expansion of mercury. The usual form of the thermometer is shown in Fig. 11, B. It may be constructed as follows: Into a carefully cleaned piece of capillary glass tube is introduced a column of mercury about 2 cms. long, and the length of this is carefully measured by means of a microscope at different parts of the tube. If the bore is uniform this length will be constant. Having by this means selected a suitable piece of tubing, bulbs of convenient size are blown on it as shown in Fig. 11, A. The cylindrical bulb at the lower

end has the great advantage over a spherical one that it can be readily passed through corks. Dry, clean mercury is placed in the upper reservoir, and the bulb is slightly heated so as to expel some air. It is then allowed to cool ; this reduces the pressure inside and mercury runs into the bulb. This procedure is necessary because, owing to the effects of surface tension, the mercury will not flow down the stem except under pressure. By alternate heating and cooling the whole is filled with mercury and this is finally boiled to expel the last traces of air and moisture. It is now heated to a temperature rather above the highest it is to be used to measure, and the glass at C is sealed off in a small blowpipe flame. By this process it is ensured that no air is present above the mercury, so that it can expand freely, and the stem being closed the mercury surface will not become fouled with dirt or moisture. If we now mark the positions of the end of the mercury thread which correspond to two fixed temperatures that can easily be reproduced, and divide the distance between them into a number of equal divisions, the instrument could be used as a means of measuring temperature. The interval between the two fixed temperatures is called the fundamental interval. Let us divide this into 100 equal divisions and number them, starting below, from 0 to 100. If when the thermometer is put into water the mercury stands at the 15th division, it could be said that the temperature is 15 degrees, meaning that the difference between the lower fixed temperature and the temperature of the water is 15/100 of the fundamental interval. In this way temperatures read by different thermometers would be directly comparable, provided, of course, that the same fixed points were used. We now proceed to show how the fixed points are chosen.

Determination of the Fixed Points.—Thermometers filled as above are usually left for some weeks in order that certain irregularities may disappear which are caused by the heating necessary to blow the bulb. If one of these thermometers is placed in pure melting ice on successive days it is found that the mercury stands at the same point on each occasion. Melting ice therefore provides us with a standard temperature which can readily be reproduced. In a similar manner, if the thermometer is closely surrounded by steam coming from boiling water it is found that the mercury always stands at another fixed point on the stem, provided that the height of the barometer is the same in each experiment. Accordingly the temperature of melting ice, and of the steam coming from water

boiling under a normal atmospheric pressure of 760 mm. of mercury are taken as the fixed points. If this fundamental interval is divided into 100 equal divisions, starting from zero at the ice point, the thermometer is called a Centigrade thermometer. This is the one usually used for scientific purposes. When the mercury expands from one division to the next we say a rise of temperature of one degree Centigrade (1° C.) has taken place. Thermometers made in this manner will evidently record temperatures that are comparable with each other, provided the expansion of the glass is regular.

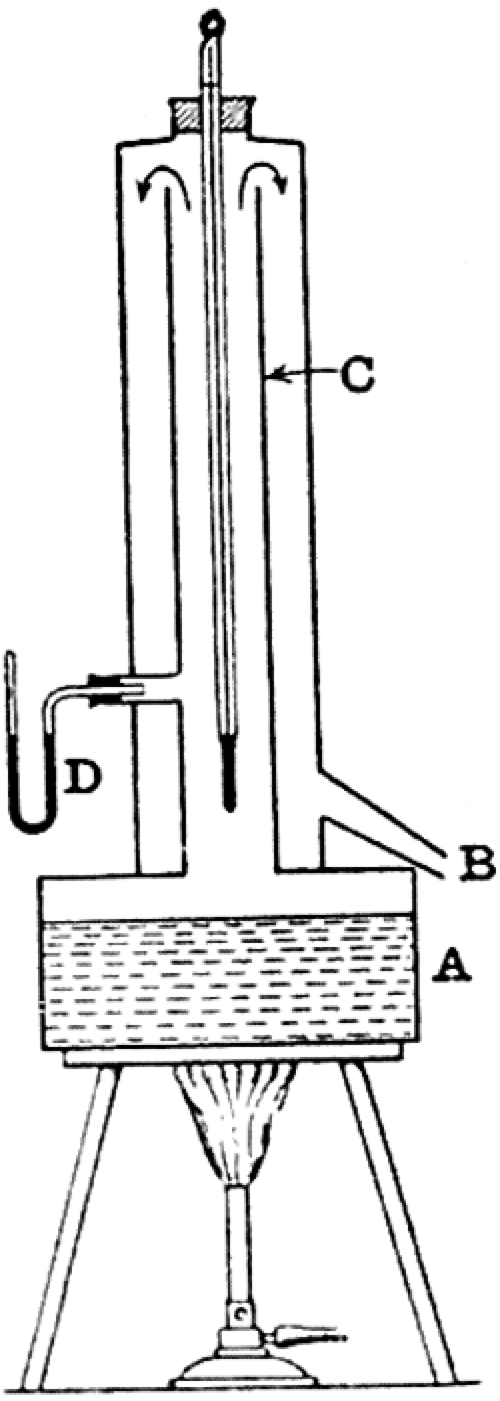


FIG. 12.—Apparatus for finding the Steam Point.

Until recent years the ice point was first determined; for reasons given later the reverse order is now commonly adopted.

EXPERIMENT.—Take two large distillation flasks and half fill one with distilled water, the other with a solution of salt in water. Make each liquid boil and place a thermometer in the neck of the first so that the bulb is 5 cms. above the water. Note the temperature, then transfer the thermometer to a similar position in the other flask; it will be found to show practically the same temperature in each case, and this coincidence will be still more exact if the flasks are replaced by the vessel shown in Fig. 12. Now put the thermometer bulb right in the liquids, the temperature is probably higher in each case than it was before, and the temperature of the salt solution may be considerably higher.

For this reason thermometers are immersed in the *steam* of boiling water when the upper fixed point is determined, since the temperature is then independent of any impurities dissolved in the liquid. If a thermometer is placed in ice, and salt is then added, the temperature falls considerably; pure ice must therefore be used for determining the lower fixed point.

A simple apparatus for determining the steam point is shown in Fig. 12.[1] Water is boiled in the copper vessel A, and steam, after circulating as shown by the arrows, escapes by the vent B, which should be wide, otherwise the rapid production of steam may create

[1] See also Barton and Black, "Practical Physics," p. 51.

a pressure in excess of that outside; the water gauge D shows whether this is the case or not. The thermometer is placed in the inner tube C with the exposed end of the mercury thread just above the cork. As this tube is surrounded by steam the whole arrangement evidently secures that its walls are at the same temperature as the steam and the thermometer, and there is no tendency for heat to pass from the bulb to the outside by radiation (p. 116). Let us assume that the barometer stands at 760 mm. After a quarter of an hour the temperature becomes steady and the position of the mercury thread is marked on the stem. The lower fixed point is now determined at once. The thermometer is placed in shavings of melting ice[1] contained in a glass funnel (Fig. 13). The mercury at first falls rapidly, then more slowly, and may finally rise by a small amount. The lowest position it reaches is marked. The interval between the two marks is then divided into 100 equal parts, and the divisions may be extended above and below to enable us to read temperatures above 100° C. or below 0° C. A small bulb is usually made at the top of the stem to diminish the risk of breakage, if, by accident, the apparatus is heated too strongly.

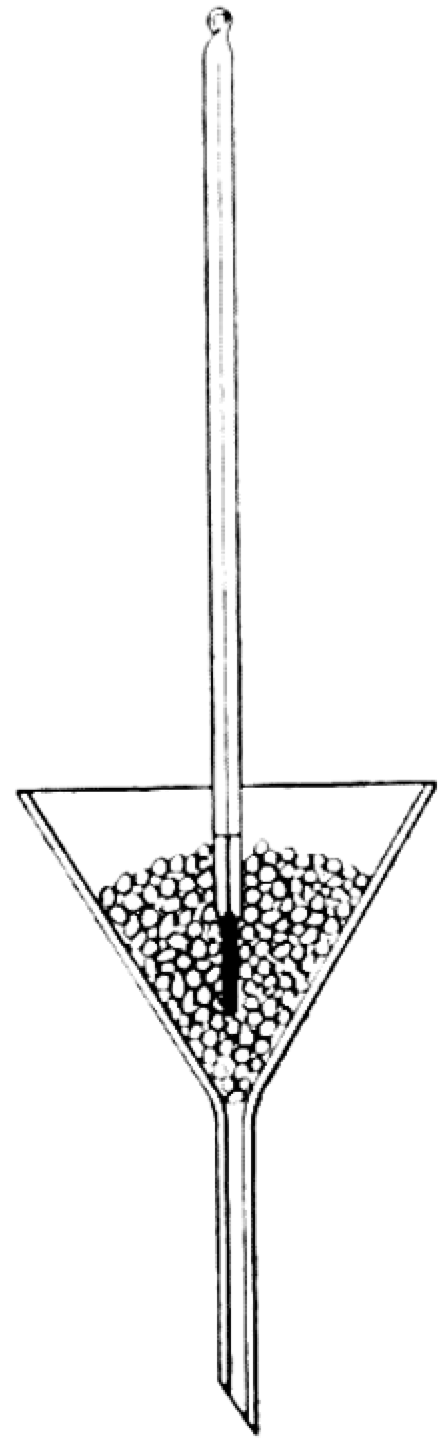


FIG. 13.—Apparatus for finding the Ice Point.

As mercury freezes at — 40° C. a mercury thermometer cannot be used below about — 30°; for lower temperatures other liquids, such as alcohol or toluene, must be employed. These thermometers should be compared with some form of standard thermometer. Mercury boils at 350°, but before this temperature is reached considerable evaporation takes place which reduces the amount of liquid in the stem; there is also a tendency for bubbles of mercury vapour to form and break the thread. For these reasons mercury thermometers cannot be used to measure temperatures accurately much above 230° C. Some thermometers, intended for high temperatures, contain an inert gas like nitrogen in the upper part of the stem.

[1] A convenient machine for breaking up ice into fine shavings is made by Messrs. Avery.

This, by its pressure, raises the boiling point (Chap. VIII.) and hinders the breaking of the column by bubbles of vapour. The bulb must be made strong to withstand this pressure.

In the determination of the upper fixed point it was assumed that the water was boiling under a pressure of 760 mm. of mercury; if this is not the case the temperature of the steam will not be 100° C., and a correction must be made. Experiments showing how the boiling point varies with the pressure are described in Chap. VIII. Let us suppose the pressure is 750 mm.; from tables giving the boiling point it is found that under this pressure water boils at 99·63° C. The distance between the marked points, supposing the ice point accurately determined, corresponds therefore to 99·63°. We may calculate by simple proportion where the mark corresponding to 100° should be placed and make it accordingly.

Errors in Mercury Thermometers.—Even when a thermometer has been constructed as described above, certain corrections have to be applied when it is used for accurate work.

Zero Correction.—EXPERIMENT.—Test the zero of a common thermometer, that has not been used for some weeks, by means of the apparatus shown in Fig. 13. The mercury will usually stand at a point above the zero of the scale. If it is kept for some hours at a temperature considerably below 0° the error will be increased. This is due to a slow contraction in the volume of the bulb, usually called the secular change, which may take years to complete.

EXPERIMENT.—Keep the thermometer at 100° for 30 mins. and again determine the freezing point. The mercury stands lower than it previously did owing to a temporary increase in the volume of the bulb. Experiment shows that the zero determined in the latter case is fairly constant on different days; it is for this reason that the lower fixed point is found immediately after the steam point. These zero changes are due to the fact that, after being heated, glass takes a long time, extending in some cases to years, to regain its initial volume. They are largely reduced by the use of special kinds of glass. It has recently been found that they are entirely absent if the envelope is made of fused silica. Evidently if the zero has risen 0·1° every temperature read on the thermometer will be too high by this amount.

Other errors which have to be considered are due to (1) change in the size of the degree owing to the distance between the two fixed points becoming slightly different from 100 divs.; (2) Irregularities in the bore of the tube; (3) Changes in the volume of the bulb caused by pressure, either internal or external; (4) The mercury in the stem being at a different temperature from that in the bulb; this is called the "exposed stem" correction.

Nos. (2) and (3) need a long and laborious series of corrections

too complicated for the present book. To test No. 4 the following experiment may be performed :—

Experiment.—Place a thermometer in the boiling point apparatus (Fig. 12), leaving the stem from 50° to 100° exposed above the cork ; note the temperature when it becomes steady. Now push the thermometer through the cork until merely the top of the mercury is visible. The reading will be slightly greater because the mercury between the 50° and 100° divisions has become hotter and has expanded.

Let t_1 be the temperature of a bath as read on a thermometer when n divisions of the thread are exposed above the surface ; let t_2 be the mean temperature of the exposed column, and σ the apparent coefficient of expansion of mercury in glass (p. 48). Then the true temperature of the bath $t = t_1 + n\sigma(t_1 - t_2)$. For a proof of this formula see p. 57. This correction is uncertain since t_2 is not known accurately; it should be made small by immersing the thermometer as far as possible; σ may be taken to be 0·00015. If $t_2 = 20°$ and $t_1 = 99·4°$ in the above experiment, the true temperature is 100°.

Experiment.—To test the trustworthiness of the correction immerse the thermometer in the last experiment to different depths, read t_2 by another thermometer placed near the middle point of the exposed stem, and calculate t. Compare the results with the reading obtained when all the stem is immersed.

Nearly all the corrections given above can be found by direct comparison with a standard thermometer. The two instruments are placed in the same bath and their corresponding readings observed at different temperatures. A table is now drawn up showing the amount that must be added to or subtracted from the reading of the incorrect thermometer to make its readings agree with those of the standard. From these a curve can be plotted as in Fig. 14, which enables us to determine the correction for any reading. The reading of the incorrect thermometer is shown on the horizontal line, and the amount to be added or subtracted to get the true temperature is indicated by the vertical distance of the curve from the axis of temperature. Thus if the reading is 80° the curve shows that 0·18° must be added.

Thermometers for Special Purposes.—*Maximum and Minimum Thermometers.* For some purposes thermometers are required which will show the highest or lowest temperature to which they have been subjected during a certain time. They are called maximum or

minimum thermometers and may take various forms. In Fig. 15, *a* is a maximum and *b* a minimum thermometer.

The bulb of *a* is filled with mercury, Q is a dumb-bell shaped piece of coloured glass. We have seen, p. 10, that the surface of a liquid offers a resistance to rupture; if the mercury expands it therefore pushes Q along. When cooling takes place the index is left in position and shows the maximum temperature reached. The liquid in *b* is coloured alcohol, and the glass index P, in a manner similar to Q above, is pulled to the right when the temperature falls. If the temperature rises afterwards P is left in position showing the minimum temperature experienced by the thermometer. The

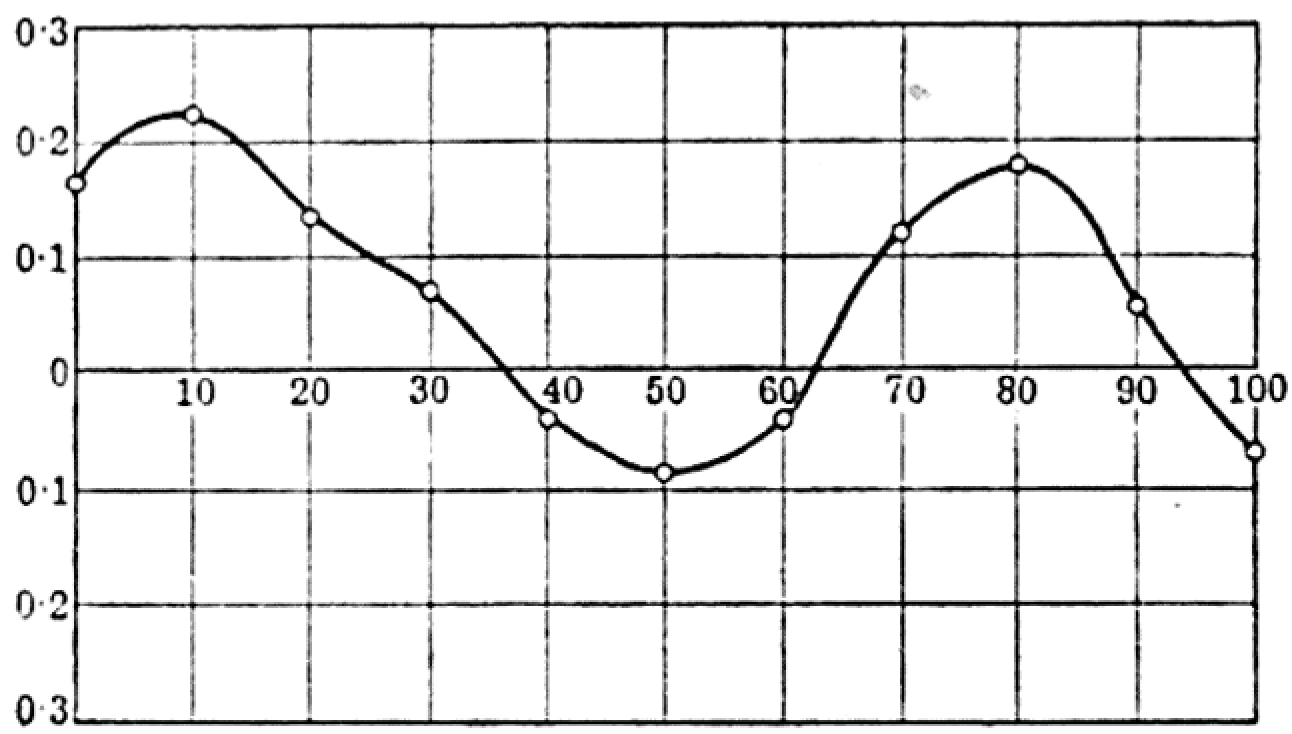


FIG. 14.—Correction Curve for a Thermometer.

instruments are set ready for use by shaking the index into contact with the surface of the liquid.

Fig. 15, *c*, shows a clinical thermometer used by doctors. At A the bore is constricted, the mercury expands past this, but when cooling takes place the liquid column breaks at the constriction and the further end of the thread shows the maximum temperature.

Six's maximum and minimum thermometer, largely used by gardeners, is shown in Fig. 15, *d*. The bulbs, A and B, containing alcohol freed from air, are separated by a column of mercury E. Two dumb-bell shaped iron indexes, D and C, are pressed lightly against the glass by weak springs. If the temperature rises the liquid in A expands and D is pushed upwards; a fall in temperature similarly causes an upward movement of C. The springs hold each index in its extreme position when the mercury retreats. A magnet

may be used to bring the indexes into contact with the mercury surface when the thermometer is set for use.

Until the invention of electrical thermometers the differential

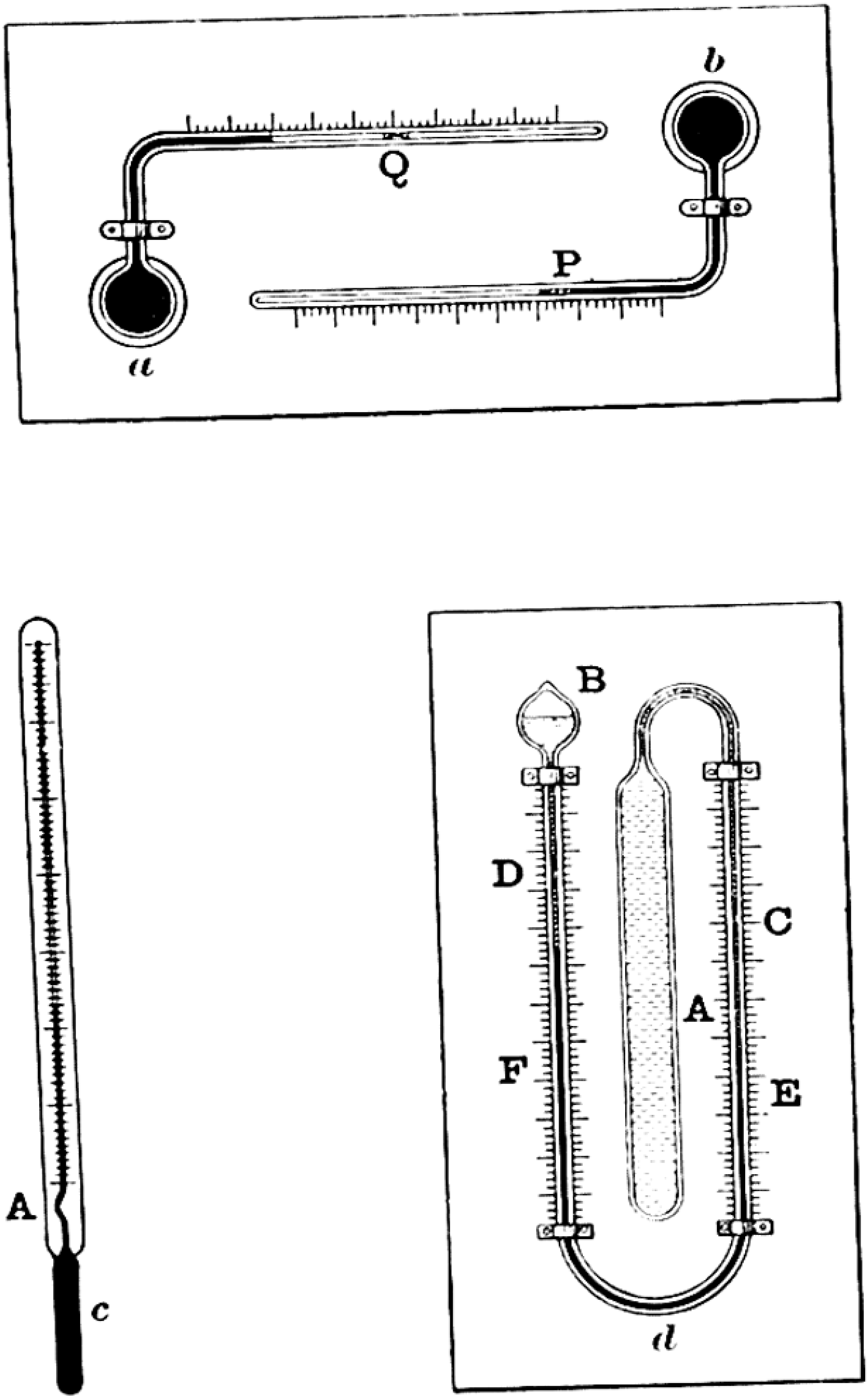


FIG. 15.—Maximum and Minimum Thermometers.

air thermometer (Fig. 16) was very largely used for measuring temperature differences, especially in radiation experiments. The two glass bulbs contain air, C is a small index of coloured liquid.

The bulbs are first put in communication with each other by the tap B, which is then closed; if now one bulb be slightly heated the air in it expands and the index moves. Other thermometers will be described later when the changes produced by heat have been further studied.

Other Thermometric Scales.—The division of the fundamental interval into 100 degrees is not the only system used. In the Fahrenheit thermometer the melting point of ice is called 32°, and the boiling point of water 212°, so that the fundamental interval is divided into 180°. This is the thermometer in common use in England for non-scientific purposes; it is also frequently used by engineers and metallurgists.

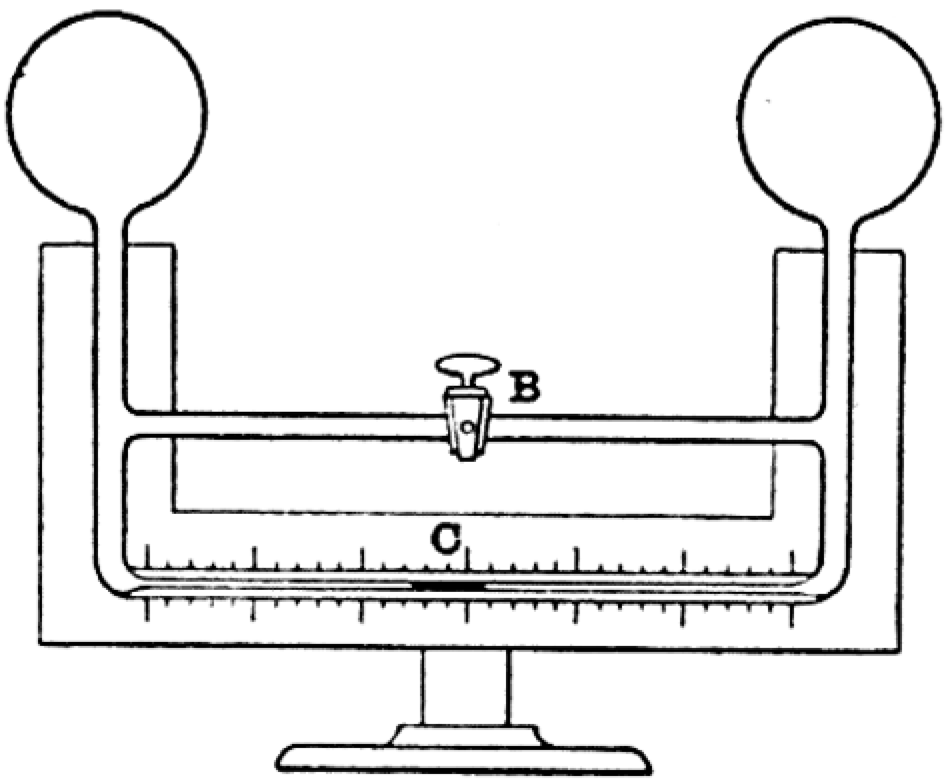


FIG. 16.—Differential Air Thermometer.

On the Réaumur scale, which is in use on the Continent, the fixed points are marked 0° and 80°, the fundamental interval being divided into 80°.

Suppose we require to convert from one scale to another. Let the temperature of the same bath, as read by Centigrade, Fahrenheit and Réaumur thermometers be C, F, and R respectively. The distance of the end of the mercury thread from the lower fixed point, measured in degrees, is C, (F − 32), and R; this distance must evidently be the same fraction of the fundamental interval in each case; hence

$$\frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

or

$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4}$$

EXAMPLE.—Convert 80° F. into degrees Cent.
In the above equation put F = 80, then

$$\frac{C}{5} = \frac{80-32}{9}$$

whence

$$C = 26{\cdot}6°$$

EXAMPLES ON CHAPTER II

1. What is meant by a scale of temperature, and on what does the definition of any particular scale depend? (L. '07.)

2. Convert the following temperatures from the Centigrade to the Fahrenheit scale: 80°, − 49°, − 273, 1000°. Also find at what temperature the two scales agree.

3. The temperature of a living room is 66° F., that of the blood is 98° F., and the temperature on a hot summer's day is 88° F. Find the corresponding readings on the Centigrade scale.

4. A thermometer which has been tested in the usual way is sunk to its 20° mark in a liquid and reads 90°. The mean temperature of the rest of the stem is 25°. Find the true temperature of the liquid, the coefficient of expansion of mercury in glass being 0·00015. (L. '08.)

CHAPTER III

CALORIMETRY AND SPECIFIC HEAT

Heat as a Quantity.—Up to the present we have considered various effects caused by changes of temperature without inquiring whether it is possible to measure the quantities of heat involved. Let us now consider this point. When 100 gms. of water at a temperature of 40° are mixed with an equal quantity at 20°, the temperature of the mixture is very approximately 30°. The hot water has lost heat and the cold water has gained it. When 300 gms. of hot water, whose temperature is 40°, are poured into 100 gms. at a temperature 20°, the resulting temperature is 35°. The cold water has gained more heat than in the first experiment; we are thus led to the idea of different amounts of heat and therefore of heat being a measurable physical quantity. The first point to be settled is what shall be taken as the heat unit. Any physical change that heat produces may be used to define this; it is merely a matter of convenience in measurement that influences our choice. Thus the heat required to melt one gram of ice might be taken as the unit; we should then be justified in assuming that it takes two units to melt two grams and m units to melt m grams. It is, however, found more convenient to define the unit quantity of heat as that required to raise the temperature of 1 gm. of some standard substance, such as water, through 1°. To raise 10 gms. through 1° will then require 10 units, but we are not justified in assuming that 50 units must be supplied to heat 1 gm. through 50°, for the heat necessary to raise the temperature from 10° to 11° might differ from that required to heat the same mass from, say, 40° to 41°. It must therefore be specified at which part of the temperature scale the 1° interval is to be taken. Although there is no general agreement on this point, that most usually chosen is from 15° to 16° C. The **unit of heat** is then defined as **the quantity of heat necessary to raise the temperature of 1 gm. of water from 15° to 16°.**

This unit is named the **calorie or therm.** When a gram of water cools from 16° to 15° it gives out, or loses, one calorie. It has been stated above that the heat lost by 100 gms. of water in cooling from 40° to 30° is just capable of raising the temperature of an equal mass from 20° to 30°. This, if strictly true, would prove that the average quantity of heat required to change the temperature of a gram of water by 1° is the same between 20°–30° as between 30°–40°. More accurate experiments, however, show that this is not quite true, but as the difference is very small, even when other temperatures are taken, it will be assumed in the following pages that the addition of one calorie will change the temperature of one gram of water by 1°, no matter what is its initial temperature, so long as it is between 0° and 100°. To raise the temperature of m gms. through 1° will then require m calories, and to heat the same mass from t_1° to t_2° $m(t_2 - t_1)$ calories must be supplied. This also is the number of units of heat given out by m gms. in cooling through the same temperature range. The measurement of quantities of heat is called **calorimetry,** and the vessels in which the measurements are carried out are called calorimeters.

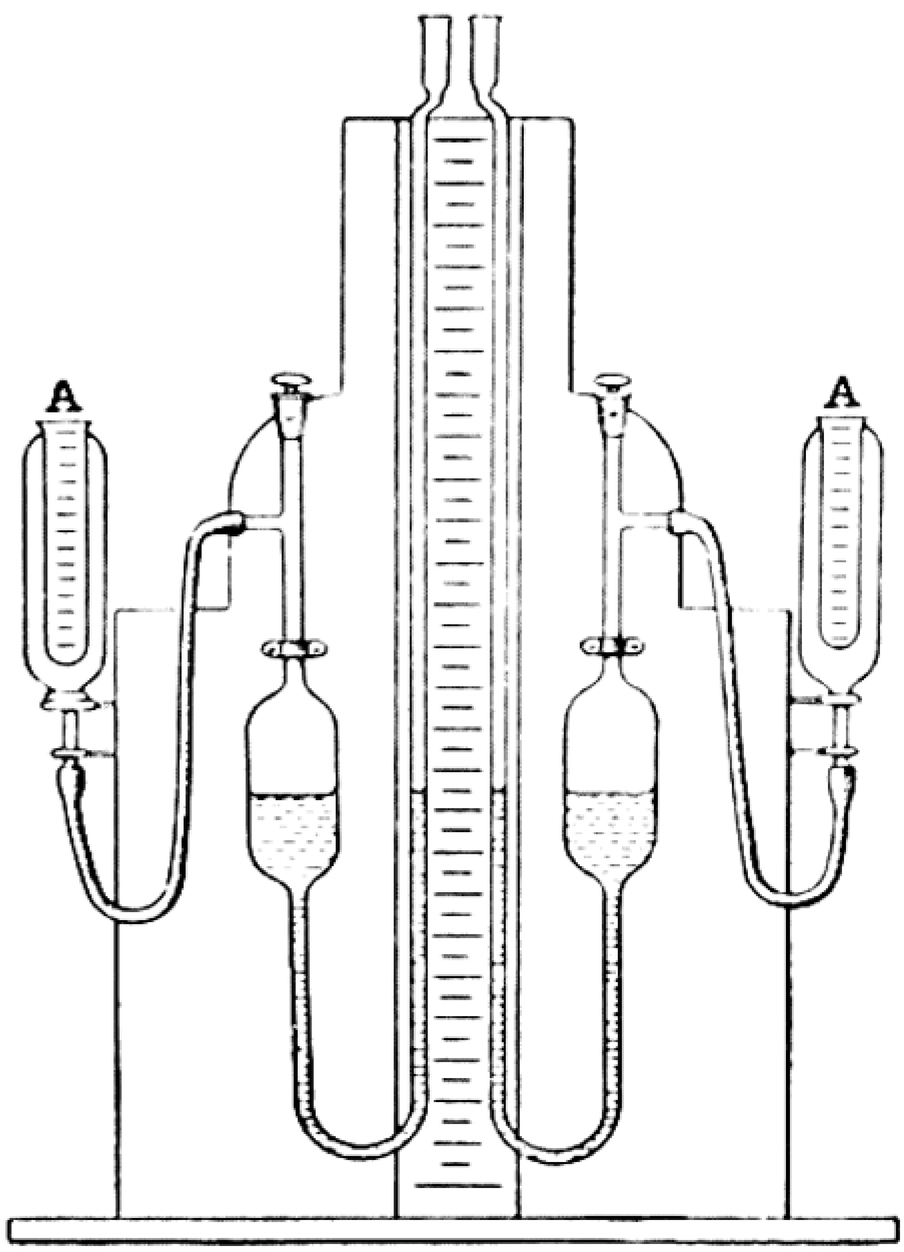


FIG. 17.—Looser's Thermoscope.

Thermal Capacity and Specific Heat.—The number of calories required to change the temperature of a body by 1° is called its **thermal capacity.** It varies with the mass of the body and depends also on the nature of the substance. A lecture experiment shows this: as we shall find the apparatus to be used very convenient at a later stage a full description of it is given here. Fig. 17 illustrates a Looser's thermoscope; it consists of two separate air thermometers, but into the bulb of each there is fused a graduated test-tube

(A in Fig.). Each bulb communicates through rubber tubing with one of two U-tubes containing coloured water for an index. These tubes are open at their further ends to the external air. By means of taps either bulb may be put directly in communication with the atmosphere when necessary. When a bulb becomes hot the air it contains expands, and the temperature change is proportional to the movement of the corresponding liquid index in the narrow limb of the U-tube, as in the mercury thermometer (Chap. II.).

EXPERIMENT.—Half fill each test-tube with cold water. Suspend equal masses of copper and lead in a beaker of boiling water; when they have taken up the temperature of the bath transfer them quickly one into each test-tube so that they are completely immersed. The hot bodies lose their heat to the water, which in turn heats the thermometer bulbs. It will be found that the rise in temperature is roughly three times greater in the bulb containing the copper than in the other, showing that when equal masses of these metals cool through approximately the same range of temperature, copper emits three times as much heat as lead.

EXPERIMENT.—Vary the experiment by putting equal masses of turpentine and water in the test-tubes and then drop into them equal masses of copper at 100°. The bulb containing turpentine rises in temperature about twice as much as the other, showing that this liquid requires less heat to raise its temperature than water does, or its thermal capacity is less, taking equal masses.

The number of calories required to change the temperature of one gram of a substance by 1° is called the specific heat of the substance at the given temperature. It follows from this definition that the specific heat of water is unity at 15°, since one calorie is necessary to change the temperature of 1 gm. by 1°; according to what has been said on p. 31, we shall assume that it is unity at all temperatures between 0° and 100°. The first of the two experiments given above shows that the specific heat of copper is greater than that of lead, and the second that turpentine has a less specific heat than water. From the definition, if the specific heat of a substance is s (assumed constant at all temperatures), we have that

To heat	1 gm. of it	through 1°		requires	s cals.
,,	m gms.	,,	1°	,,	ms cals.
,,	m ,,	,,	from $t_1°$ to $t_2°$	,,	$ms(t_2 - t_1)$ cals.

This expression is a fundamental one in calorimetry. When put into words it tells us that :—When the temperature of a body changes, the number of calories absorbed or emitted is obtained by multiplying the mass of the body by the specific heat and by the

temperature change. From the definition given above it follows that the thermal capacity of a body is $m \times s \times 1 = ms$.

Measurement of Specific Heat.—The following example will best illustrate how the specific heat of a solid can be found by what is called the method of mixture.

Example.—A block of copper weighing 93·5 gms. was heated in boiling water to 100°. It was then dropped into a calorimeter containing 200 gms. of water at 16·4°. The temperature of the mixture was 20°; find the specific heat, s, of the copper.

We have to express that all the heat given out by the copper goes into the water in the calorimeter.

The heat absorbed by the water $= 200(20 - 16{\cdot}4) = 720$ cals.

Heat lost by the copper $= 93{\cdot}5(100 - 20)s = 93{\cdot}5 \times 80 \times s$ cals.

and these quantities are equal.

$$\therefore 93{\cdot}5 \times 80 \times s = 720$$

and

$$s = 0{\cdot}096$$

There are several sources of error in this experiment which must be eliminated in accurate work: (1) The metal cools while it is being transferred from the hot to the cold water; it also carries with it some of the hot liquid so that all the heat given up does not come from the copper; (2) Part of the heat emitted by the copper goes to raise the temperature of the calorimeter itself; (3) Directly the calorimeter and its contents become hotter than surrounding bodies they begin to lose heat by conduction and radiation (p. 115).

To eliminate the first error as far as possible the substance must be heated without coming in contact with the hot liquid, and a more convenient method of transferring it from the heater must be employed. The apparatus described below shows how this is done. The second source of error can be allowed for in the calculation, for if m_2 is the mass of the calorimeter and s_2 the specific heat of its material, the heat absorbed by the calorimeter alone when its temperature is raised from $t_1°$ to $t_2°$ is $m_2 s_2(t_2 - t_1)$ cals. The heat absorbed by the cold liquid is $m_1 s_1(t_2 - t_1)$, if m_1 is its mass and s_1 its specific heat. The heat emitted by the hot body is similarly $Ms(T - t_2)$, if T is its initial temperature and t_2, as before, the temperature of the whole calorimeter after mixture. Equating the heat emitted to the heat absorbed, we have the equation

$$Ms(T - t_2) = m_1 s_1(t_2 - t_1) + m_2 s_2(t_2 - t_1)$$

from which s can be found if s_1 and s_2 are known. The last term is

usually small, hence an approximate value of s_2 will suffice; for copper calorimeters it may be taken as 0·095. That mass of water which has the same thermal capacity as the calorimeter is called the **water equivalent** of the calorimeter. From the last paragraph it is seen that the thermal capacity in question is m_2s_2 calories. This quantity of heat raises the temperature of the calorimeter 1°, it would also raise the temperature of m_2s_2 gms. of water by 1°; the water equivalent is therefore m_2s_2 gms. When we calculate the heat absorbed by the calorimeter and the water it contains, we may therefore add m_2s_2 gms. to the weight of the water and multiply this quantity by the rise in temperature. The product will give the heat absorbed by the calorimeter and its contents. If the whole of the calorimeter is not raised from $t_1°$ to $t_2°$ the water equivalent must be found experimentally under the conditions in which it is to be used. The following experiment, in which it is assumed that the calorimeter is to be used two-thirds filled, will show the method.

EXPERIMENT.—The weight of the empty calorimeter was 74·8 gms. Some cold water was poured in and a reweighing showed that the amount added was 81·7 gms. The liquid was well stirred, so as to take up the temperature of the vessel, and the temperature found to be 15·4°. Hot water at a temperature 36·6° was then poured in from a beaker until the calorimeter was two-thirds full, after stirring well the temperature was 24·6°. A final weighing showed that 67·2 gms. of hot water had been added.

The heat given out by the hot water in cooling from 36·6° to 24·6° = 806·4 cals.

And the heat absorbed by the cold water = 81·7 × 9·2 = 751·6 cals.

∴ Heat absorbed by calorimeter = 806·4 − 751·6 = 54·8 cals.

∴ The calorimeter requires 54·8 cals. to raise its temperature 9·2°, and the heat required to raise its temperature 1° = 5·9 cals.

The water equivalent is therefore 5·9 gms.

The third error mentioned above is reduced by hanging the calorimeter by three threads inside a larger vessel; this screens it from air currents, and, as will be understood later, lessens the heat conduction. The radiation losses are smaller if both vessels have well-polished surfaces (p. 125). In addition it is arranged that the initial temperature of the calorimeter is slightly below and its final temperature nearly an equal amount above that of its surroundings. During the early stages of the experiment the calorimeter thus receives heat from the room, but, as its temperature rises, it gives out heat and the two may be made to balance approximately.

A convenient form of heater is shown in section in Fig. 18. It consists of two concentric brass tubes; in the inner one the substance

to be heated is suspended in contact with the bulb of a thermometer, and the tube is closed by corks at each end to prevent air currents. Steam is made to circulate in the annular space between the tubes. The substance is thus heated without being wetted. It should be used in the form of thin sheet in order that it may acquire more quickly the temperature of the heater and of the cold water in the calorimeter. When the temperature of the hot body has remained steady for 15 mins. the temperature of the calorimeter is taken, the vessel is brought under the heater, the lower cork is removed, the upper one loosened, and the body is lowered rapidly into the water. The calorimeter is then removed some distance away, and, after stirring, the temperature of the mixture is noted. The specific heat is calculated from the equation already given. If the solid is in the form of a powder, or is soluble in water, it is enclosed in a copper case and the heat emitted by this is allowed for in the calculation.

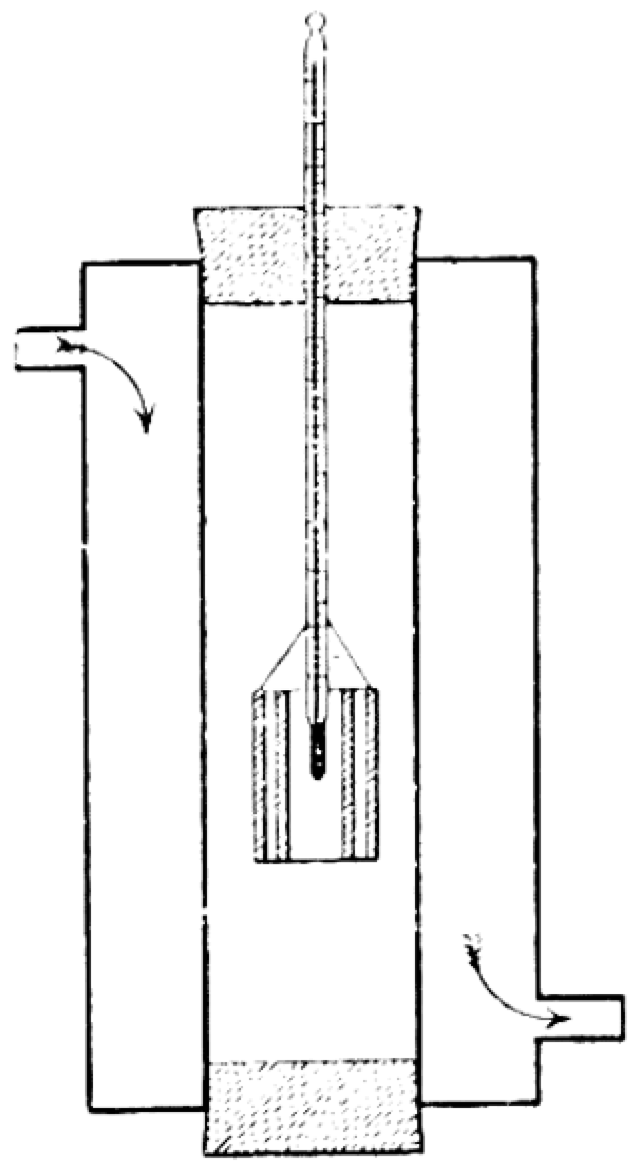


FIG. 18.—Heater for use in Specific Heat Determinations.

Specific Heat of Liquids.—If the specific heat, s, of the solid is known the same method may be used to find that of a liquid; the liquid in this case replaces the water in the calorimeter and it is s_1 which is calculated from the equation. When the liquid does not react chemically with water direct mixture may be used. A known weight of liquid is placed in the calorimeter and its temperature observed, water at a known temperature near 35° is added, and the temperature of the mixture is found. The calorimeter is then weighed to get the amount of water added and the calculation performed as in the previous case. Other methods which can be used for liquids are given on pp. 127 and 406.[1]

Specific Heats of Gases.—Consider a quantity of gas placed in a cylinder which is closed by a movable piston; the gas expands as its temperature is raised and pushes back the piston against the

[1] Barton and Black, "Practical Physics," pp. 63–66.

atmospheric pressure, *i.e.* it does work. It would push back the surrounding atmosphere just the same if the piston were removed, hence whenever a gas is heated at constant pressure it performs work. As will be seen in Chap. X it can do this only by using up some of the heat supplied to it, and this heat does not go to raise its temperature. It follows that when a gas is allowed to expand, more heat must be supplied to it to raise its temperature 1° than is necessary when its volume is kept constant, the excess is expended in the performance of work. In other words, we must consider two specific

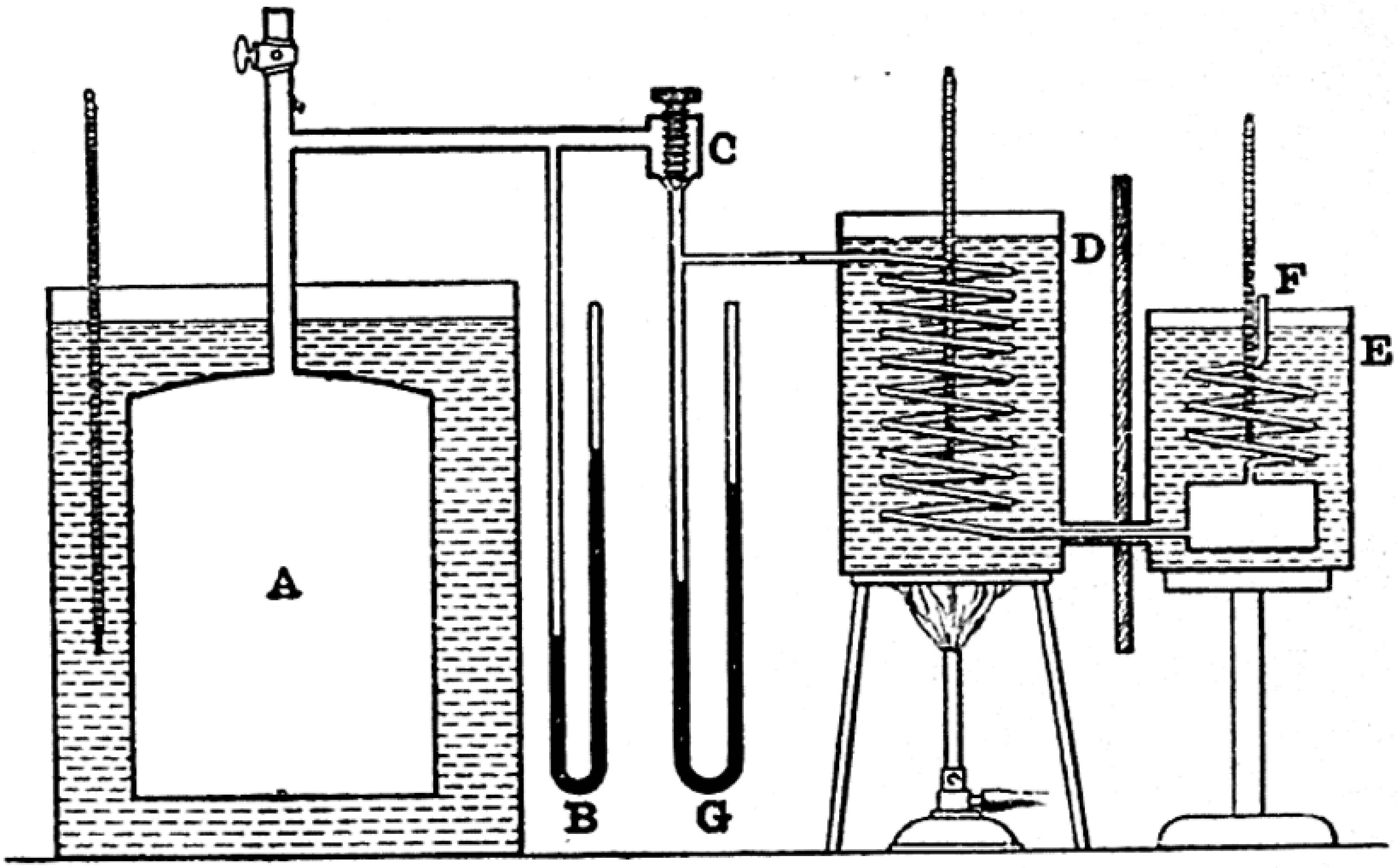


FIG. 19.—Regnault's Apparatus for measuring the Specific Heat of Gases.

heats in the case of a gas: (1) that at constant pressure, C_p; (2) that at constant volume, C_v, the former being the larger of the two. For solids and liquids the expansions are so small that the two specific heats are practically equal.

Specific Heat of a Gas at Constant Pressure.—As in so many other cases the classical experiments on this subject are those of Regnault, his apparatus is shown, partly in section, in Fig. 19. The gas to be used was compressed in a reservoir, A, which was kept at a constant temperature in a tank of water. Experiments were first made to determine how the quantity of gas in the reservoir varied with the pressure, so that from any future reading of the manometer, B, the mass of the contained gas would be known. From A the gas flowed in succession through a regulating cock, C, the heater D, the

calorimeter E, finally escaping into the atmosphere at F. The gas flow was kept steady by altering the cock C so that the pressure indicated by the manometer G was constant. The heater consisted of a long spiral of fine copper tubing immersed in a liquid maintained at a steady, high, temperature. The gas thus entered the calorimeter at a known temperature T. Here it passed through another copper spiral immersed in water and was cooled to the temperature of the calorimeter. Let t_1 and t_2 be the initial and final temperatures of the calorimeter, then the first portion of the gas was cooled from T° to t_1° and the final portion from T° to t_2°; the average temperature of the calorimeter during the passage of the gas was therefore $\frac{t_1+t_2}{2}$, and the heat lost by the gas was $m\left(T-\frac{t_1+t_2}{2}\right)s$, if m is its mass and s its specific heat. The heat gained by the calorimeter and its contents was calculated in the usual way and hence s was found. Owing to the time the experiment lasted the losses by conduction and radiation were large; an error in their determination appears to have caused a 2 per cent. inaccuracy in the final result. (See also p. 406.)

Table of Specific Heats.

Air (C_p)	0·2417	Ice	0·502
Air (C_v)	0·1715	Iron	0·119
Aniline	0·514	Lead	0·032
Bismuth	0·0304	Mercury	0·033
Copper	0·094	Turpentine	0·43
Glass	0·19	Zinc	0·093
Hydrogen	3·402		

Applications.—Calorimetry has some important scientific and technical applications other than the determination of specific heats. For example, it is important to the chemist to know how much heat is absorbed or evolved when chemical changes take place, and for the engineer it is necessary to know how much heat is evolved by burning a known weight of different kinds of fuel. These processes are made to take place in special kinds of calorimeter, where the heat evolved may be measured as in the preceding pages.

Dulong and Petit's Law.—Dulong and Petit, from their investigations of the specific heats of various chemical elements, were able to

deduce the law that the product of the specific heat and the atomic weight is constant. Regnault found that the law was approximately true if the substances were in the solid state; the mean value of the product (atomic weight × specific heat) is 6·2. Since the specific heat of a substance is found to vary with the temperature it is clear that the law cannot be universally true; in fact, recent experiments show that the specific heats of many substances are very much smaller at −250° C. than they are at the temperature of the laboratory.

EXAMPLES ON CHAPTER III

1. A mass of 200 gms. of copper, whose specific heat is 0·095, is heated to 100° and placed in 100 gms. of alcohol at 8° contained in a copper vessel whose mass is 25 gms. and the temperature rises to 28·5°. Find the specific heat of alcohol. (L. '89.)

2. A copper vessel contains 100 gms. of water at 12°. When 56 gms. of water at 30° are added the resulting temperature of the mixture is 18°. What is the water equivalent of the vessel? A calorimeter with water equivalent 12 contains 100 gms. of water at 12°. When 100 gms. of metal at 100° are added the resulting temperature of the mixture is 20°. Find the specific heat of the metal. (L. '93.)

3. Why is it difficult to measure the specific heat of a gas by the method of mixtures? What weight of gas of specific heat 0·25 entering at 100° would require to pass through an apparatus of which the heat capacity was 50 cals. per degree before raising the temperature from 15° to 17°? (L. '10.)

4. Eighty gms. of water at 35° are poured into a calorimeter containing 120 gms. of turpentine whose temperature is 15°. The calorimeter weighs 70 gms. and its specific heat is 0·1. The specific heat of turpentine is 0·45; find the temperature of the mixture.

CHAPTER IV

LINEAR EXPANSION

Coefficient of Expansion.—It has already been shown that an increase in the temperature of a body is frequently accompanied by expansion; there are, however, exceptions to this rule. Below $-80°$ a rod of silica decreases in length when heated, and silver iodide contracts in volume up to a temperature of 142°. In the case of solids we may have to consider changes in length, area, or volume; with fluids we are concerned with volume changes alone. This arises from the fact that a fluid takes the shape of the containing vessel and an increase in one dimension depends upon how much it is allowed to alter in the other two. The linear expansion of a liquid is therefore an indefinite quantity, but its volume is independent of the shape of the vessel and is perfectly definite at a given temperature.

When a bar is heated experiment shows that the increase in length is proportional to the original length and to the temperature change, provided the latter is not too large. Suppose we have a bar whose length at 0° is L_0 cms., and that when heated to $t°$ its length becomes L_1, then each cm. has expanded $(L_1-L_0)/L_0$, and for 1° the expansion is $(L_1-L_0)/L_0t$. **The ratio of the increase in length for 1° rise in temperature to the length at 0° is called the coefficient of linear expansion.** Denoting this by l,

$$l=\frac{L_1-L_0}{L_0t}$$

or

$$L_1=L_0(1+lt)$$

When the temperature decreases, t must be put negative in this formula. If it is desired to compare the relative expansibilities of different solids we have merely to compare their coefficients of linear expansion. These coefficients are very small quantities, *e.g.* a bar

of brass 100 cms. long at 0° if heated to 50° expands to about 100·09 cms., whence we find that $l = 0{\cdot}000018$. This smallness makes it possible to simplify the calculations, for by division or the binomial theorem we find

$$\frac{1}{1+x} = 1 - x + x^2 - \text{ etc.}$$

$$\frac{1}{1-x} = 1 + x - x^2 + \text{ etc.}$$

also $$(1+x)(1+y) = 1 + x + y + xy$$

Suppose now $x = 0{\cdot}00002$ and $y = 0{\cdot}00003$, *i.e.* quantities of the same order of magnitude as l above, then x^2, y^2, x^3, y^3, xy, etc., are very small quantities indeed, and we may neglect them in comparison with x and y.

$$\therefore \frac{1}{1 \pm x} = 1 \mp x$$

$$(1+x)(1+y) = 1 + x + y$$

and $$\frac{1}{(1+x)(1+y)} = \frac{1}{1+x} \cdot \frac{1}{1+y} = (1-x)(1-y) = 1 - x - y$$

As an example, let the lengths of a bar at 0°, t_1° and t_2° be L_0, L_1, and L_2 respectively, and l its coefficient of linear expansion.

Then $$L_1 = L_0(1 + lt_1)$$

$$L_2 = L_0(1 + lt_2)$$

$$\therefore \frac{L_2}{L_1} = \frac{L_0(1+lt_1)}{L_0(1+lt_2)} = \frac{1+lt_2}{1+lt_1}$$

accurately, or, using the above approximations, when lt_2 and lt_1 are small quantities,

$$\frac{L_2}{L_1} = (1 + lt_2) \cdot \frac{1}{1+lt_1} = (1+lt_2)(1-lt_1) = 1 + l(t_2 - t_1)$$

Suppose we require the ratio of the length at 100° to that at 20° of a bar whose coefficient of linear expansion is 0·00002. The accurate formula gives 1·001599, the approximate formula 1·001600, practically the same result. From the last equation we get

$$l = \frac{L_2 - L_1}{L_1(t_2 - t_1)}$$

showing that, with these approximations, we need not refer the original length to 0° in order to calculate l; all that is required are the lengths at two known temperatures. How these are found is explained in the next paragraph.

Measurement of Coefficient of Linear Expansion.—A simple apparatus is shown in Fig. 20; it will also illustrate what errors are likely to arise in such an experiment. The rod AB to be experimented upon passes through corks up the centre of a wider tube C. Near the end A it is clamped between two metal knife-edges projecting slightly from wooden blocks fixed to a base-board. The end B, which is flat, can be made to touch a screw D of known pitch, say 0·5 mm. The large circular head of this screw is divided into 100 equal divisions; if it is turned through one division it will advance 1/200 mm. Steam is passed through C by the side tubes E, F, and a

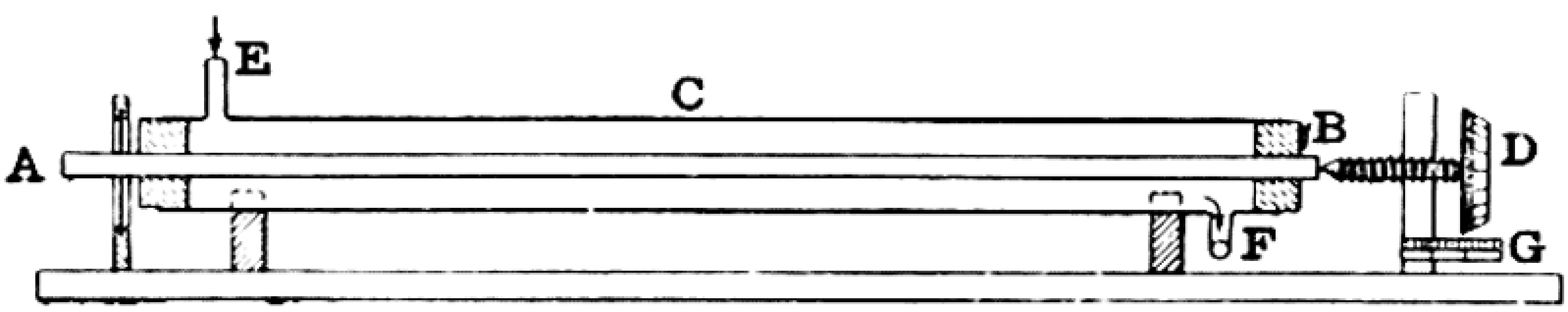


FIG. 20.—Simple Apparatus for Linear Expansion.

thermometer placed in the exit tube gives the temperature. The screw D is now brought into contact with B and the reading at the pointer G is taken. Cold water is next passed through tube C, causing the rod to contract. The contraction can be measured by noting how far the screw D has to be turned to bring it again in contact. The temperature of the water is noted, and finally the length of the rod from the knife-edges to B is measured by a scale; l can then be calculated from the last formula.

EXAMPLE.—For a glass rod 80 cms. long the screw had to be turned through 110 divisions on the circular head; the temperature of the steam was 100° and that of the water 15°. Hence $L_2 - L_1 = 110 \times 0{\cdot}005 = 0{\cdot}55$ mm. and $l = \dfrac{0{\cdot}055}{80 \times 85} = 0{\cdot}0000081$.

There are several sources of error in this experiment: (1) Part of the rod is exposed to the outside air and will probably not reach the proper temperature; (2) The screw may become heated and so alter in length, this is minimised by having it in contact only when the reading is being taken, especially at the higher temperature; (3) The base-board may expand and alter the position of the screw.

Lost time in the screw is avoided since it is always turned in the same direction. These errors are eliminated in the method now to be described, which, in principle, is that used at the International Bureau of Weights and Measures.[1]

Comparator Method.—The experimental bar (Fig. 21) is placed on rollers in one compartment of a metal trough, which is divided throughout the greater part of its length by a vertical division; in the other compartment a screw, worked by a motor, keeps water circulating past the bar. The temperature is read by two or more thermometers placed horizontally in the liquid. A second similar

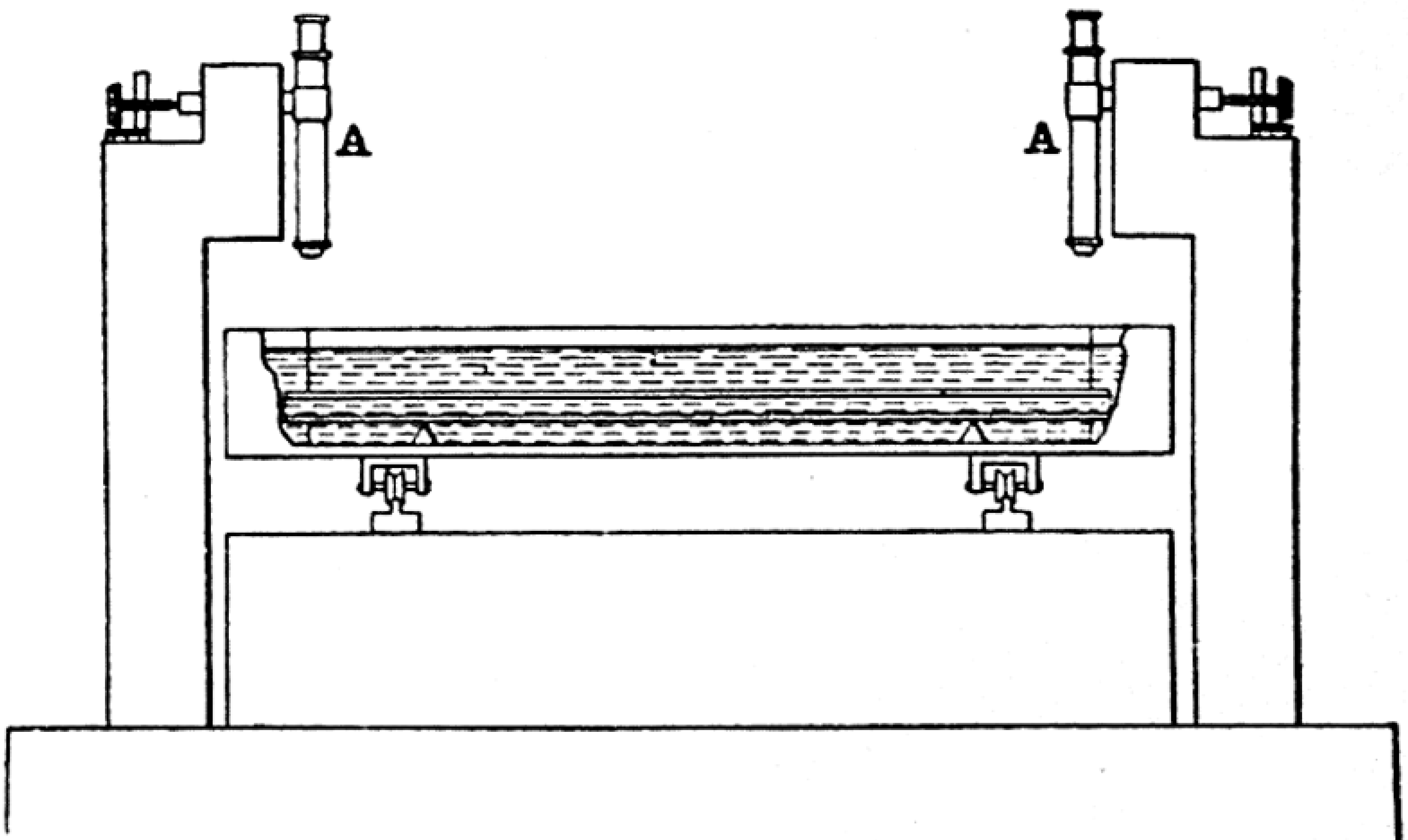


FIG. 21.—Comparator Method of measuring Expansions.

trough contains a standard bar having fine marks near the ends exactly one metre apart. Marks separated by approximately the same distance are also made on the experimental bar. There are, in addition, arrangements for levelling and for giving slight lateral or longitudinal displacements to either bar. The troughs can be run to and fro on a small tramway so that either rod can be brought beneath two vertical microscopes A, A, which are supported independently of the rest of the apparatus. Each microscope carries cross-wires in the eye-piece which can be moved by a micrometer screw with divided head, similar to the one already described. A movement of the marks as small as 0·001 mm. can be measured. The troughs having been filled with ice-cold water, the standard bar

[1] For a simple modification, see Barton and Black, "Practical Physics," p. 52.

is placed beneath the microscopes and the cross-wires are adjusted so that they appear to coincide with the marks when seen through the instruments. The second trough is now brought into position and the cross-wires adjusted as before by moving the micrometer screws. The amount of this movement shows at once by how much the distance between the marks on the experimental bar differs from one metre and hence the length at 0° is found. The water is next heated to a known temperature and the resulting expansion of the bar is measured by the cross-wires and screws. The standard bar, still at 0°, is finally brought under again to ensure that the distance apart of the

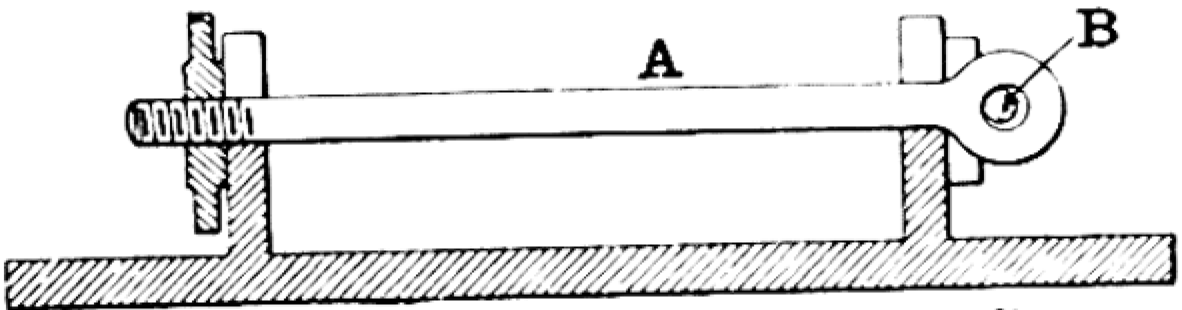


FIG. 22.—To show Stress caused by Cooling.

microscopes has remained unaltered. The length being known at two temperatures the coefficient of linear expansion can be calculated.

TABLE I.

Coefficients of Linear Expansion.

Brass	0·0000188	Nickel steel (45% nickel)	0·0000082
Copper	0·0000172	Platinum	0·0000084
Glass (tube)	0·0000084	Porcelain	0·0000088
Iron (soft)	0·0000122	Fused silica	0·00000059
Invar. (Nickel steel, 36% nickel)	0·00000087	Steel (untempered)	0·0000108
		Zinc	0·0000294

(The above numbers represent average values.)

Applications.—The fact that bodies expand when heated has frequently to be allowed for, or is made use of, in industrial applications. Thus when the metals on a railway are laid a small space is left between successive sections so that on hot days expansion may take place; otherwise the rails would buckle. The iron tyres of cart wheels are fitted on while they are red hot; when they cool they grip the wheel much more tightly. The forces exerted on account of expansion or contraction may be very large. The apparatus shown in Fig. 22 illustrates this. The bar A is heated and then screwed up

as tightly as possible; when it cools the stress is so great that the small bar, B, which passes through a hole in the end of A, is broken.

The time of swing of a pendulum depends upon its length; if, therefore, the temperature changes, the rate of a clock will be altered unless we can arrange to keep the pendulum bob at a fixed distance from the point of suspension. This is done in various ways. Fig. 23 shows a form of Harrison's gridiron pendulum. The middle and two outer rods are made of iron, the remaining pair of zinc. The middle rod passes freely through the cross-piece B. Expansion of the iron alone will evidently lower the bob, expansion of the zinc alone will raise it, since Z can only expand upwards. When the temperature of the whole alters these changes may be made to balance each other. Denoting the lengths of the rods by the letters on them in the figure, the distance of the bob from $A = I - Z + S$.

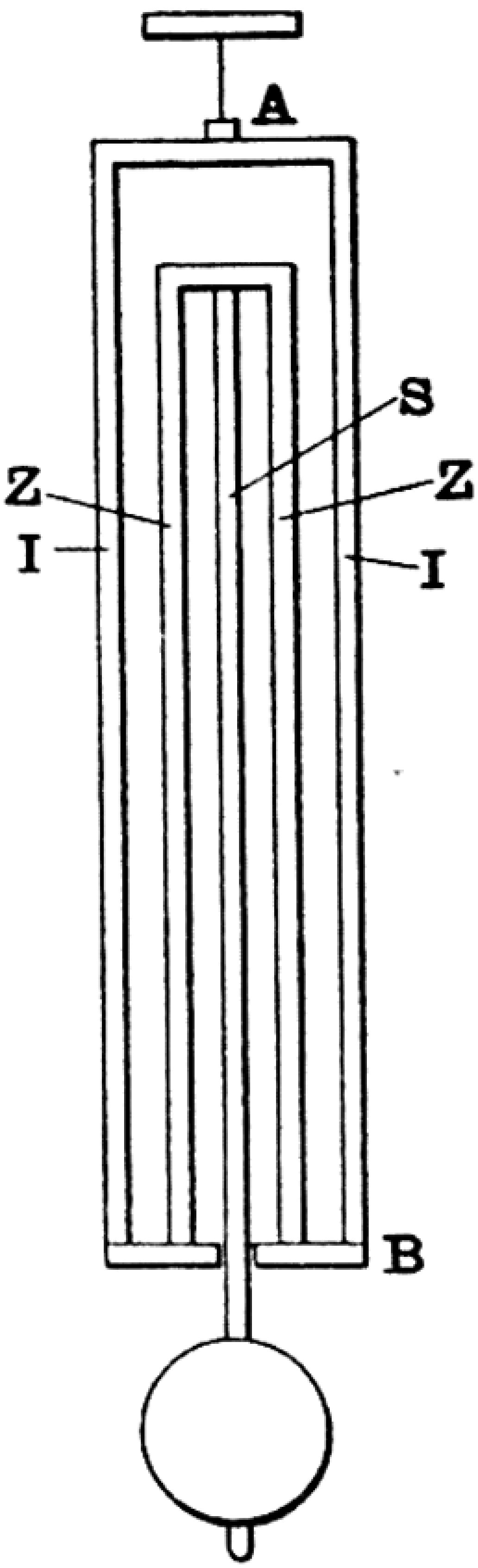


FIG. 23.—Gridiron Pendulum.

When the temperature rises by $t°$ this becomes

$$(I + S)(1 + 0{\cdot}0000122t) - Z(1 + 0{\cdot}0000294t)$$

from Table I.

This must be equal to the original length if the time of vibration is to remain unaltered. Hence

$$(I + S)(1 + 0{\cdot}0000122t) - Z(1 + 0{\cdot}0000294t) = I - Z + S$$

whence we find readily

$$\frac{I + S}{Z} = \frac{294}{122}$$

showing that the ratio of the lengths of the iron and zinc rods is inversely as their coefficients of linear expansion.

The rate of a watch is regulated by the elasticity of the spring and the size and mass of the balance wheel. A rise in temperature decreases the elasticity and increases the size of the wheel, each of which causes the watch to lose time, though it is the former which is

chiefly effective. This is counterbalanced by causing the expansion to bring the weight of the wheel rim nearer the centre.

EXPERIMENT.—Rivet a strip of zinc 20 cms. long and 1 cm. wide to a similar strip of iron to form a compound strip of the same length and heat it in a flame. It becomes curved with the zinc on the outside. This is because zinc expands more than iron and the metals can only take up their appropriate lengths by curving in this manner.

The same principle is applied to the balance wheel of a watch (Fig. 24). The rim is made of a compound strip of two or more metals with the more expansible metal outside; as the temperature rises the strip curves inwards, and compensation may be attained by properly distributing the weight of the rim.

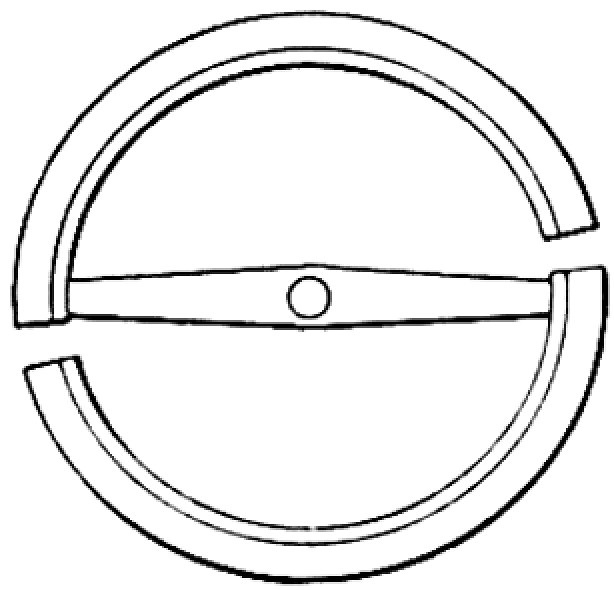


FIG. 24.—Balance Wheel of a Watch.

It will be noticed from Table I. that invar has a very small coefficient of expansion; it should, therefore, prove useful for clock pendulums, standards of length, etc. If a piece of iron wire is sealed through a glass tube, the joint usually fractures as it cools owing to the unequal contraction of the two substances. Platinum and nickel steel (45 per cent. nickel) have, as shown in Table I., about the same expansion as glass; hence they may be used more safely for the purpose, *e.g.* in the construction of incandescent electric lamps.

Thick-bottomed drinking-glasses frequently crack if hot water is poured into them owing to the unequal expansion of the inner and outer layers. Glass is a bad conductor of heat (p. 115) and so hinders the rapid equalisation of temperature in the different portions. Fused quartz or silica has a very small expansion; vessels made of this substance may be plunged into a very hot bath without fear of breakage.

EXAMPLES ON CHAPTER IV

1. A base line 2 miles long is to be laid out by means of an iron chain whose length is known at 0°. Find the percentage error caused by neglecting the expansion of the iron if the coefficient of linear expansion is 0·000012 and the average temperature is 16°.

2. A rod is found to be 100 cms. long at 50° and 100·1 cms. long at 100°.

Assuming that it expands uniformly, obtain a formula giving its exact length at any temperature and calculate its coefficient of cubical expansion. (L. '06.)

3. If it takes a force of 20,000 kilos./cm.^{2} to produce a 1 per cent. diminution of length in an iron bar, what force would you expect it to require to prevent a bar 8 cms. long, 3 cms. wide, and 2 cms. deep, from expanding lengthways when raised 500°? Coefficient of expansion of iron = 0·0000122. (L. '04.)

4. The height of the barometer at 18° is found to be 76 cms. when measured with a brass scale which is correct at 0°. Find the actual length of the mercury column. Coefficient of expansion of brass is 0·0000182.

CHAPTER V

CUBICAL EXPANSION

Volume Expansion of Solids and Liquids.—When a homogeneous body is heated it is found, as the result of experiment, that the increase in volume due to a small rise of temperature is proportional to the temperature change and to the original volume of the body. **The increase in volume when the temperature is raised 1° divided by the volume at 0° is called the coefficient of cubical expansion.** Denoting this by c, if $V_0 =$ vol. at 0°, $V =$ vol. at t°,

$$c = \frac{V - V_0}{V_0 t}$$

or

$$V = V_0(1 + ct)$$

In the case of linear expansion, on account of the smallness of l, we saw that it did not introduce serious error if the original length was measured at some temperature other than 0°. The coefficient of cubical expansion is a much larger quantity, especially in the case of liquids and gases; it is better, therefore, to refer our definition to the volume at 0°, although the difference will not be large in the case of liquids and will be still smaller for solids.

Consider a cube of a solid body 1 cm. in side at 0°, and let the coefficients of linear and cubical expansion of the material be l and c respectively. If the temperature be raised 1° each side becomes $(1 + l)$, and the volume becomes $(1 + l)^3 = 1 + 3l + 3l^2 + l^3$. From what has been said about small quantities we may take this as being $(1 + 3l)$, and the increase in volume is thus $3l$. But this, from definition, is the coefficient of cubical expansion c, hence $c = 3l$. **For solid homogeneous bodies the coefficient of cubical expansion is three times the coefficient of linear expansion.**

Suppose we have a spherical glass flask filled with a solid core of

the same material; when heated they will expand as if they formed a solid body. The flask would expand by an equal amount if the core were absent. It is seen from this that hollow bodies increase in volume by the same amount as solid bodies of the same dimensions. Thus a glass tube and a glass rod of the same diameter at one temperature will still have equal diameters if the temperature of each is changed by the same amount.

Effect of Temperature on Density.—The density of a body is defined as the mass of unit volume. If d_0 is the density of a body at 0° when the volume is V_0, the mass is $V_0 d_0$. At another temperature, $t°$, let the volume be V and the density d. The mass being constant,

$$V_0 d_0 = Vd$$

or

$$\frac{d}{d_0} = \frac{V_0}{V}$$

But

$$V = V_0(1 + ct)$$

$$\therefore \frac{d}{d_0} = \frac{V_0}{V_0(1 + ct)} = \frac{1}{1 + ct}$$

This result is important in dealing with the expansion of fluids; it shows that we can calculate the coefficient of cubical expansion c if we can compare the densities at 0° and $t°$.

Apparent and True Coefficient of Expansion of a Fluid.—We have already seen, p. 19, that the expansion of a liquid is partly masked by that of the vessel containing it. The expansion observed under such conditions is called the apparent expansion. Similarly the coefficient of apparent expansion of a fluid, which we will denote by c_a, is the apparent increase in volume for 1° rise of temperature divided by the volume at 0°. The true coefficient c will be greater than this since it takes into account the increase in volume of the vessel. Suppose we have a glass bulb, whose volume at 0° is V_0 cm.^{3}, surmounted by a stem graduated in c.cms., and let the coefficient of cubical expansion of the glass be g. Let the bulb be filled at 0° with a liquid whose true coefficient is c. At 1° the true volume of the liquid is $V_0(1 + c)$, but owing to the expansion of the glass the graduations are incorrect and the volume as read on the stem will be less than this; it will apparently be $V_0(1 + c_a)$. At this temperature each c.cm. of the vessel has expanded to $(1 + g)$, so that we

may also get the true volume of the liquid by multiplying its apparent volume by $(1+g)$.

$$\therefore \text{ true volume of the liquid } V_0(1+c) = V_0(1+c_a)(1+g)$$

or

$$1+c = 1+c_a+g+c_ag$$

The coefficients being small the term c_ag may be neglected and

$$c = c_a + g$$

To get the true coefficient of expansion of a liquid we must add the apparent coefficient to the coefficient of cubical expansion of the vessel.

The apparent coefficient can readily be observed in the dilatometer, Fig. 25. This consists of a glass bulb, whose volume is known at 0° C., attached to a stem graduated in c.cms. Liquid is placed in the apparatus and its apparent volume at two different temperatures is noted by the graduations, the apparent coefficient of expansion can then be calculated. If we add to this apparent coefficient three times the linear coefficient for glass we should expect to get the true coefficient for the liquid. This procedure, however, is faulty, since glass is usually far from homogeneous, and after determining l for a glass tube a bulb must be blown on it, which might greatly alter its expansibility. The method is therefore inaccurate. If c could be determined for some liquid independently of the envelope, we could afterwards observe c_a for the same liquid in the dilatometer, and, using the above equation, calculate g. The dilatometer could then be used to find c_a for any other liquid, and hence c.[1] We proceed to show how the true (or absolute) expansion of a liquid is found.

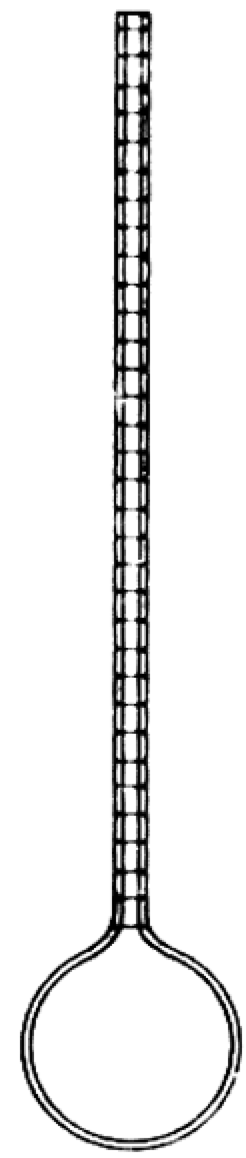


FIG. 25.—Dilatometer.

Absolute Expansion of Mercury.—The apparatus shown in Fig. 26 is a simplified form of that used by Dulong and Petit. BAA′B′ represents a glass tube containing mercury which is at different temperatures in the upright limbs; AA′ is horizontal. From a well-known hydrostatical principle, the intensity of pressure at A must equal that at A′ when there is equilibrium, independently of whether the tubes have equal

[1] Barton and Black, "Practical Physics," p. 54.

diameters or not. If H_0 and H are the lengths of the columns in AB and A′B′ respectively, d_0 and d the corresponding densities, then

$$Hd = H_0 d_0$$

or

$$\frac{H}{H_0} = \frac{d_0}{d} = 1 + ct \qquad \text{from p. 48,}$$

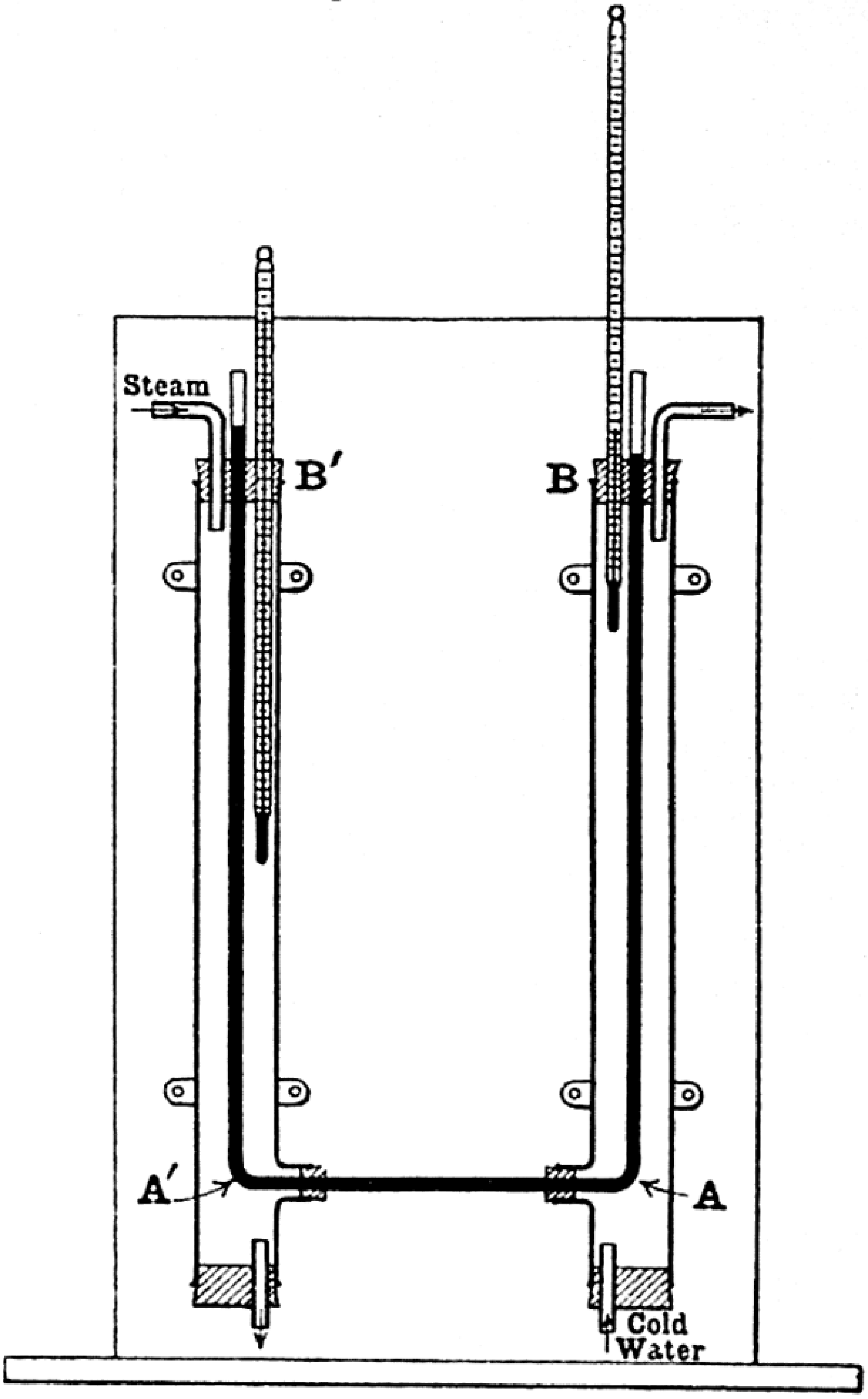


FIG. 26.—Dulong and Petit's Apparatus.

if the temperatures of the limbs are 0° and $t°$ and c is the true coefficient of expansion of the liquid;

$$\therefore c = \frac{H - H_0}{H_0 t}$$

Hence if the heights of two balancing columns are measured when they are at different temperatures, c can be found without any knowledge of the expansion of glass.

To carry out the measurements in a form suitable for a simple laboratory experiment the columns are surrounded by wider tubes as in the figure. Through these a stream of ice-cold water is first run, and the quantity of mercury is adjusted until the ends of the columns are just visible above the corks B, B′. The heights of the surfaces above the axis of AA′ are measured, corresponding with H_0 above. Steam is next passed round A′ B′; the temperature in each case is given by thermometers projecting through the corks at B′, B. The length of the warm column is again measured, giving $H - H_0$, and c is calculated as above.

The apparatus in this simple form has several disadvantages: (1) It does not give a continuous series of temperatures in the hot limb; this can be obviated by having the column A′B′ surrounded by an oil bath, which must be well stirred; (2) The thermometer may not give the mean temperature of the hot column; (3) The surfaces are exposed and may change in temperature while the observations are being made.

Fig. 27 shows the principle of an apparatus used by Regnault, and more recently improved by Callendar, to overcome these defects.

The vertical tubes AB, A′B′, from one to two metres long and a cm. in diameter, are bent twice at right angles so that the portions BC, B′C′ are horizontal. AA′ is made of narrow bore to prevent the circulation of currents of mercury from one vertical tube to the other. Water cooled to 0° by ice in M is steadily passed through the wide tube surrounding AB; we will suppose also that it drips on blotting paper wrapped round CD and C′D′. A′B′ is surrounded by oil, which is first heated by passing an electric current through the wire coil Q and is then forced past the mercury column in the direction of the arrows by a small centrifugal pump R. The mean temperatures of the long columns are given by platinum thermometers P′ and P (p. 389), whose bulbs extend the whole lengths of AB, A′B′; that of the short tubes CD, C′D′ by mercury thermometers placed in contact with them. The heights of the various columns are measured by a cathetometer. This consists of a horizontal telescope, having cross-wires in the eye-piece, which moves up and down a vertical graduated bar. The telescope is first focussed

so that the cross-wires appear to coincide with the axis of AA′; it is then moved so that they appear to coincide with the axis of B′C′. The distance it has been displaced gives the vertical length A′B′, and similarly for the other columns. Let h', H, h and H_0 be the

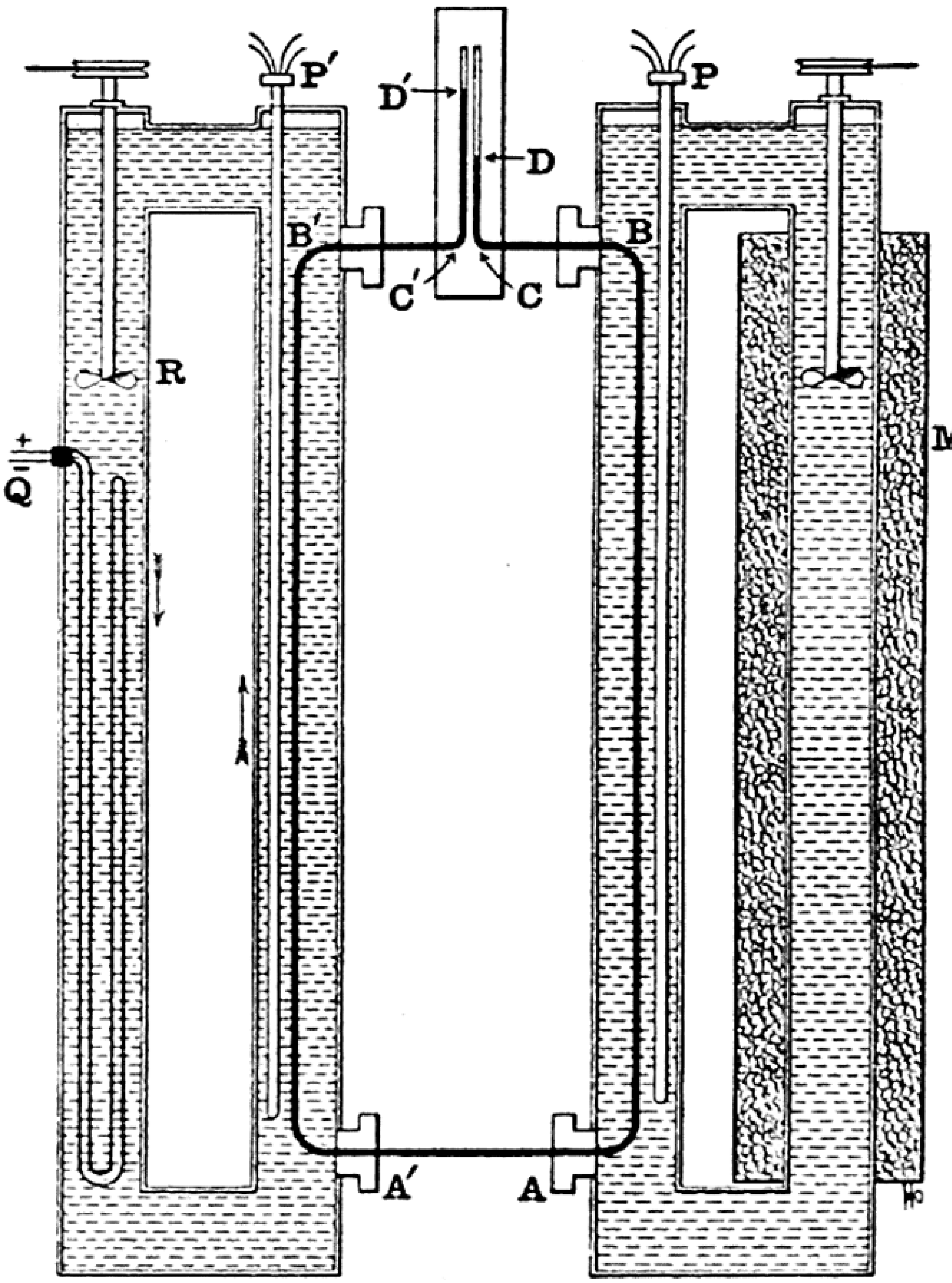


FIG. 27.—Callendar's Apparatus.

lengths of C′D′, A′B′, CD and AB respectively, when A′B′ has a temperature $t°$ and the others are at 0°. If d and d_0 are the densities of mercury at these temperatures the pressure at A′ is $(h'd_0 + Hd)$, and that at A is $(hd_0 + H_0d_0)$.

$$\therefore h'd + Hd = hd_0 + H_0d_0$$

that is,

$$Hd = (H_0 + h - h')d_0$$

or $$\frac{H}{H_0 + h - h'} = \frac{d_0}{d} = 1 + ct \quad \text{(p. 48)}$$

whence $$c = \frac{H - H_0 - h + h'}{(H_0 + h - h')t}$$

Between 0° and 100° c is found to be 0·000182, as the temperature is raised the coefficient increases.

Methods of determining the Apparent Expansion of Liquids.—(1) *The Dilatometer method* already mentioned is chiefly used when volatile liquids are concerned, as there is little opportunity for evaporation owing to the small surface exposed.

(2) *Weight thermometer.* The glass apparatus shown in Fig. 28 has a reservoir about 6 cms. long which is continued by a capillary tube bent twice at right angles. It is cleaned, dried, and weighed, and is then supported by copper gauze with the end of the tube under the surface of the liquid. It is filled by alternate heating and cooling, and during the final cooling is placed in melting ice; it is thus completely filled with liquid at 0°. It is next transferred to a bath which can be heated to any suitable temperature $t°$; on account of expansion some of the liquid overflows and is received in a weighed beaker. A further weighing of the beaker and its contents gives the mass that has overflowed. The bulb and remaining liquid are also weighed; subtracting the weight of the glass we have the mass of liquid left in.

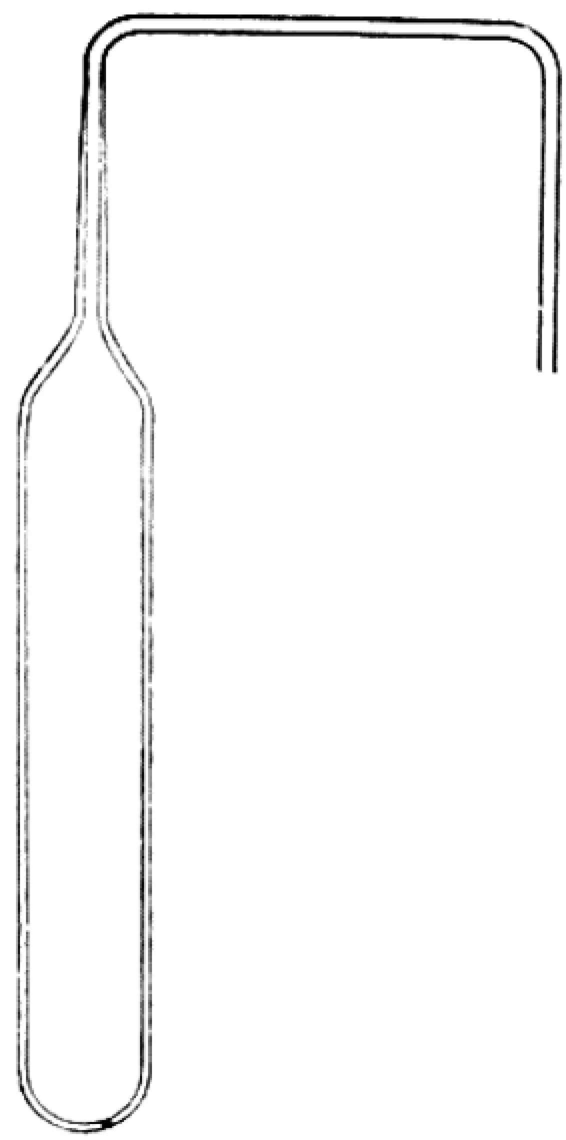


FIG. 28.—Weight Thermometer.

Let m be the mass that overflows, M the mass left in, and d_0 the density of the liquid at 0°. Since we are finding the apparent expansion the increase in volume of the glass is neglected. The mass of liquid filling the thermometer at 0° is $(M + m)$,

$$\therefore \text{ volume of thermometer} = (M + m)/d_0$$

But a mass M of liquid, whose volume at 0° is M/d_0, fills the thermometer at $t°$ when the volume is $(M + m)/d_0$. Hence the apparent expansion of this mass between 0° and $t° = (M + m)/d_0 - M/d_0 = m/d_0$.

Dividing by the volume of the liquid at 0°, M/d_0, and by the temperature change $t°$, we get the coefficient of apparent expansion c_a

$$\therefore c_a = \frac{\frac{m}{d_0}}{\frac{M \cdot t}{d_0}} = \frac{m}{Mt}$$

The method is useless for volatile liquids, since obviously a large proportion of the mass overflowing will evaporate.

The apparatus may be replaced by a small flask with a narrow neck, or a specific gravity bottle, and the liquid adjusted to a fixed mark upon it by means of a pipette. If mercury is the liquid used, since c is known from the last paragraph, we can calculate the cubical expansion of the glass from $c = c_a + g$. The apparatus can then be used to find c_a for any other liquid, and as g is known the true coefficient can be found.

(3) *Hydrostatic method.* Let a solid be weighed (1) in air, (2) completely immersed in a liquid at 0°, (3) in the same liquid at $t°$. Let the loss of weight at 0° be m_0 and at $t°$ be m; these represent the masses of liquid having the same volume as the solid at 0° and $t°$ respectively. The volume of liquid displaced at 0° is m_0/d_0, that at $t°$ is m/d, where d_0 and d are the corresponding densities of the liquid. Hence the volume of the solid at 0° is m_0/d_0, and at $t°$ this has expanded to $m_0(1 + gt)/d_0$, g being the coefficient of cubical expansion of the solid. (See equation p. 47.)

Equating the volume of the solid at $t°$ to that of the displaced liquid at the same temperature,

$$\frac{m}{d} = \frac{m_0}{d_0}(1 + gt)$$

$$\therefore \frac{m}{m_0} = \frac{d}{d_0}(1 + gt)$$

and
$$\frac{m}{m_0} = \frac{1 + gt}{1 + ct}$$

since
$$\frac{d}{d_0} = \frac{1}{1 + ct}$$

where $c =$ coeff. of expansion of the liquid.

We can therefore find either c or g, provided the other is known. The solid can be made in the form of a glass bulb and its coefficient g

determined by the dilatometer method. It is then partially filled with lead shot to make it sink, the neck is sealed, and the whole hung from the arm of a balance by a thin wire and immersed in the liquid. In these methods it would be better to replace the glass by quartz since its expansion is more definite.[1]

Expansion of Water.—If a dilatometer containing water at 0° is gradually heated the liquid is seen to contract until a temperature near 4° is reached, after which it continually expands, showing that water has a maximum density near 4°. The exact temperature observed will depend on the expansion of the glass. Joule's method gives the temperature of maximum density directly. Two cylindrical vessels, A, B (Fig. 29), containing water, communicate below through a tube which can be closed by a tap, above through an open trough. Their temperatures are adjusted so that one is below and the other above 4°. If the density of the liquid is greater in A than in B, when the tap is opened a current sets in in the direction ACDBA, because the pressures at C and D are unequal. This is rendered evident by a small bead floating in the trough. Two temperatures are found at which water has the same density; these are altered until they are nearly equal and their mean taken as the temperature of maximum density.

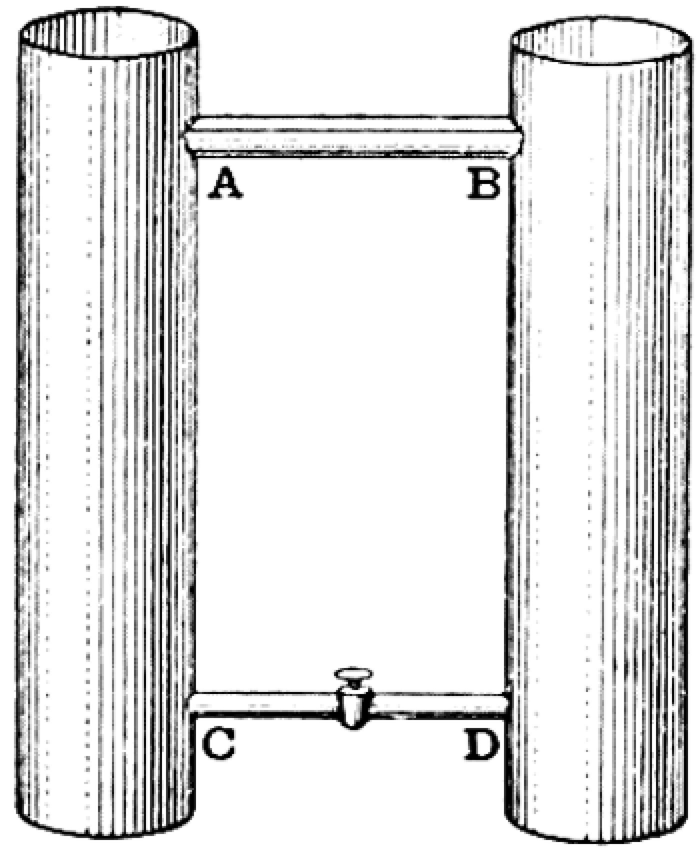


FIG. 29.—Joule's Method of finding the Temperature of Maximum Density of Water.

This singular behaviour of water has an important influence on animal and vegetable life. During a frost the surface layers of a pond are first cooled, they increase in density and sink, and are replaced by the warmer layers from below. This proceeds until the whole is reduced to 4°, when any further cooling produces a decrease in density. The water which is cooled below 4° consequently floats on the surface and is finally frozen, while the lower portions have a uniform temperature of 4°, thus protecting plants and animals from being frozen.

The currents set up in a cooling liquid are well shown in Hope's

[1] See also Barton and Black, "Practical Physics," p. 56.

apparatus, Fig. 30. Water at the room temperature is placed in the upright cylinder into which two thermometers project, and a freezing mixture of ice and salt is placed in the trough A. This causes the temperature of the lower thermometer to fall to 4°, without, at first, seriously affecting the upper one. The reading of the lower thermometer then remains stationary and that of the upper falls rapidly until it reaches zero.

Correction of a Barometer for Temperature.—The observed height of a barometer will vary with the temperature on account of the expansion of the scale and of the mercury; it must therefore be reduced to a standard temperature of 0°. Let H, H_0 cms. be the observed heights at t° and 0° respectively, d and d_0 the corresponding densities of mercury, l the coefficient of linear expansion of the scale, c the coefficient of cubical expansion of mercury. The graduations on the scale being supposed correct at 0°, the true length of the warm mercury column at t° is $H(1 + lt)$. The atmospheric pressure in gms. per cm.^{2} is thus $H(1 + lt)d$, while if it were measured by a barometer at 0° it would be $H_0 d_0$.

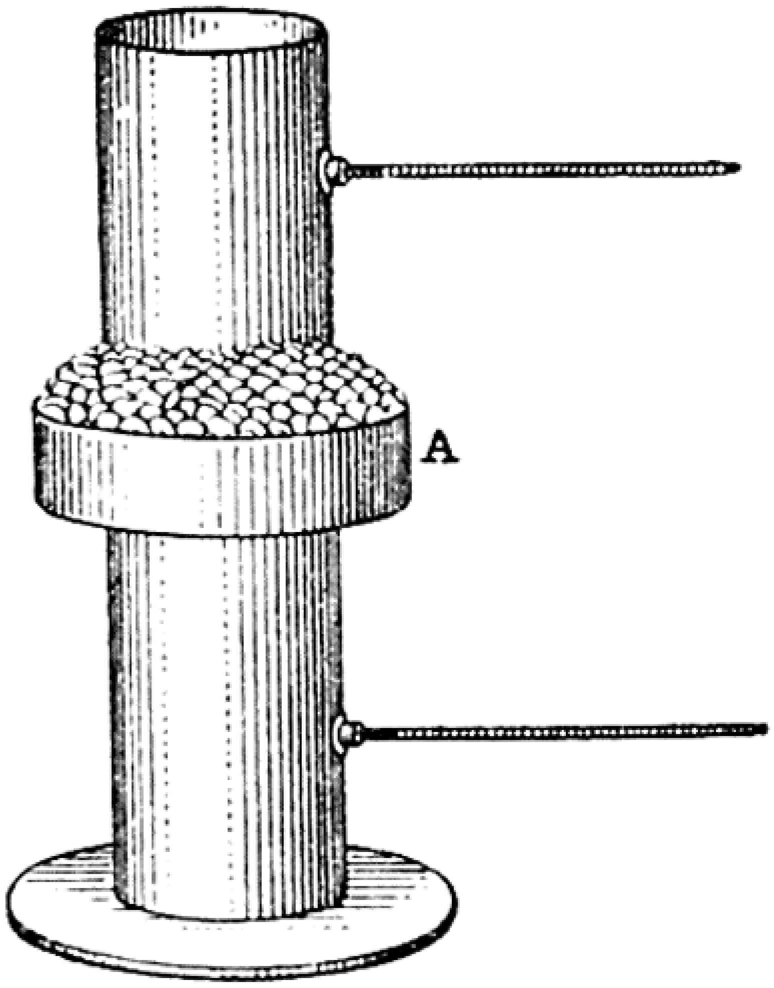


Fig. 30.—Hope's Apparatus.

$$\therefore H_0 d_0 = H(1 + lt)d$$

and
$$H_0 = H(1 + lt)\frac{d}{d_0}$$

But
$$\frac{d}{d_0} = \frac{1}{1 + ct}$$

$$\therefore H_0 = \frac{H(1 + lt)}{1 + ct}$$

or approximately
$$H_0 = H\{1 - t(c - l)\} \quad \text{(p. 40)}$$

To get some idea of the magnitude of this correction suppose the scale is brass, for which $l = 0{\cdot}000018$, the temperature 15°, and take the coefficient of cubical expansion of mercury to be 0·000181.

Then when the observed height H is 76 cms. the true height, reduced to 0° *c*, is 75·81 cms.[1]

Exposed Stem Correction for a Thermometer.—With the notation of p. 25 it is seen that if the mercury occupying n divisions is heated from t_2 to t, so as to be at the same temperature as the rest of the liquid, it will expand $n\sigma(t - t_2)$ divisions. This is the amount to be added to the observed temperature t_1 to get the true temperature t, or $t = t_1 + n\sigma(t - t_2)$. The correction should never be large as it is not very trustworthy, hence it will be sufficiently near the truth if t on the right side is replaced by the nearly equal temperature t_1, and the formula becomes $t = t_1 + n\sigma(t_1 - t_2)$.

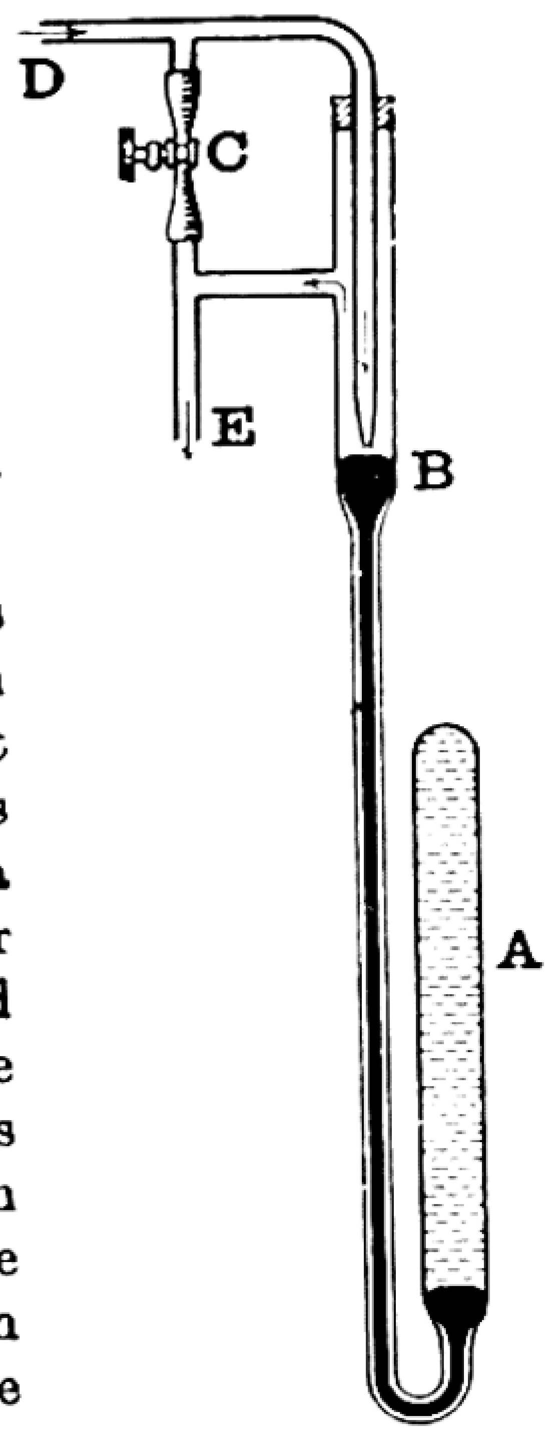


FIG. 31.—Gas Regulator.

Applications.—The expansion of a liquid is used to regulate the supply of heat to a bath which it is desired to maintain at a constant temperature. Fig. 31 shows one form of gas regulator. The glass bulb A is filled with a highly expansible liquid like toluene, the lower part of A and the narrow tube to B are filled with mercury. The apparatus is placed in the bath, which should be well stirred, and gas from the main enters at D, travels in the path shown by the arrows, and goes from E to the burner underneath the bath. When a certain temperature is reached the expansion of the toluol causes the mercury to close the tube B and cut off the gas. To save it from being extinguished a small bye-pass is provided at C which allows sufficient gas to pass to keep the flame alight. When the temperature falls the toluol contracts and the full supply of gas again passes through B. The temperature can be kept nearly constant for days by this device.

EXAMPLES ON CHAPTER V

1. Describe a method of measuring the coefficient of expansion of a metal rod. A solid at 0° when immersed in water displaces 500 cub. in.; at 30° it

[1] For further corrections see Barton and Black, "Practical Physics," p. 58.

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displaces 503 cub. in. Find its mean coefficient of linear expansion between 0° and 30°. (L. '80.)

2. Find the value in grams weight, and in dynes per cm.^{2}, of a pressure able to sustain a 50 cm. column of mercury at 0°. Find what pressure would be exerted by the same height of mercury at 100°, if its density at 0° be 13·6 and its mean coefficient of expansion be 0·00018. (L. '90.)

3. A glass bulb with a fine uniform stem weighs 10 gms. when empty, 117·3 gms. when the bulb only is filled with mercury, and 119·7 gms. when a length of 10·4 cms. of the stem is also filled with mercury. Calculate the relative coefficient of expansion for temperature of a liquid which, when placed in the same bulb, expands through the length from 10·4 to 12·9 cms. of the stem when warmed from 0° to 28°. The density of mercury is 13·6 gms. per cm.^{3} (L. '89.)

4. A mercury thermometer at 0° contains 2 c.c. of mercury and the distance between the fixed points is 30 cms. Calculate the diameter of the tube at 0° given the coefficient of cubical expansion of mercury is 0·00018 and of glass is 0·00003. (L. '09.)

5. A specific gravity bottle holds 50 gms. of water at 4°. How much will it hold at 40° if the mean coefficients of cubical expansion for glass and water between 4° and 40° are 0·00003 and 0·00027 respectively?

6. Describe some method by which the expansion of water has been studied. If S be the expansion of water between 4° and 0° and Δ its expansion between 4° and t°, show what is the density of water at t° referred to water at 0°. (L. '84.)

7. The height of a barometer as read by a brass scale at a temperature 18° was 760 mm. Find the true height reduced to 0°. The coefficients of cubical expansion of brass and mercury respectively are 0·0000552 and 0·000181.

CHAPTER VI

EXPANSION OF GASES. GAS THERMOMETERS

Volume and Pressure Coefficients.—Since the volume of solid or liquid bodies depends but slightly on the pressure to which they are subjected, any pressure variations can be neglected when we are dealing with their thermal expansion. This is not so for gases, as Boyle's law shows, hence the effect of a rise in temperature on the state of a gas is usually observed under two different conditions: (1) The pressure is kept constant and the alteration in volume is observed, or (2) the volume is kept constant and the variation in the pressure is measured. In the first case we measure the coefficient of expansion at constant pressure, or, more briefly, the volume coefficient; in the second we find the coefficient of pressure increase at constant volume, or the pressure coefficient. Experiment shows that these coefficients are much larger than any with which we have dealt in the preceding chapters, hence the approximate methods of calculation (p. 40) are no longer applicable; the increase in volume or pressure must be compared with the volume or pressure, as the case may be, at 0° C. **The coefficient of expansion at constant pressure** is defined as **the ratio of the increase in volume for 1° rise in temperature to the volume at 0°**, the pressure remaining constant. If V is the volume at a temperature $t°$ and V_0 that at 0°, the coefficient $\alpha = \frac{V - V_0}{V_0 t}$, or $V = V_0(1 + \alpha t)$. Similarly **the pressure coefficient is the ratio of the increase of pressure for 1° rise of temperature to the pressure at 0°**, the volume being kept constant. Denoting this by β, and the pressures at 0° and $t°$ by P_0 and P respectively, we have $\beta = \frac{P - P_0}{P_0 t}$, or $P = P_0(1 + \beta t)$.

Expansion at Constant Pressure.—Before describing the more elaborate experiments of Regnault we will give two simple laboratory

methods for measuring the expansion of air at constant pressure; the first corresponds to the dilatometer method of p. 49, the second to the specific gravity method of p. 54.

EXPERIMENT.—The apparatus consists of a piece of fairly wide capillary tube, about 70 cms. long, closed at one end; to the other end a wider tube is attached as in Fig. 32. The tube is heated in a Bunsen flame to dry it thoroughly before the end is closed, after closing it is slightly heated and a few drops of mercury or strong sulphuric acid are introduced into the wide portion. As the air cools a thread of liquid about 2 cms. long is sucked into the capillary; this serves as a piston to confine the gas and as an index to show its volume. The tube is now fastened to a graduated scale and placed horizontally in a long trough where it is surrounded with melting ice. The length of the air column is noted

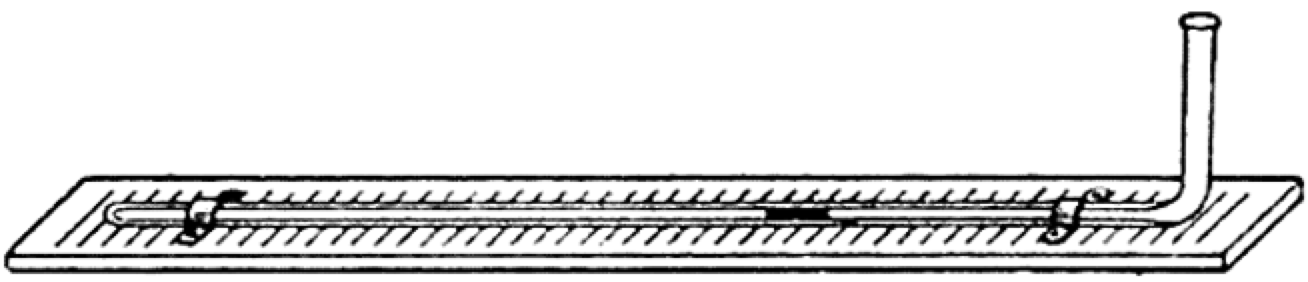

FIG. 32.—Apparatus to determine the Coefficient of Expansion of a Gas at Constant Pressure.

at this temperature, then the trough is heated and the volume of the air is noted at different temperatures. The coefficient may be calculated from the preceding formula. Measure in this manner the expansion of air, hydrogen, and carbon dioxide.

Gay Lussac used apparatus which, in principle, was the same as this; the figure he obtained for α was 0·00375. Regnault afterwards showed that this was too high, chiefly on account of the insufficient drying of the tube. When damp air is heated the particles of water are converted into steam which occupies a much larger volume, the expansion is therefore too large.

EXPERIMENT.—A flask of about 60 cm.^{3} capacity is tightly closed by a rubber stopper through which is passed a short piece of capillary tube. The latter carries at its outer end a piece of rubber tube about 3 cms. long which may be closed by a pinch-cock. The apparatus is weighed after thoroughly drying, and is then placed, neck upwards, in a large beaker of water which is gradually brought to a temperature of about 60° C. as measured by a thermometer. Keeping the temperature as steady as possible the flask is pushed down until the hot water rises just below the level of the stopper, it is held there for a minute and the pinch-cock is closed. It is then taken out and inverted as quickly as possible, with the neck well immersed, in a large vessel of cold water. The pinch-cock is then opened. Owing to the fall of temperature the contained air contracts and water runs into the flask. Ice is added to the vessel until a large excess remains unmelted, when the temperature should be 0°. The flask is

now immersed to such a depth that the level of the water is the same inside and outside and the pinch-cock is closed. (To prevent heating by the hand during this operation the flask should be held by a cloth which has been thoroughly wetted in the ice-cold water.) The confined air is now at atmospheric pressure and at 0°. The flask is removed, dried carefully on the outside and reweighed; finally it is completely filled with water and its weight again determined. Let m be the mass of water in gms. that runs in when the cock is opened in the vessel at 0°, and M that required to fill the flask. Then the volume of air in the flask at 0° is $(M - m)$ c.cms., and this completely filled it when the cock was closed at 60°. Hence a mass of air, whose volume at 0° is $(M - m)$ c.cms. expands to M c.cms. at 60°, the pressure remaining constant. The expansion is $M - (M - m) = m$ c.cms., and the coefficient $a = \frac{m}{(M - m)60}$.[1]

Regnault used a more elaborate apparatus based on this principle, but great precautions were taken to dry the air and the flask. He obtained a value $a = 0{\cdot}00366$. In each of the above experiments the coefficient of cubical expansion of glass should be added to the calculated result, p. 49. We will now describe one method used by Regnault to determine a.

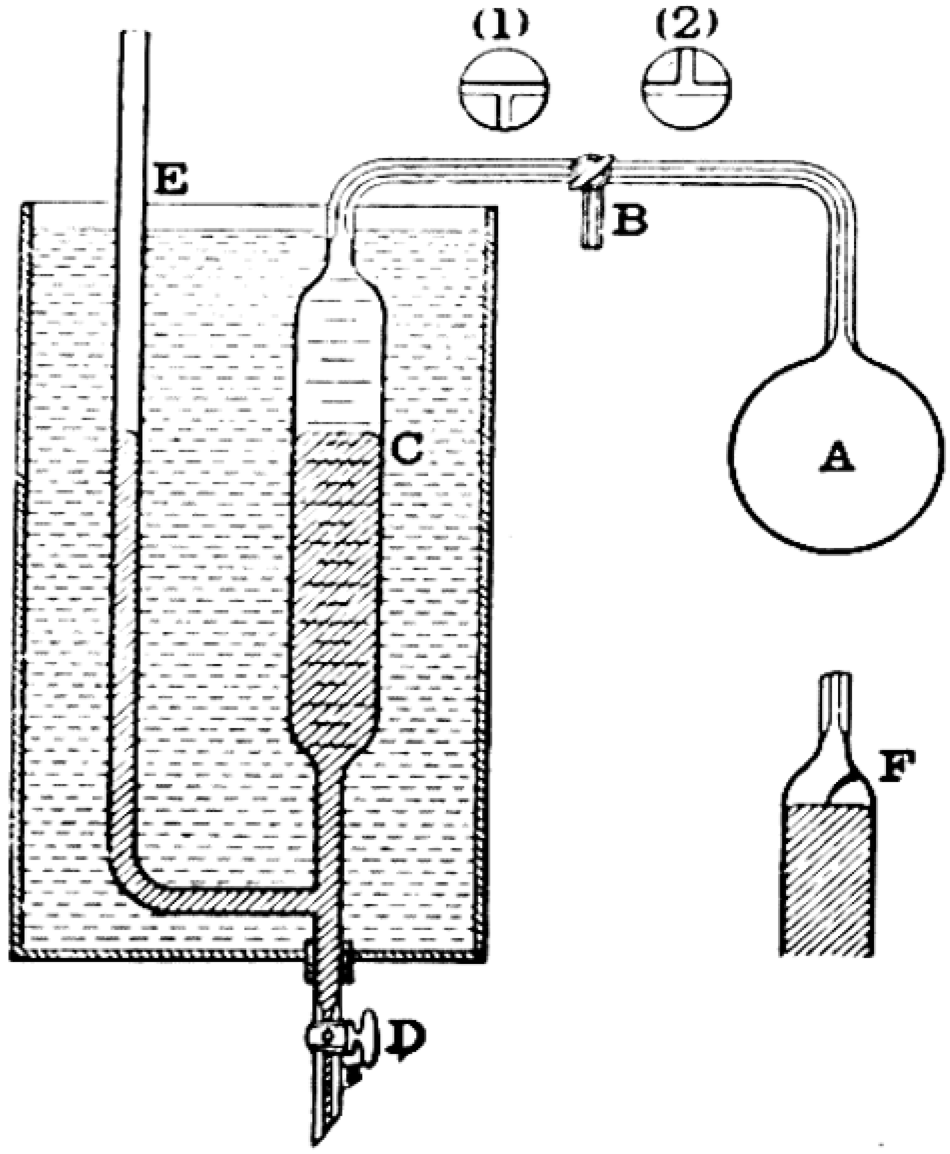

FIG. 33.—Regnault's Apparatus to determine the Expansion of Gases at Constant Pressure.

Regnault's Experiments.—The bulb A, whose volume had previously been found by filling it with mercury, was attached by capillary tubing to a three-way cock B (Fig. 33), and a graduated glass cylinder C immersed in water. The cylinder could be made to communicate with the external air by means of a cock D. E was closed, B was turned into the position shown at (1), and the bulb A and the space above the mercury in C was thoroughly exhausted of air through the tap. Air was then readmitted through drying tubes, and the process of exhaustion and refilling was repeated

[1] See also Barton and Black, "Practical Physics," pp. 59–63.

several times, the bulb being heated in the meantime to ensure that the last traces of moisture were removed. Tap B was next turned into the position shown at (2), so that the tubes E and C and bulb A were all in communication and E was opened. The bulb was then surrounded with melting ice, and, by adding mercury to E or running some out through D, the level in C was adjusted to be near the top of the graduations. The difference in level of the mercury in E and C was finally read by a cathetometer; by adding this to or subtracting it from the barometric height the pressure of the gas at 0° was found. A was then placed in steam; the air expanded and mercury was run out through D into a vessel placed immediately below until the pressure was the same as before. The amount of expansion was then given directly by the graduations on C. Corrections had to be made: (1) Because the gas that expanded into C was not at the same temperature as that in the bulb; (2) For the air in the capillary tube; (3) For the expansion of the bulb. Evidently the initial pressure could be increased by adding more mercury to E, hence the expansion under different pressures could be measured. Some of the results at atmospheric pressure are given on p. 63.

Increase of Pressure at Constant Volume.—Regnault measured the increase of pressure at constant volume by a similar apparatus, except that the water-bath and the graduations on C were unnecessary. The bulb was filled with dry gas in the manner already described, and the mercury in C was adjusted so that the surface just touched the tip of a small enamel pointer when the temperature was 0°. This pointer is shown at F, Fig. 33. The pressure on the gas at 0° was then found as in the previous experiment. Steam was next passed round the bulb, and, as the pressure increased mercury was poured into E until the surface in C resumed its former position. The gas was thus brought back to its initial volume, except for a small correction arising from the expansion of the bulb, and the new pressure was noted. The pressure coefficient was calculated from the formula $P = P_0(1 + \beta t)$. Its value for different gases is shown in the table below.

Table showing α and β for Different Gases.

Name of gas.	Coefficient at constant pressure (α).	Coefficient at constant volume (β).
Air	0·00367	0·00367
Nitrogen	0·00367	0·00367
Hydrogen . . .	0·00366	0·00366
Oxygen	—	0·00367
Carbon dioxide . . .	0·00374	0·00372
Sulphur dioxide . .	0·00391	0·00386
Helium	—	0·00366

A glance at the table shows that the coefficient of expansion under constant pressure is practically the same for all gases. This was first noted by Charles, who expressed his results in the law known by his name: **At constant pressure the coefficients of expansion of all gases are equal.** As the table shows, this common coefficient is 0·00366 or 1/273. The law is not quite true as the numbers show; those gases which depart most widely from Boyle's law are more expansible than the law requires, *e.g.* carbon dioxide and sulphur dioxide. More extensive experiments establish the fact that at lower pressures or higher temperatures these gases also approximate to a condition in which both Boyle's and Charles' laws are obeyed. We are thus led to the notion of an ideal gas which obeys each of these laws accurately; such a substance is called a **perfect gas.** Although there is no substance which actually fulfils these conditions, yet for many purposes the more permanent gases like air, hydrogen, nitrogen, oxygen and helium may be treated as such. Accordingly in the following pages, unless otherwise stated, we shall regard these gases as perfect. Further reference to the above table also brings out the fact that for perfect gases the volume and pressure coefficients are equal.

Gas Thermometers.—The expansion of a gas at constant pressure may be used to measure temperature. In this respect it possesses various advantages over mercury; the expansion being larger it is more easily observed and the possibly irregular expansion of the bulb has less effect. In addition it could be used for much higher or lower temperatures, and two thermometers in which different gases

were used would agree over a very wide range of temperature. The latter is not usually the case with liquids. For example, let two liquid-in-glass thermometers be constructed and graduated as in Chap. II., one containing mercury, the other, say, heavy petroleum oil. From the method of graduation they will, of course, agree at the fixed points, but very probably not at any other temperature. This is because the ratio of the coefficients of expansion of the two liquids varies with the temperature, and hence a rise in temperature sufficient to make the mercury expand from the 100th to the 200th degree mark on the scale might be insufficient to cause the oil in its thermometer to do likewise. The ratio of the coefficients of expansion of gases is much nearer being constant. The student is especially warned against making the statement that mercury is used as the usual thermometric substance because its expansion is uniform. If the expansion of mercury is measured, *using a gas thermometer to read the temperatures*, its coefficient increases as the temperature rises. The apparatus of Fig. 33 could be used as an air thermometer, but it would be very cumbersome, and a considerable fraction of the gas, viz. that in C, would be at a lower temperature than the bulb, thus involving a troublesome correction similar to that for the exposed stem (p. 25). The latter disadvantage is largely overcome if the increase of pressure at constant volume is measured instead of the volume expansion. For these reasons the constant volume hydrogen thermometer is used as the standard to which all other thermometers are referred for comparison; its unwieldiness is not then a serious disadvantage, since it can be set up once for all in the standardising laboratory. Fig. 34 shows a simple apparatus for measuring the pressure of a gas at different temperatures, the volume being kept constant. The mercury reservoir B, which is connected to C by rubber tubing, can be raised or lowered in order to keep the volume of the gas constant. The difference in level of the mercury in the two limbs can be read directly off the graduated scale. This difference added to or subtracted from the barometric height gives the gas pressure in cms. of mercury.

EXPERIMENT.—Measure with this apparatus the coefficient β for air between 0° and 100° cent. Then exhaust with an air-pump through the three-way tap, fill with dry hydrogen and repeat the observations. Neglect in each case the expansion of the bulb. The volume of the space above the mercury in C should be small in order that practically all the gas may be at the same temperature.

Assuming we have no other thermometers let us see how such an

apparatus can be used to construct a temperature scale and act as a thermometer whose indications shall be comparable with those of any other similar arrangement. Having filled the bulb with dry air or hydrogen, immerse it first in melting ice, then in the steam from boiling water, and observe the pressure at constant volume in each case. Let these pressures be represented on the diagram (Fig. 35) by AC and BD respectively. As in Chap. II., we will call these temperatures 0° C. and 100° C. Join CD by a straight line and draw CO parallel to AB; then DO represents the increase in pressure due to a rise in temperature of 100°. Just as we did with mercury thermometers we may now divide this fundamental interval DO into one hundred equal parts, and define 1° of temperature as that necessary to raise the pressure by one of these divisions. Let the bulb be now immersed in a bath whose temperature it is desired to read, and suppose the pressure of the gas to be represented by QB; then the temperature is above 0° by an amount measured by OQ, where DO represents 100°. It is therefore equal to $\frac{OQ}{OD} \cdot 100°$. We should get the same result by drawing QE parallel to AB and EF parallel to CA, the temperature would then be represented by AF, where AB represents 100° and A is the zero point, for OQ/OD = CE/CD = CM/CO = AF/AB. If P_0, P_{100}, and P are the pressures at 0°, 100°, and t° respectively, then the unknown

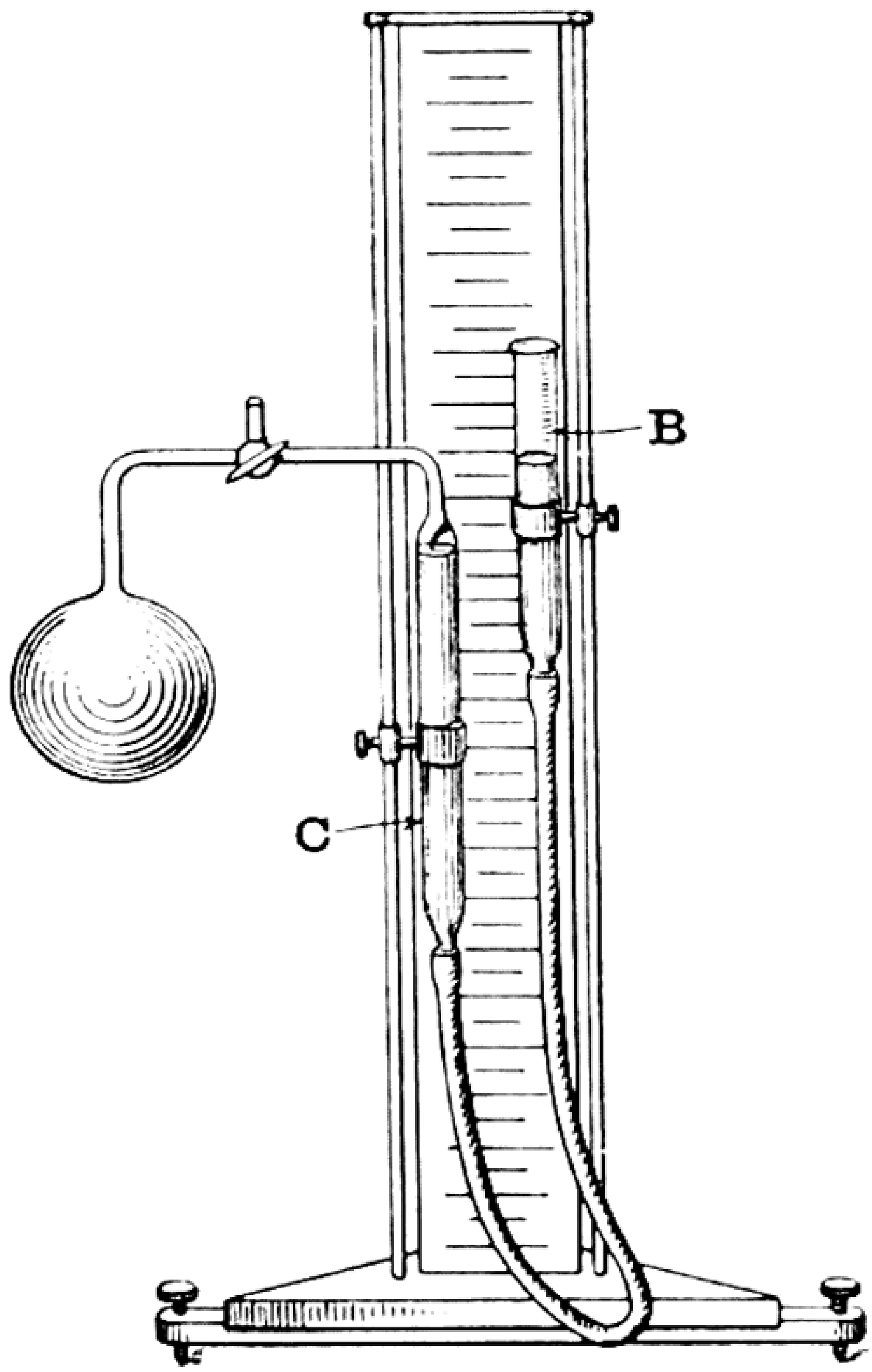

FIG. 34.—Simple Constant Volume Air Thermometer.

temperature $t = \frac{OQ}{OD} \cdot 100$[1] $= \frac{P - P_0}{P_{100} - P_0} \cdot 100$; hence to determine it we have merely to observe the pressure P after the fundamental interval has been determined once for all. Evidently the same formula could be used to measure temperatures beyond the limits 0°–100°; *e.g.* if QB is greater than BD or less than OD the point F will lie on AB produced. A thermometer based on this principle is the standard to which all others are referred; one such is described on p. 67.

EXPERIMENT.—Place a few scraps of paraffin wax in a thin capillary tube, fasten it to the bulb of a constant volume air thermometer (Fig. 34) and immerse the two in a beaker of water. Heat the water slowly, keeping the volume of

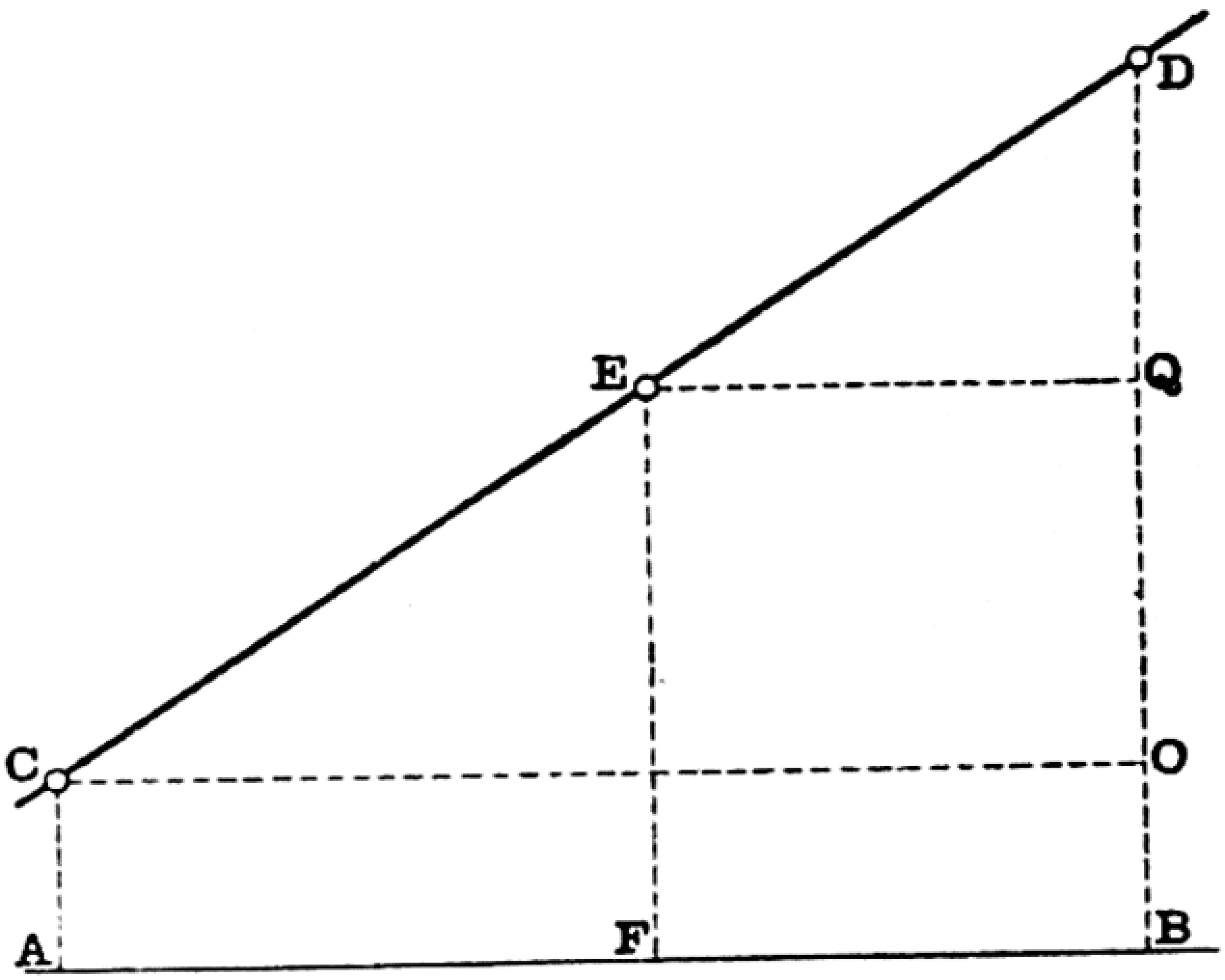

FIG. 35.

the air constant, and note the difference of level of the mercury in the two limbs at the moment the wax melts. Let it cool, or add small amounts of cold water if this is too slow, and read the pressure when the wax solidifies. Take the mean of these as the pressure of the gas at the melting point of paraffin wax. Determine the fundamental interval ($P_{100} - P_0$) as in the last experiment and calculate the temperature at which wax melts from the formula $t = \frac{P - P_0}{P_{100} - P_0} \cdot 100$.

Absolute Zero.—Whether we use the volume or the pressure of a gas to measure temperatures it is easily seen that theoretically there is a minimum temperature below which the thermometer cannot be used. Thus if P_1 and P_0 are the pressures at $t°$ and 0° respectively,

[1] $OQ = BQ - OB = P - P_0$, $OD = BD - OB = P_{100} - P_0$.

$P = P_0(1 + \frac{1}{273} \cdot t)$; let the temperature be supposed to decrease to $-273°$ C., then the pressure would be $P = P_0(1 - 1) = 0$, *provided the gas were perfect throughout this range.* This temperature is therefore the lowest that could possibly be read on such a thermometer, it is called **the absolute zero of the perfect gas thermometer.** Temperatures within a few degrees of this have actually been reached in recent years, but such extreme cold is found to liquefy or even solidify all gases when, of course, they cease to behave in the manner supposed. No stage is realised in practice at which the pressure of a gas is zero, nevertheless the idea of such a zero of temperature is found to be very useful. According to the kinetic theory the gas molecules at this temperature are absolutely devoid of heat, it is therefore the lowest conceivable. The temperature of a body reckoned from this point as zero is called its **absolute temperature**; on this scale ice melts at 273° absolute, water boils at 373° absolute, and generally, if T and t are the corresponding absolute and Centigrade temperatures of a substance, $T = 273 + t$.

Standard Gas Thermometer.—To measure the pressure with the apparatus shown in Fig. 34 four readings of mercury surfaces are necessary, viz. those in B and C, and the zero and upper surface of the barometric column. They are reduced to two in the standard gas thermometer shown in Fig. 36, and the possible error is thus halved. In this apparatus the vessel B of the simpler form is made the reservoir of the barometer H, and the barometer tube is bent round so that the upper end of the mercury column is vertically above the index at C. When the gas pressure in the bulb is equal to that of the atmosphere the mercury stands at the same level in the three limbs C, B, E, and the vertical distance CH is the height of the barometer. As the temperature of the bulb A increases, the reservoir E is raised to keep the volume of the gas constant; the level of the mercury therefore rises to B′, E′ and H′ respectively in the other tubes, and B′H′ is now the barometric height. The gas pressure is then CH′ cms. of mercury, and this can readily be measured by a cathetometer. The bulb A, about one litre in capacity, is made of an alloy of platinum and iridium; it communicates with C through a very fine metal tube. When it is desired to obtain a correction curve for a mercury thermometer the bulb A and the thermometer are immersed in a well-stirred bath and their readings at different temperatures are compared: a curve can then be constructed as on p. 26.

The Gas Equation.—Let v_1 and v_2 be the volumes of a given mass of gas at temperatures $t_1°$ and $t_2°$ Cent., the pressure remaining constant, then from Charles' law, if v_0 is the volume at 0°,

$$v_1 = v_0(1 + \tfrac{1}{273} \cdot t_1)$$
$$v_2 = v_0(1 + \tfrac{1}{273} \cdot t_2)$$

or
$$\frac{v_1}{v_2} = \frac{273 + t_1}{273 + t_2} = \frac{T_1}{T_2}$$

where T_1, T_2 are the absolute temperatures corresponding to t_1 and t_2. Thus at constant pressure the volume of a perfect gas is proportional to its absolute temperature. Similarly if P_1 and P_2 are the pressures at t_1 and t_2, the volume being kept constant,

$$P_1 = P_0(1 + \tfrac{1}{273} \cdot t_1)$$
$$P_2 = P_0(1 + \tfrac{1}{273} \cdot t_2)$$
$$\therefore \frac{P_1}{P_2} = \frac{273 + t_1}{273 + t_2} = \frac{T_1}{T_2}$$

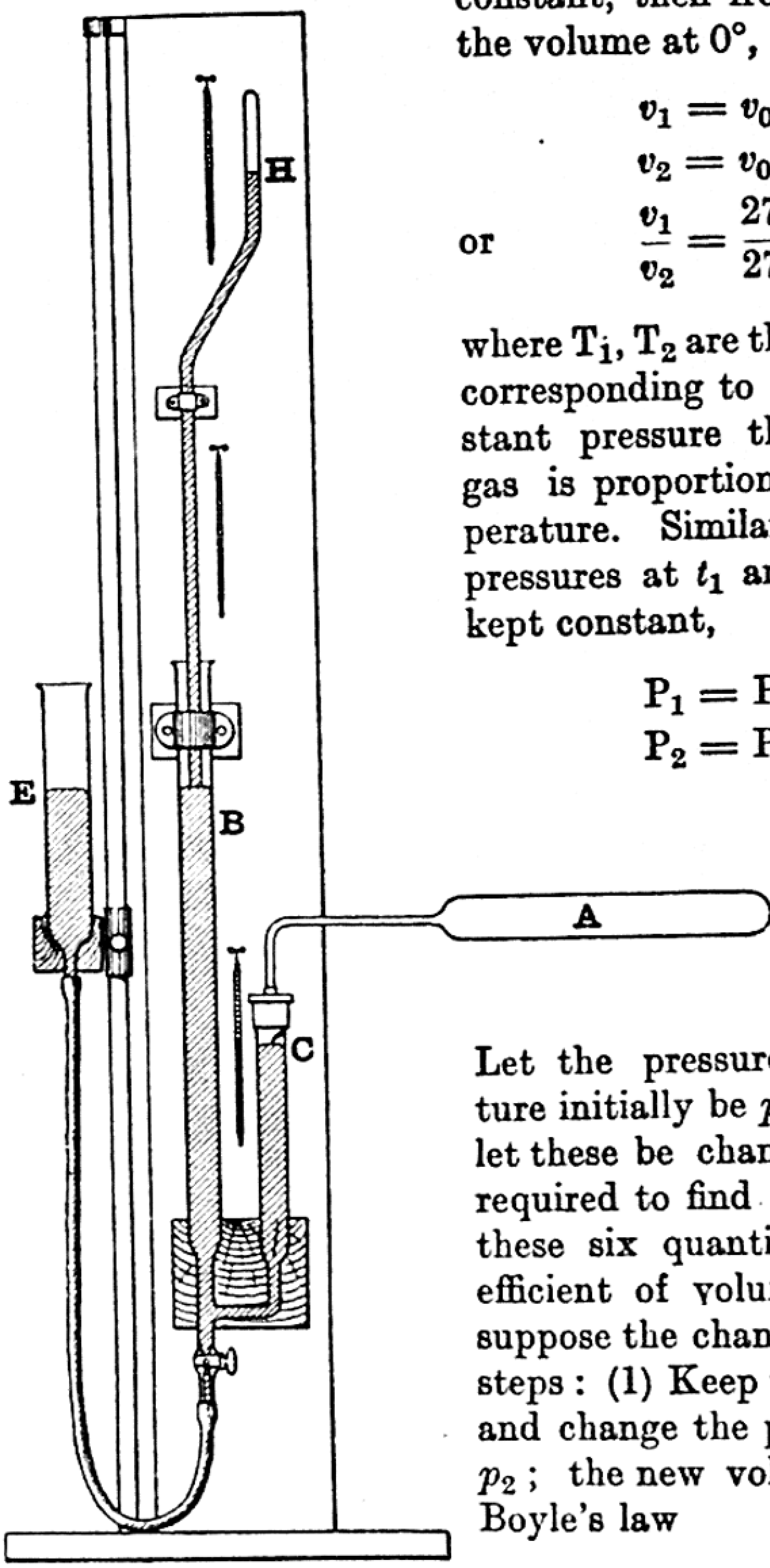

FIG. 36.—Standard Hydrogen Thermometer.

These results may be combined with Boyle's law in one equation. Let the pressure, volume and temperature initially be p_1, v_1, t_1 respectively, and let these be changed to p_2, v_2, t_2; it is required to find an equation connecting these six quantities. Let a be the coefficient of volume expansion. We may suppose the change to take place in two steps: (1) Keep the temperature constant and change the pressure to its final value p_2; the new volume v will be given by Boyle's law

$$p_2 v = p_1 v_1$$

or
$$v = \frac{p_1 v_1}{p_2}$$

(2) Keep the pressure constant at p_2 and heat the gas to a temperature t_2, thus changing the volume from v to its final value v_2. Then, as above,

$$\frac{v}{v_2} = \frac{1 + at_1}{1 + at_2}$$

or

$$v = \frac{1 + at_1}{1 + at_2} \cdot v_2 = \frac{p_1 v_1}{p_2}$$

when the value previously obtained for v is substituted. This may be written

$$\frac{p_1 v_1}{1 + at_1} = \frac{p_2 v_2}{1 + at_2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If $a = \frac{1}{273}$ this becomes $\dfrac{p_1 v_1}{273 + t_1} = \dfrac{p_2 v_2}{273 + t_2}$

or

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where T_1 and T_2 are absolute temperatures. If the quantities on the right-hand side refer to 0° Cent.

$$\frac{p_1 v_1}{T_1} = \frac{p_0 v_0}{273}$$

The second fraction is constant for a given mass of gas, hence the pressure, volume, and absolute temperature of a perfect gas obey the equation

$$\frac{pv}{T} = R$$

where R is a constant quantity called the gas constant. The equation (2) is called the gas equation; it should be used for all questions relating to the pressure, volume, or temperature of a perfect gas which arise on the subject matter of this chapter. If a is not equal to 1/273 equation (1) above must be used instead. It is easily seen to contain all the results we have previously obtained, for if T is constant the equation becomes $pv = \text{const.}$, which is Boyle's law. Similarly if p is constant the equation shows that the volume is proportional to the absolute temperature, or if v is constant the pressure is proportional to the absolute temperature.

EXAMPLE.—A mass of gas which occupies one litre at 20° C. under a pressure 76 cms. of mercury is heated to 100° C. and the pressure is reduced to 75 cms.; find the new volume.

The absolute temperatures are 293° and 373°. Using the gas equation and calling v_2 the volume required,

$$\frac{75v_2}{373} = \frac{76 \times 1}{293}$$

whence $$v_2 = 1{\cdot}29 \text{ litres.}$$

EXAMPLES ON CHAPTER VI

1. A litre of air at 0° and 76 cms. pressure weighs 1·293 gms. Find the weight of 5 litres when the temperature is 20° and the pressure 75 cms.

The weights of a litre of air under the two sets of conditions are proportional to the densities. Writing the gas equation in the form

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \quad \left(\text{since } v \propto \frac{1}{\rho}\right)$$

if x is the weight of a litre under the second conditions

$$\frac{x}{1{\cdot}293} = \frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \cdot \frac{T_1}{T_2} = \frac{75}{76} \cdot \frac{273}{293}$$

$\therefore$ the weight of 5 litres $= 5x = 5 \times 1{\cdot}293 \times \frac{75}{76} \times \frac{273}{293}$

2. Determine the height of the barometer when a mgm. of air at 27° C. occupies a volume of 20 cms.^{3} in a tube over mercury, the mercury standing at 73 cms. higher inside the tube than outside. [1 gm. of air at N.T.P. measures 773·4 cms.^{3}.] (L. '85.)

3. At the sea-level the barometer stands at 750 mm. and the temperature is 7°, while on the top of a mountain the barometer stands at 400 mm. and the temperature is —13°. Compare the weights of a cubic metre of air in the two places. The barometer readings may be taken as corrected for temperature. (L. '89.)

4. A given volume of air is at 740 mm. pressure at 17° C. What is the temperature when its pressure is 1850 mm.? (L. '93.)

5. State in symbols and in words the two laws which, if a gas obeys, it is called a perfect gas. One lb. of air at a temperature 0° and at a pressure of 1033 gms. per cm.^{2} has a volume of 0·3555 cub. metres. At what pressure will its volume be 403,700 c.c. if measured at 27° C.? (L. '97.)

6. Explain how the apparent weight of a body in air varies with its rise of temperature. A piece of iron measuring 1000 c.c. is weighed at 0° and again at 100° C. What will be its apparent change in weight? Coefficient of expansion of air = 0·00367, of iron (linear) = 0·000012, mass of 1000 c.c., of air at 0° = 1·293 gms. (L. '86.)

7. Define the coefficient of increase of pressure of a gas. Show that, if a gas obeys Boyle's and Charles' laws, this coefficient is equal to the coefficient of expansion. (L. 1900.)

8. Find the number of feet in a steel bottle to hold at 120 atmospheres pressure, when the temperature is 25° C., 20 cub. ft. of oxygen under normal conditions. (L. '03.)

9. The mercury in a barometer containing some air stood at a height of 70 cms. and the volume of the tube above the mercury was 20 c.c. The tube was then lowered into the reservoir until the volume above the mercury was 10 c.c., when the barometer indicated 65 cms. only. Calculate (1) the true barometric height, and (2) what the reading of the barometer in question would be if its tube were raised until the volume above the mercury became 100 c.c. (L. '06.)

10. A sample of gas was found to have a volume of 100 c.c. at 18° and 72 cms. pressure, and a volume of 200 c.c. at 90° and 45 cms. pressure. Assuming that the gas obeys Boyle's law and expands uniformly at constant pressure, calculate at what temperature it would have a volume of 400 c.c. at 100 cms. pressure. (L. '07.)

11. A volume of 50 c.cs. of air at 15° is expelled from the bulb of a constant pressure air thermometer by changing the temperature from 0° to 100° C. Given the coefficient of expansion of air is 1/273, calculate the temperature of the thermometer when 10 c.cs. are expelled, neglecting the expansion of the bulb. (L. '08.)

CHAPTER VII

CHANGE OF STATE

Melting and Boiling Points.—When a solid like ice or paraffin wax is continually heated a temperature is finally reached at which it liquefies; this temperature is called the **melting point** of the substance. Provided the pressure is unchanged a substance always melts at the same temperature, this provides the chemist with a means of identification. Similarly when a liquid is continually heated a stage is reached where the temperature remains steady and the liquid is continuously converted into vapour.[1] The liquid is then said to **boil**. The temperature at which boiling takes place is called the **boiling point**. It also is characteristic of the substance but varies greatly with the pressure. These changes can take place in the inverse order; thus if steam is cooled it finally condenses into liquid, and the water so formed, if its temperature is sufficiently reduced, at last solidifies or freezes. Except when chemical change is produced the condensing point coincides with the boiling point, and the freezing point with the melting point. Certain substances, such as glass, have no well-defined melting point, in changing from the solid to the liquid state they pass through an intermediate pasty condition; it is this property which makes it possible to work with glass in the blow-pipe flame. Other substances, such as iodine, when heated pass directly from solid to vapour without becoming liquid; they are said to **sublime**. The reverse change, directly from vapour to solid, occurs in the case of hoar frost.

EXPERIMENT.—Put a quantity of melting ice in a beaker, in a second beaker place an equal weight of cold water, and place each of them over Bunsen burners. It will be found that the temperature of the water rises continuously to the boiling point, but the temperature in the other beaker remains steadily at 0° until all the ice is melted, after which it increases as in the other vessel.

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[1] A vapour may be defined as the gaseous form of a liquid.

This is a typical case of melting; the temperature of the melting substance is constant during the process. As heat is entering from the flame it is clear that the solid absorbs heat without changing its temperature; this heat is said to be **latent.**

The number of calories required to convert one gram of a solid into a liquid without changing its temperature is called the latent heat of fusion of the substance.—For example, the latent heat of fusion of ice is 80 calories; *i.e.* the heat necessary to liquefy one gram of ice would raise the temperature of a gram of water from 0° to 80°. Other solids behave in a similar manner, but the actual value of the latent heat of fusion varies with the substance. Before a liquid at the freezing point can solidify it must part with its latent heat of fusion, while it is doing this its temperature remains steady; upon this fact is founded a method of determining the freezing point (p. 74). Similar absorptions or evolutions of heat are shown at the boiling point.

EXPERIMENT.—Heat over Bunsen burners two small flasks, one containing mercury, the other water. When 100° C. is reached the water is gradually converted into steam and its temperature is constant, that of the mercury steadily rises beyond this point. Heat is absorbed in each case, that entering the mercury causes a rise in temperature, while that absorbed by the water at 100° becomes latent; this latent heat is used in producing steam. Finally at a temperature near 350° the mercury also boils and absorbs latent heat.

The number of calories required to convert one gram of a liquid into vapour without changing its temperature is called the latent heat of vaporisation of the liquid.

Methods of determining the Melting Point.—When a substance shows a clearly defined melting point, or is obtainable only in small quantities, the following method may be used.

EXPERIMENT.—A few small particles of the substance are placed in a thin walled capillary glass tube which is attached to a thermometer bulb by rubber bands. This is mounted in a test-tube and placed in a beaker of water as in Fig. 37. The water is slowly heated and the temperature at which the substance melts is observed; the whole is then allowed to cool and the temperature of solidification noted. These observations are repeated until the two temperatures differ by only a few tenths of a degree, when their mean is taken as the melting point. The air currents in the test-tube ensure that the bulb and capillary tube are at the same temperature.

Another method is used where it is difficult to tell by the eye when melting actually takes place. The substance is thoroughly melted

and allowed to cool slowly, the temperature being observed every 10 secs., a cooling curve is then plotted showing the temperature at different times. When solidification begins the liquid gives out its latent heat of fusion and the temperature remains steady for some time, this is shown clearly on the cooling curve. The method is largely used by metallurgists to find the freezing points of metals; as the temperatures are much higher than can be read by a mercury thermometer a thermo-couple is used in place of it. (See p. 431.)

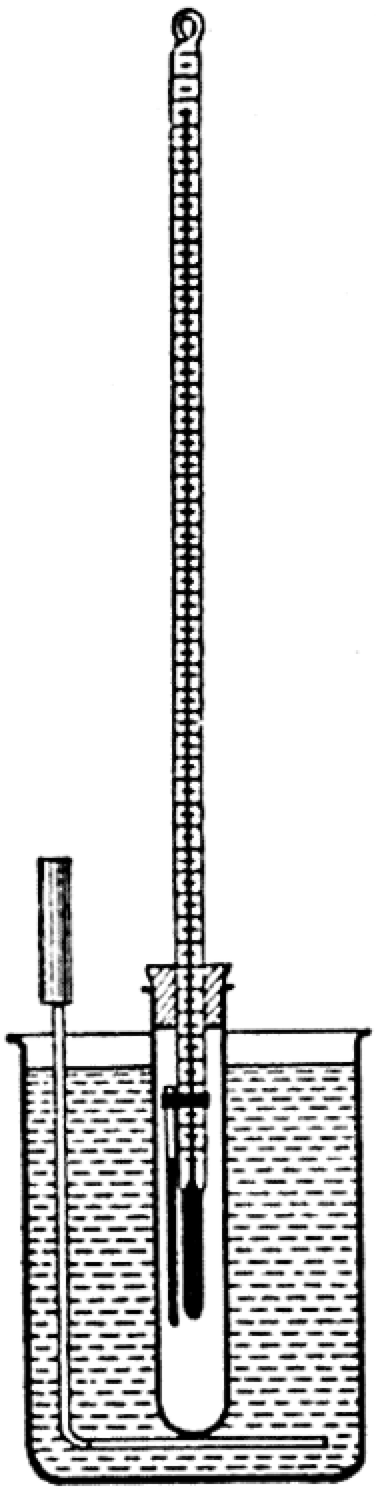


FIG. 37.—Apparatus to determine Melting Points.

EXPERIMENT.—Some tinman's solder was melted in a large crucible and a hard-glass test-tube containing mercury was pushed into it. A thermometer was placed in the mercury and the cooling curve shown in Fig. 38 was obtained. The mercury provided good contact with the solder and yet preserved the thermometer from breakage when solidification took place. Owing to solder containing two metals, freezing takes place in two steps at temperatures near 194° and 178°. The student should obtain by this method the melting point of paraffin wax.

The curve shows that it is possible to cool a liquid below its freezing point without causing it to solidify, but directly solidification begins the temperature rises to that of the normal freezing point and remains steady until the change of state is completed (at 194° in Fig. 38). A substance cooled below its freezing point, yet remaining fluid, is said to be **supercooled.** The "hypo" used by photographers shows supercooling exceedingly well.

EXPERIMENT.—Powder some "hypo" in a mortar and place it round the bulb of a thermometer in a test-tube. Heat it in a beaker of water until it melts in its own water of crystallisation; this takes place at about 48°. The test-tube may then be removed from the beaker and allowed to cool, a cooling curve being obtained in the usual way. If it is not shaken the temperature may fall to 30° without the "hypo" solidifying. Finally when solidification begins the temperature rises suddenly to 48° and remains steady for some minutes. If a solid crystal is dropped into the supercooled liquid solidification begins at once. The absence of dissolved air from a liquid makes it more capable of being supercooled.

Latent Heat of Fusion.—The latent heat of fusion of ice can be

determined by the method of mixtures. A known weight of water is placed in a calorimeter, which is protected in the usual way, and its temperature is taken. Lumps of melting ice half the size of a walnut are carefully dried with blotting paper and are then shot into the calorimeter. The lowest temperature of the mixture is noted when all the ice has melted, and the calorimeter is reweighed to get the amount of ice added. Let m_1 and m_2 be the masses of the

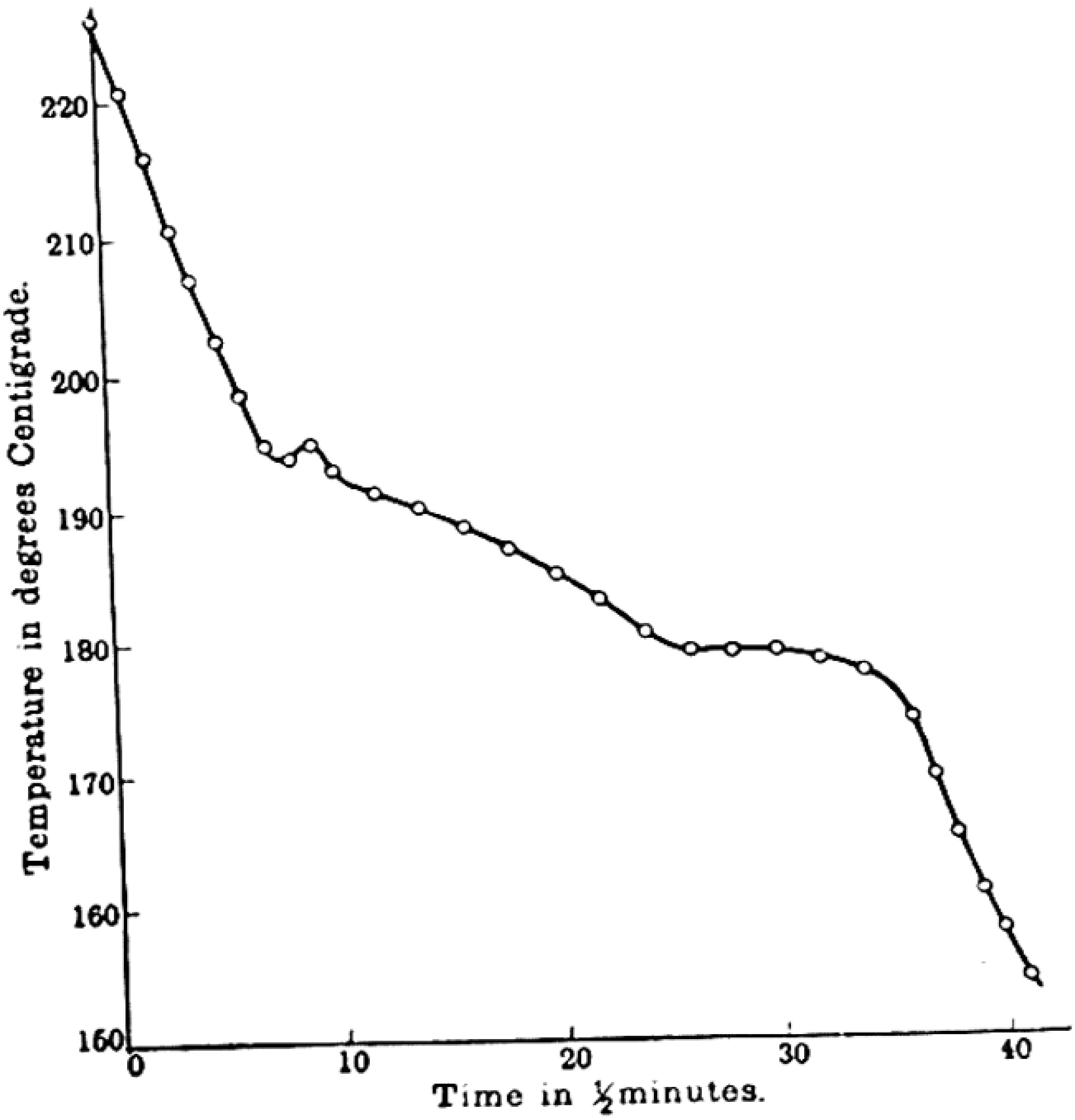


FIG. 38.—Cooling Curve for Solder.

water and the calorimeter respectively, s_2 the specific heat of the latter, L the latent heat of ice, t_1 and t_2 the initial and final temperatures of the calorimeter and its contents, M the mass of ice added. The ice absorbs heat from the water to melt it and to raise its temperature from 0° to t_2°. We may suppose this absorption takes place in two steps:—

(1) To melt M gms. without changing its temperature requires ML cals.

(2) To raise the water so formed from 0° to t_2° requires Mt_2 cals.

Also heat lost by the water originally in the calorimeter $= m_1(t_1 - t_2)$ cals.

And heat lost by the calorimeter $= m_2s_2(t_1 - t_2)$ cals.

Hence $ML + Mt_2 = m_1(t_1 - t_2) + m_2s_2(t_1 - t_2)$, from which L can be found. Accurate experiments show that L for water is very nearly 80 cals. If the ice is not thoroughly dry the water added with it will not absorb its latent heat and the final result will be too low. To render the radiation correction small the calorimeter should be 4° above the room temperature and sufficient ice should be added to cool it by an equal amount below.

Change of Volume produced by Melting.—Since ice floats in water at 0° its specific gravity must be the smaller of the two; in other words, a c.cm. of water at 0° will expand to more than a c.cm. when it freezes. It is due to this expansion that water-pipes are frequently burst during a frost. On the other hand solid paraffin wax sinks when thrown into its liquid at a temperature just above the melting point. Paraffin therefore contracts when it solidifies. Substances which are to be cast should expand at the moment of solidification in order to retain the shape of the mould.

Experiment.—*To find the Specific Gravity of Ice.* Pour about 50 c.cms. of methylated spirit into a small beaker and drop into it a small lump of ice. Add water until the ice is nearly wholly immersed. Stir the mixture; ice gradually melts, at the moment it sinks below the surface remove it as quickly as possible. By mixing two liquids, one having a greater the other a less specific gravity, a mixture has been made in which the ice floats, its specific gravity, which equals that of ice, may now be found by means of a specific gravity bottle.[1]

According to Bunsen 1 gm. of ice at 0° occupies 1·0908 cm.^{3}, and a gm. of water at the same temperature has a volume 1·0001 cm.^{3} The expansion when a gm. of water freezes is therefore 0·0907 cm.^{3}

Bunsen's Ice Calorimeter.—Bunsen has utilized this volume change in the construction of a very delicate calorimeter. A tube P (Fig. 39) is fused into the upper end of a wider tube Q, shaped as shown in the figure, and the space between them is filled with water from which the dissolved air has been removed by boiling. Mercury is then poured in until it rises to the top of the narrow tube at S. R is a capillary tube, graduated in c.cms., which is pushed through a rubber stopper at S until the mercury extends to near its middle point. A block of ice is next made to form round the bottom of the

[1] Barton and Black, "Practical Physics," p. 34.

tube P. With this object the apparatus is placed in ice, but as the water is free from air it may be greatly supercooled before freezing begins. To start the freezing a little ether is placed in P and is caused to evaporate quickly by bubbling air through it. The evaporating liquid absorbs its latent heat of vaporisation from the water and so causes it to freeze; once begun this will continue for some hours. When sufficient ice has been formed, water cooled down to 0° is placed in P, and the hot body whose specific heat is required is dropped into it after noting the position of the mercury thread. It cools from T° to 0° and emits MsT calories of heat, thereby melting some ice. The volume of the water in Q is thus altered and the change is read off on the graduated tube R. Let v be the volume change, then the number of gms. of ice melted is, from the last paragraph, $v/0{\cdot}0907$, and the heat it absorbs is $80v/0{\cdot}0907$, since the latent heat is 80.

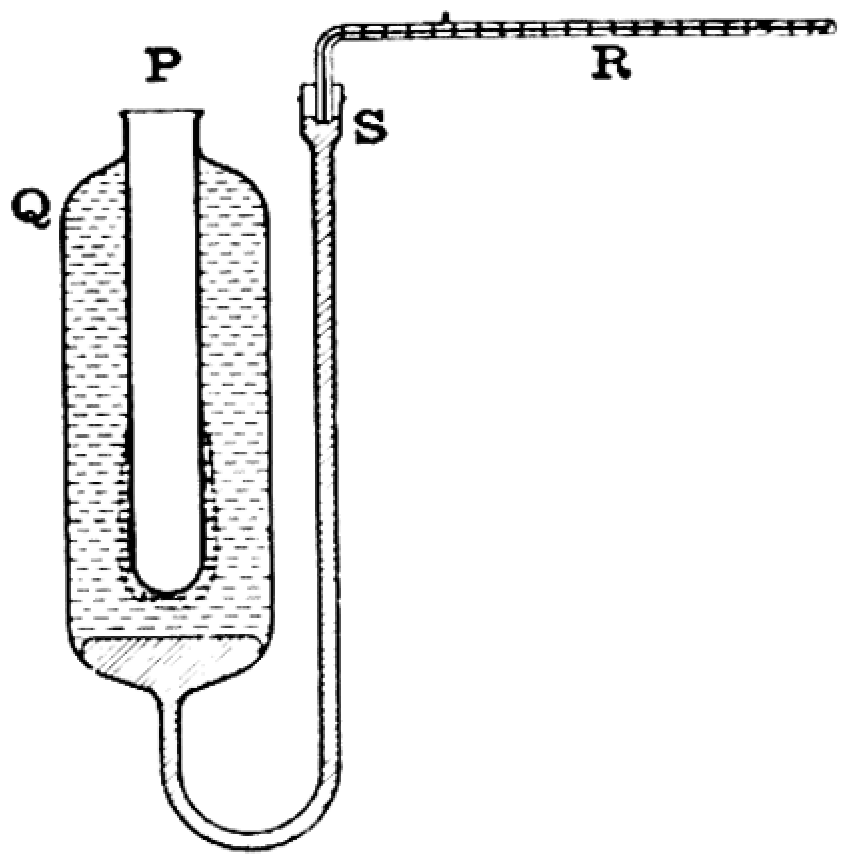


FIG. 39.—Bunsen's Ice Calorimeter.

Hence $$MsT = \frac{80v}{0{\cdot}0907}$$

and $$s = \frac{80v}{0{\cdot}0907MT}$$

Instead of bringing into the calculation the latent heat of ice, a quantity about which there is some uncertainty, the instrument may be standardised by pouring into the tube P a mass m gms. of water at a temperature t°. The heat it emits in cooling to 0° is mt cals.; if this causes the mercury in R to move over n divisions then one division corresponds to an emission of mt/n cals., hence the heat emitted in any subsequent experiment is known from the movement of the mercury column.

Solution. Freezing Mixtures.—When a solid substance is dissolved in a liquid it absorbs its latent heat of fusion from the solvent, and, unless chemical actions occur, the temperature falls. To ensure

that this temperature change is entirely due to the act of solution the solid must be initially at the same temperature as the solvent.

EXPERIMENT.—Place some powdered "hypo" in a test-tube and immerse it for 30 mins. in water contained in a calorimeter. The temperatures of solid and liquid should then be equal. Note the temperature of the water and pour the contents of the test-tube into the calorimeter; the temperature falls. It is for this reason that the fixing solution used in photography should be made up some time before it is required for use, otherwise its temperature will be low and its action correspondingly slow.

When snow and salt are mixed together they mutually dissolve each other, and, in accordance with what has just been said, a very low temperature results. This is the principle of freezing mixtures. After a fall of snow salt is frequently thrown on the pavements to make the snow melt; it produces the attendant discomforts of a slush whose temperature is below 0°. If 33 parts by weight of sodium chloride are mixed with 100 parts of ice a temperature as low as — 20° C. can be reached.

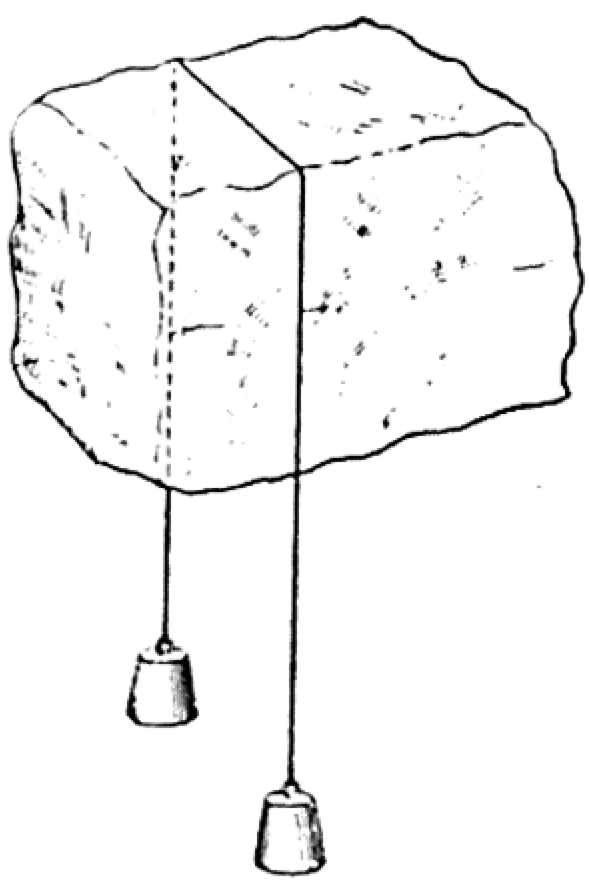


FIG. 40.—Tyndall's Experiment on Regelation.

Effect of Pressure on the Melting-Point.—Since ice contracts in volume when it melts we should perhaps expect that an increased pressure, which renders a contraction more liable to take place, would cause it to melt more easily, or, in other words, would cause it to melt at a temperature below 0°. Similarly when a substance expands on melting it is possible that an increased pressure would raise the melting point. This, in fact, is what actually takes place. When a strong steel cylinder at 0° is filled with pieces of ice and these are subjected to great pressure they partially melt, if the pressure is then relieved the water freezes, since its temperature is 0°, and the whole is found to have formed a solid block of ice. This effect of pressure is called **regelation.** The following experiment, due to Tyndall, may be explained in the same manner. Weights are hung over a block of ice by means of a copper wire in a room where the temperature is 0° (Fig. 40). Owing to the pressure under the wire ice melts, and the water which is formed escapes to the upper side of the copper, where, being relieved from the pressure, it freezes

again, giving out its latent heat. This heat, if transmitted through the wire, will assist the pressure in melting more ice ; the wire thus works its way through the block which nevertheless remains whole. If an iron wire is substituted for copper its rate of progress is slower owing to its being a worse conductor of heat (p. 119). It is due to regelation also that skating on ice is possible ; the pressure of the steel edge causes ice to melt and so allows the skate to "bite." Similarly the lower portions of glaciers melt under the great pressure to which they are subjected. In the case of ice the lowering of the melting point is very small, about 0·0072° per atmosphere, hence it is unnecessary to allow for variations in the barometric height when the lower fixed point of a thermometer is being found.

Boiling Point. Latent Heat of Vaporisation.—When liquid is heated in a beaker bubbles of air and vapour of the liquid are formed on the glass which finally rise to the surface and burst, causing a sound. The "singing" of a kettle is due to this. When a certain temperature is reached the supply of bubbles is very copious and the temperature remains steady ; the liquid is then said to **boil.** If it has been previously freed from dissolved air its temperature may rise above the normal boiling point before it actually commences to boil, it is then said to be **superheated**; finally a bubble of air or vapour is formed and violently bursts. This "bumping" may be hindered if a supply of air bubbles is provided by putting into the liquid some broken pieces of earthenware. As the temperature of a boiling liquid depends slightly on the vessel in which it is contained, the boiling point is determined by a thermometer whose bulb is placed in the vapour above the liquid. The latent heat of vaporisation is most easily determined by Berthelot's apparatus (Fig. 41). The liquid is heated in a special form of flask through the bottom of which projects a glass tube open at both ends, this is connected by a ground joint to a glass bulb and spiral immersed in water in a calorimeter. The calorimeter and its contents are protected from heat coming from the gas burner by a wooden cover, and a stirrer and thermometer are passed through holes in the wood. When the liquid boils its vapour passes down the vertical tube and is condensed in the spiral, at the same time giving up its latent heat. The amount condensed may be found by weighing the spiral at the beginning and the end of the experiment. As the vapour passes down through the upper part of the tube any drops of liquid that it carries with it are vaporised, if this did not happen the particles

would not have to part with their latent heat to the calorimeter, and the final result obtained would be too low. Let M be the weight of liquid condensed, T° its boiling point, L its latent heat of vaporisation, and s its specific heat. Let m be the total water equivalent of the calorimeter and its contents, including the spiral and stirrer, $t_1°$ its initial and $t_2°$ its final temperature.

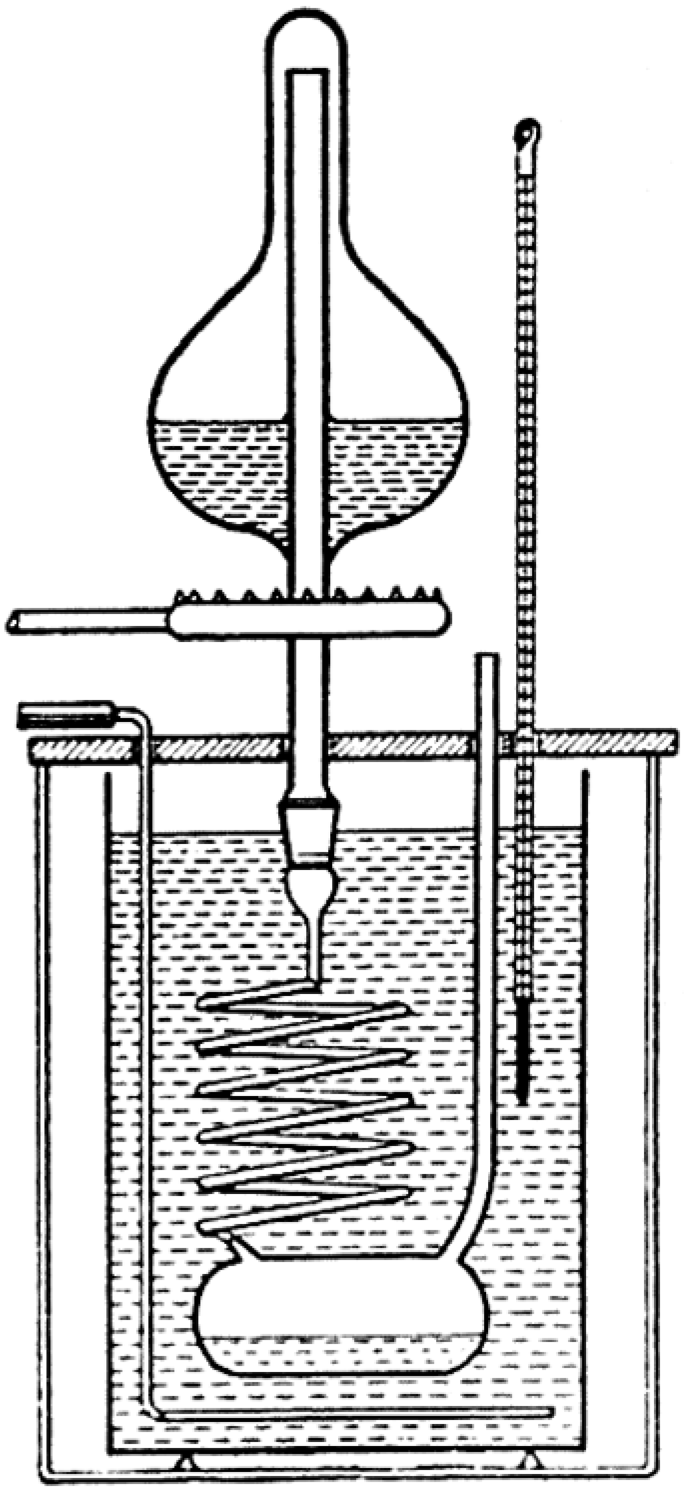


FIG. 41.—Berthelot's Apparatus.

Then the heat given out by the vapour in condensing = ML cals. and the heat emitted by M gms. in cooling from T° to $t_2°$

$$= Ms(T - t_2) \text{ cals.}$$

Also the heat absorbed by the calorimeter and its contents

$$= m(t_2 - t_1) \text{ cals.}$$

Hence

$$ML + Ms(T - t_2) = m(t_2 - t_1)$$

from which L can be found if the specific heat of the liquid and its boiling point are known. The latent heat of vaporisation of water is 539 calories. This large latent heat is turned to a useful purpose in the heating of buildings by steam. Evidently much less material is required than if hot water alone were used.

EXPERIMENT.—Drop a little ether on the hand and note that a cooling sensation is experienced. The liquid is very volatile and evaporates very quickly; to do this it absorbs its latent heat of vaporisation.

Joly's Steam Calorimeter.—Prof. Joly in his steam calorimeter has worked out a very simple and accurate method of determining specific heats which depends on a knowledge of the latent heat of steam. A simple form of the calorimeter is shown in Fig. 42. It consists of a metal enclosure, called the steam chamber, into which a rapid supply of steam can be admitted through a wide tube near the top; at the bottom of the chamber the exit tube is placed. A thin vertical wire passes through a small hole at the top and is

attached at its upper end to one arm of a balance. The body whose specific heat is required is placed on a small copper pan which hangs from the lower end of the wire in the middle of the chamber; a thin copper guard shields it from drops of condensed water which might otherwise fall on to it from the roof of the enclosure. The substance is allowed to hang in the calorimeter for some minutes and its temperature t_1 is then taken. Steam is now admitted through the wide tube and condenses on the body and pan; after a few minutes the mass condensed is found from the increased weight of the pan and its contents. Let m be the mass of steam which condenses, m_1 and m_2 the masses of the substance and the pan, s_1 and s_2 their specific heats, and L the latent heat of steam. The enclosure is finally at the temperature t_2 of the steam, hence the heat given out during condensation is mL, and the heats absorbed by the body and pan respectively are $m_1s_1(t_2 - t_1)$ and $m_2s_2(t_2 - t_1)$. Therefore

$$mL = m_1s_1(t_2 - t_1) + m_2s_2(t_1 - t_1)$$

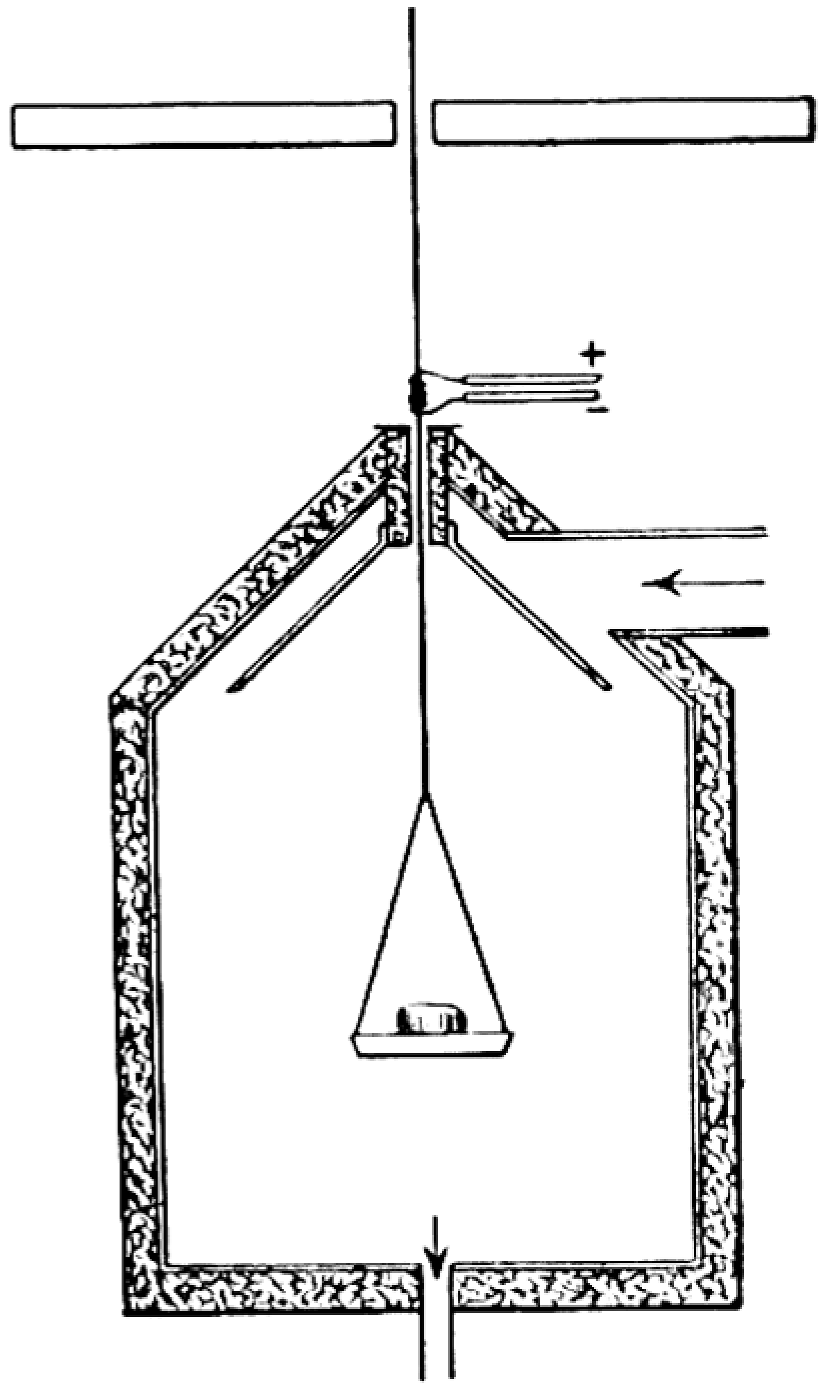


FIG. 42.—Joly's Steam Calorimeter.

The specific heat s_1 can be calculated from this equation if s_2 is determined by a preliminary experiment. In practice it is found that steam condenses on the suspending wire where it leaves the steam chamber; surface tension then makes an accurate weighing impossible. To overcome this difficulty the wire is passed along the axis of a small spiral of platinum which is heated by passing an electric current through it; sufficient heat is thus developed to hinder condensation. When a liquid is to be experimented on it is enclosed in a small copper sphere. The most novel application of the apparatus was in the determination, for the first time, of the specific heats of gases at constant volume. For this purpose two equal copper spheres were hung from the opposite arms of the

balance in the same steam chamber. One sphere contained the experimental gas while the other was exhausted; the difference in the weights of steam condensed was thus due to the contained gas. The specific heat of air at constant volume under a mean pressure of 19·5 atmospheres was found to be 0·1721, hence from the table on p. 37 $C_p/C_v = 1{\cdot}4$.

EXAMPLES ON CHAPTER VII

1. The open end of the capillary of a Bunsen calorimeter is placed under the surface of mercury. When 25 gms. of water at 15° are placed in the inner tube of the calorimeter it is found that 6·8 gms. of mercury are drawn in. Assuming the density of mercury to be 13·6 and the latent heat of ice as 79, determine the density of ice. (L. '86.)

2. A calorimeter whose capacity for heat is 48 water-gms.-degrees has 352 c.c. of water in it and the whole weighs 882 gms. Into this steam at atmospheric pressure is condensed till its temperature rises from 12·2° to 18·7°, and on weighing again the calorimeter weighs 886·2 gms. Calculate the latent heat of vaporisation of water.

3. If a boiler receives 30,000 units of heat per min. through every square metre of its surface, the total surface being, say, 5 sq. m., and if its temperature be 140° while it is fed with condenser water at 45°, what weight of steam would you expect to be able to draw off regularly per hour? The latent heat of vaporisation of water at 140° is 509. (L. '91.)

4. One hundred gms. of iron at 50° C. are placed in a vessel containing 1000 gms. of water at 0°; how many gms. of ice at 0° must be added to reduce the temperature of the mixture to 0°? All the ice is supposed to be melted. [Sp. ht. of iron = 0·113; lat. ht. of fusion of ice = 80.] (L. '96.)

5. One gm. of metal heated to 100° is dropped into a Bunsen ice calorimeter in which the weight of mercury required to fill 1 cm. of the index tube has been found to be 0·026 gm. The thread of mercury moves through 52·5 mms. What is the mean specific heat of the metal? One gm. of water in freezing expands 0·0907 c.c. and its latent heat of fusion is 80·02. The density of mercury is 13·6. (L. '02.)

6. A mass of 200 gms. of copper (sp. ht. 0·1) is hung in a closed chamber at a temperature of 60° F. Steam is then admitted at the normal atmospheric pressure. Calculate the mass of water condensed by the copper. [Lat. ht. of steam = 536.] (L. '03.)

7. Steam at 100° is passed into a copper calorimeter, weighing 100 gms. and containing 500 gms. of water at 15°, until the temperature of the calorimeter and its contents rises to 25°. Calculate the weight of steam condensed, given the sp. ht. of copper = 0·1 and latent heat of steam = 536. (L. '06.)

8. If the latent heat of fusion of ice is 80 and its density at 0° is 0·917, find the travel of the mercury in the tube of a Bunsen's ice calorimeter when 10 cals. are given to the ice, the diameter of the tube being 0·4 mm. (L. '08.)

9. The boiling point of a liquid is 156°, its mean specific heat is 0·46 and its latent heat is 68 gm. cals. Find the quantity of vapour at the boiling point that must be passed into a copper vessel (sp. ht. 0·1) weighing 30 gms., which contains 250 gms. of the liquid at 15°, in order to raise the temperature of the latter to 27°. (L. '09.)

CHAPTER VIII

VAPOUR PRESSURE. CHANGE OF STATE (*continued*)

Vapour Pressure.—A solid changes into liquid at one temperature only, the melting point, but a liquid can assume the form of vapour at any temperature. Thus a pool of water on the road dries up under the sun's rays; although the temperature is below the boiling point the water evaporates, *i.e.* is converted into vapour.

EXPERIMENT.—Set up a barometer and introduce a small drop of alcohol at the lower end of the tube by means of a curved pipette; the bubble of liquid rises up the column and is at once converted into vapour when it reaches the surface. The vapour exerts a pressure just as a small quantity of air would do and the mercury is depressed by an amount equal to the vapour pressure. If additional alcohol is introduced more vapour is formed and the height of the column is decreased still further, but at length a stage is reached when the added liquid floats on the surface and no further evaporation takes place.

When a space contains the maximum amount of vapour it can hold under the given conditions of temperature the vapour is said to be **saturated** and the pressure it exerts is called the **saturated or maximum vapour pressure.** If less than this maximum amount is present the vapour is said to be **unsaturated or superheated.** The term "vapour tension" is sometimes used instead of "vapour pressure."

EXPERIMENT.—Use barometer tubes of different diameters and lengths so that the volume occupied by the vapour is varied; it will be found that while more liquid is evaporated in the larger tubes the maximum depression is the same in every case provided the temperature is constant.

If different liquids are used it will be found that the maximum pressure varies from one to another; also when the temperature is raised more liquid evaporates and the maximum vapour pressure is increased. These results show that to every liquid there corresponds a maximum vapour pressure varying with the nature of the

substance but otherwise depending on the temperature alone. When an unsaturated vapour is gradually cooled a point is reached where its pressure is equal to the maximum vapour pressure corresponding to that temperature; the vapour is then saturated and any further cooling is accompanied by partial condensation. Similarly if an unsaturated vapour is compressed at constant temperature, as, for example, by pushing down the barometer tube containing it into a deep cistern of mercury, a state of things is eventually arrived at where the vapour actually present is sufficient to saturate the space; further compression then causes condensation but *the vapour pressure remains constant.*

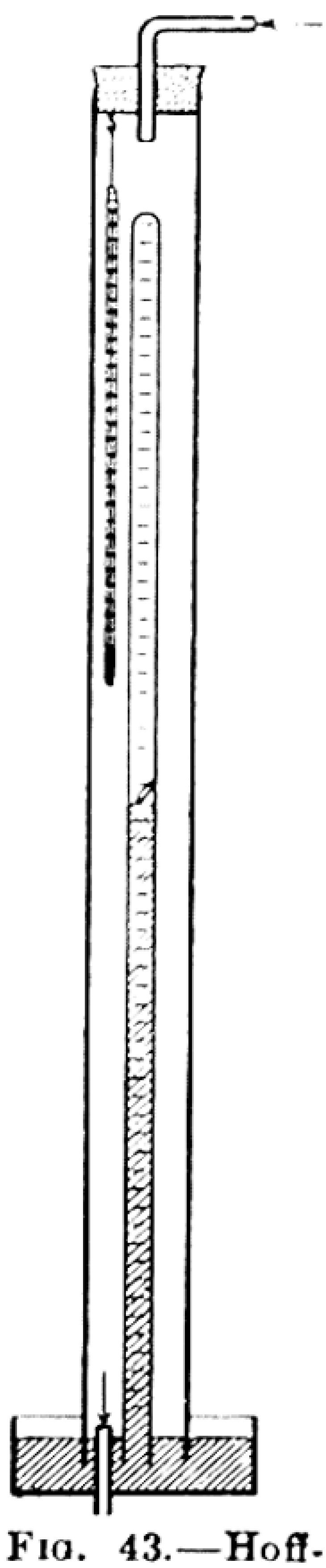


Fig. 43.—Hoffman's Vapour Density Apparatus.

Vapour Density.—Unsaturated vapours obey Boyle's and Charles' laws very approximately if the temperature is well above that at which condensation would begin. This is best proved by measurements of the vapour density at different pressures and temperatures. The method is to find by experiment the mass of vapour in a c.cm.—this is the density as usually defined; the volume v of a gram of vapour can then be calculated under the pressure and temperature prevailing in the experiment. It will be found, as in the case of gases, that $\frac{pv}{T}$ = constant. One method of experiment is shown in Fig. 43. A long barometer tube graduated in c.cms. contains mercury and is surrounded by a steam-jacket. A known weight of liquid enclosed in a small stoppered bottle is placed in the open end of the tube and rises to the top of the mercury column where, owing to the diminished pressure and the high temperature, the stopper is ejected and the liquid forms an unsaturated vapour. The depression of the mercury column measures the vapour pressure; this may be found at once by a cathetometer. The volume is read off from the graduations and the temperature is given by a thermometer in the steam-jacket, hence as the mass of the vapour is known, being equal to the liquid introduced, its density can be calculated. Using different amounts of the substance, and the vapours of different

boiling liquids in the steam-jacket, it can be proved that $pv/T = \text{const.}$, where v is the volume of one gm. of vapour at a pressure p and temperature (absolute) T.

Similar experiments in which different substances are used at the same temperature, bring out further the important fact that the vapour density is proportional to the molecular weight of the substance used. This result is important from the chemical

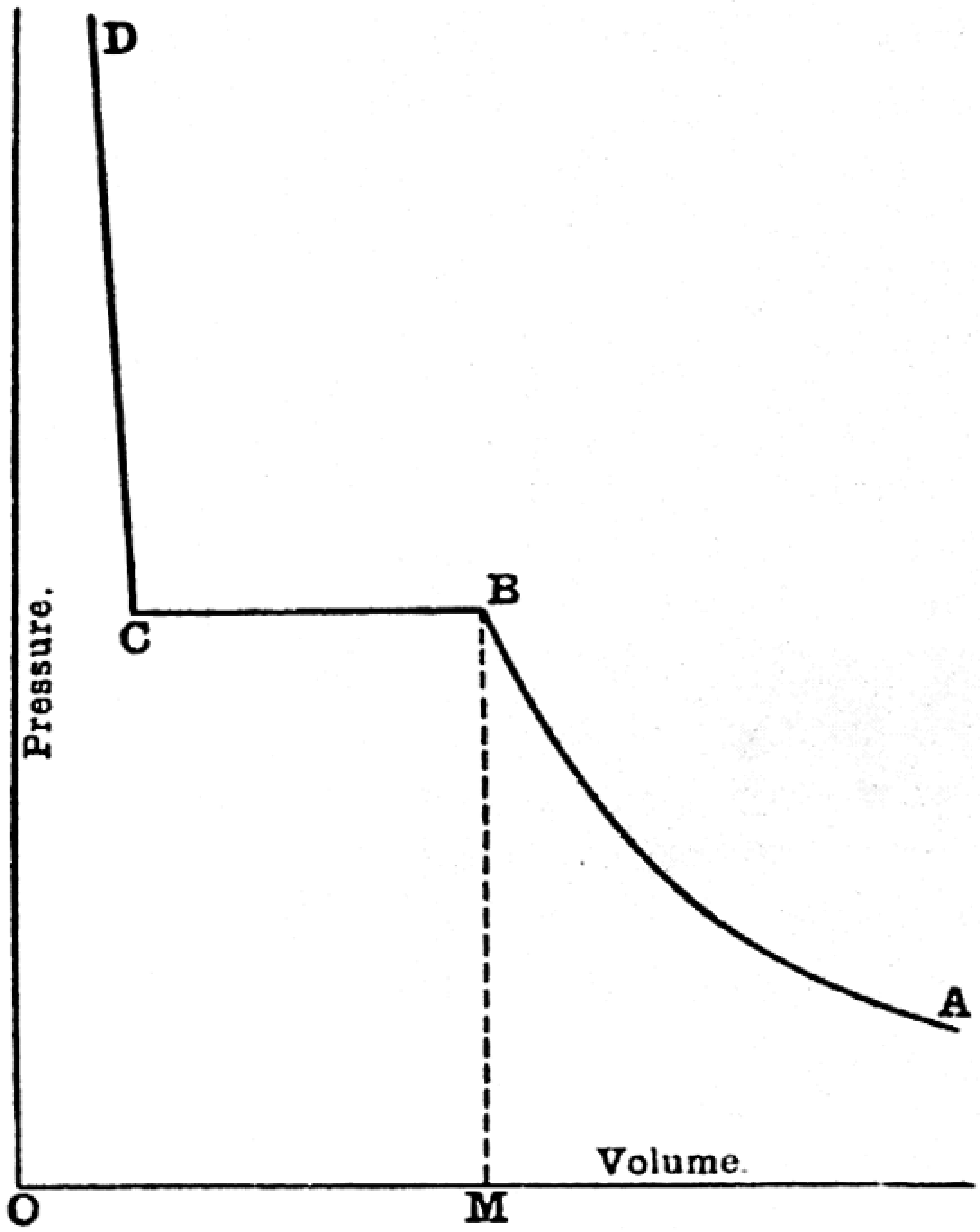


FIG. 44.—Isothermal Curve.

point of view as it enables molecular weights to be found from vapour density determinations.

Isothermal Curves.—We can now show the general shape of an isothermal curve which gives the relation between the pressure and volume of a substance at a constant temperature. Starting with the vapour in the unsaturated state $pv = \text{const.}$ until saturation is nearly reached; this part of the curve is shown at AB (Fig. 44). When the volume is reduced to the amount represented by OM condensation begins and the vapour pressure remains constant until all the vapour is converted to liquid. This part of the curve, shown

at BC, is parallel to the axis of volume. At C all the substance has condensed, and, as the volume of a liquid varies very little with pressure, the remaining part CD of the curve is very nearly parallel to the pressure axis. Summarising these results we see that along AB the substance is wholly vapour, along BC liquid and vapour in contact, and along CD wholly liquid.

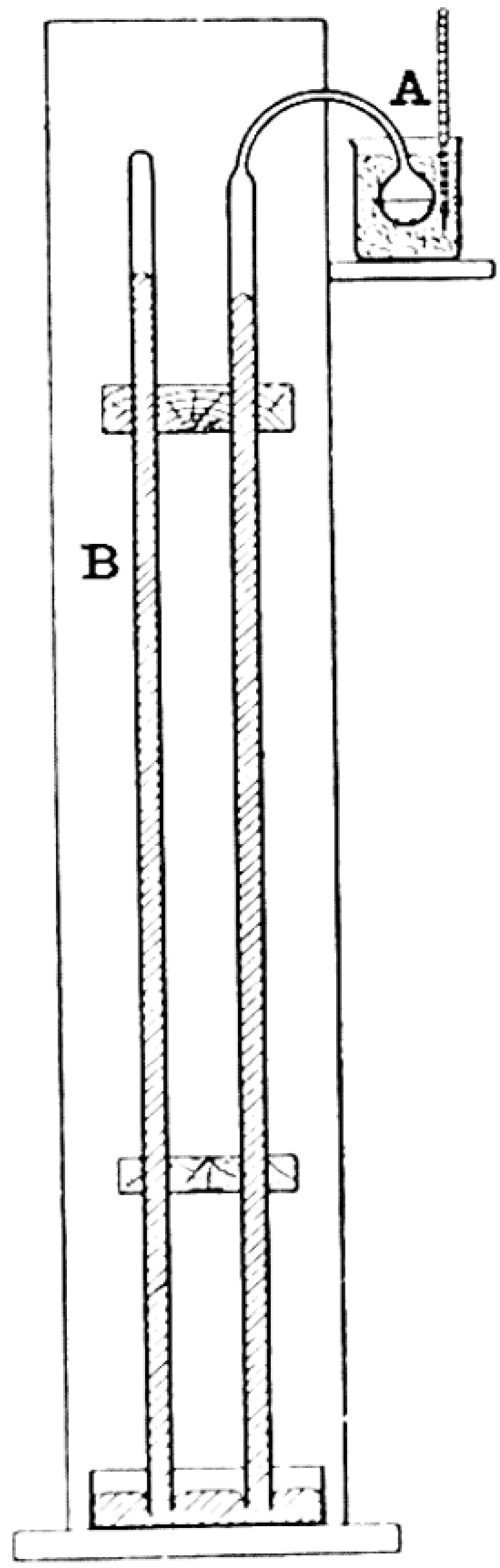


FIG. 45.—Regnault's Apparatus for measuring the Vapour Pressure of Water below 0°.

Methods of measuring Maximum Vapour Pressure.—The methods used to measure the maximum vapour pressure vary with the temperature, a procedure which is useful at 20° may be inconvenient at 80°. The apparatus shown in Fig. 45 was used by Regnault to measure the vapour pressure of water below 0°. Bulb A, which contains the water, forms the upper part of a barometer tube; it is placed in a freezing mixture of calcium chloride and snow. The vapour pressure is the same at all points in this space and is equal to the maximum pressure corresponding to the temperature of the freezing mixture; if it were higher than this condensation would take place in A. On the left is shown an ordinary barometer; the difference in heights of the two columns gives the vapour pressure in cms. of mercury. The vapour pressure of ice, which is quite appreciable, can be measured by this means.

Fig. 46 shows Regnault's apparatus for water between 0° and 50°. One vertical tube forms a standard barometer, in the other a little water floats above the mercury. The upper part of each tube is surrounded by a water-bath which is kept well-stirred and can be heated from below. The difference in heights of the two columns is read by a scale, or, in Regnault's experiments, by a cathetometer; this gives the vapour pressure at the temperature of the bath in cms. of mercury. As the mercury is warm the pressure must be given in terms of the length of a mercury column at 0°;

the difference in level must therefore be divided by $(1 + ct)$, where c is the coefficient of cubical expansion of mercury (p. 50). It is difficult to get accurate results by this method as a small amount of impurity in the tube may greatly affect the pressure; in addition it is very difficult to ensure that the vapour is not mixed with air. For higher temperatures a different principle is used (see next paragraph).

Vapour Pressure of a Liquid at its Boiling Point.—If a barometer

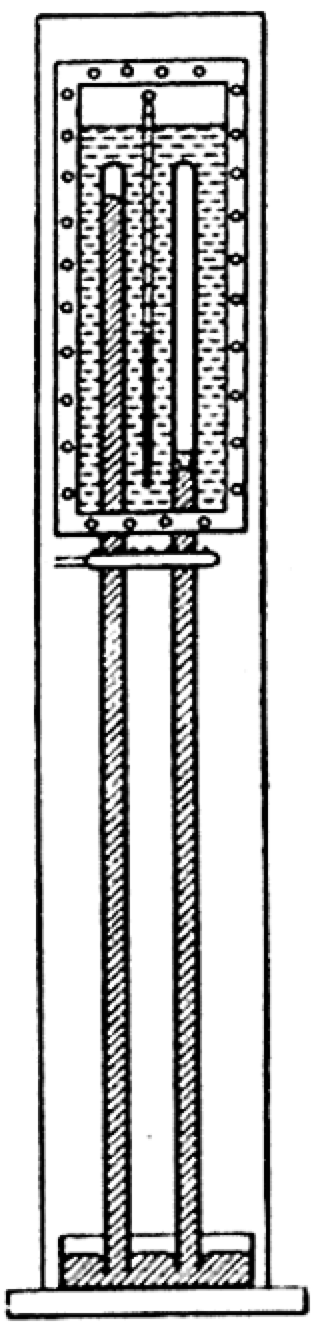


FIG. 46.—Regnault's Apparatus for Temperatures between 0° and 50°.

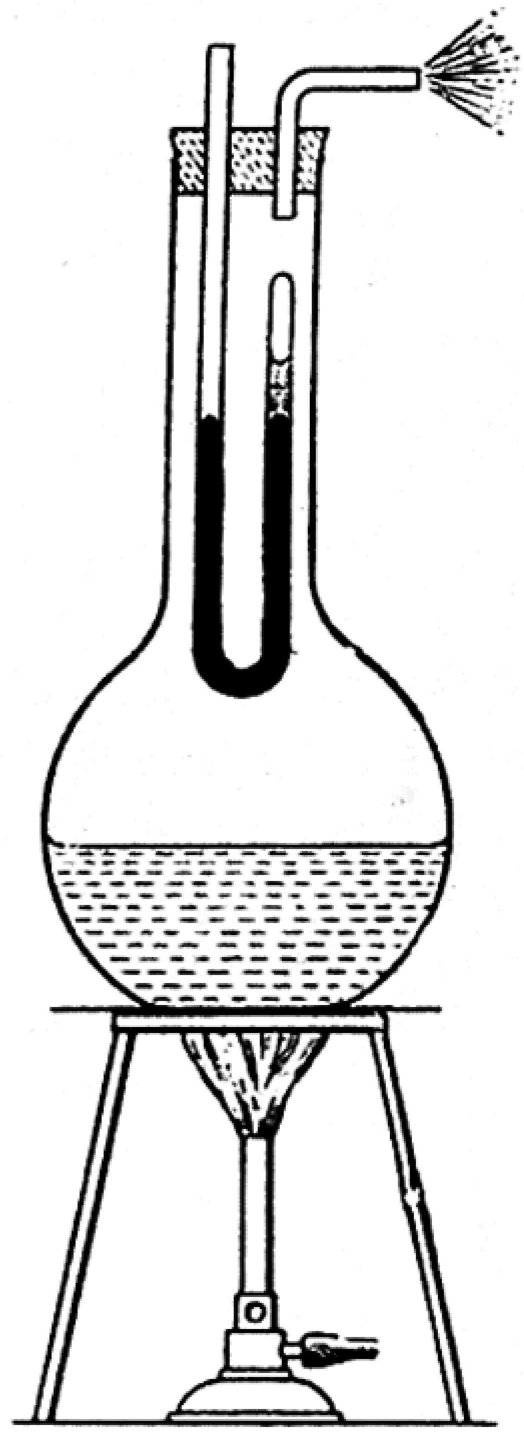


FIG. 47.

containing a little water above the mercury is surrounded by a steam-jacket and heated it is found that, when the liquid boils, the mercury stands at the same level inside and outside the tube, *i.e.* the vapour pressure of water at its boiling point is equal to the external pressure of the atmosphere. The boiling point may thus be defined as the temperature at which the pressure of the vapour is equal to the external pressure on the liquid.

EXPERIMENT.—The shorter closed limb of the U-tube shown in Fig. 47 contains water in its upper part, the lower portion is filled with mercury which

reaches just beyond the bend. It is placed in the steam rising from water boiling in a wide-necked flask. When the water in the tube reaches its boiling point the mercury is depressed until it stands at the same level in each limb, showing that the pressure of the steam is equal to the atmospheric pressure.

If the pressure on a water surface is sufficiently reduced the liquid will boil at temperatures much lower than 100°.

EXPERIMENT.—Boil water in a flask for some minutes until most of the air is expelled; while boiling is still in progress close the flask tightly with a rubber

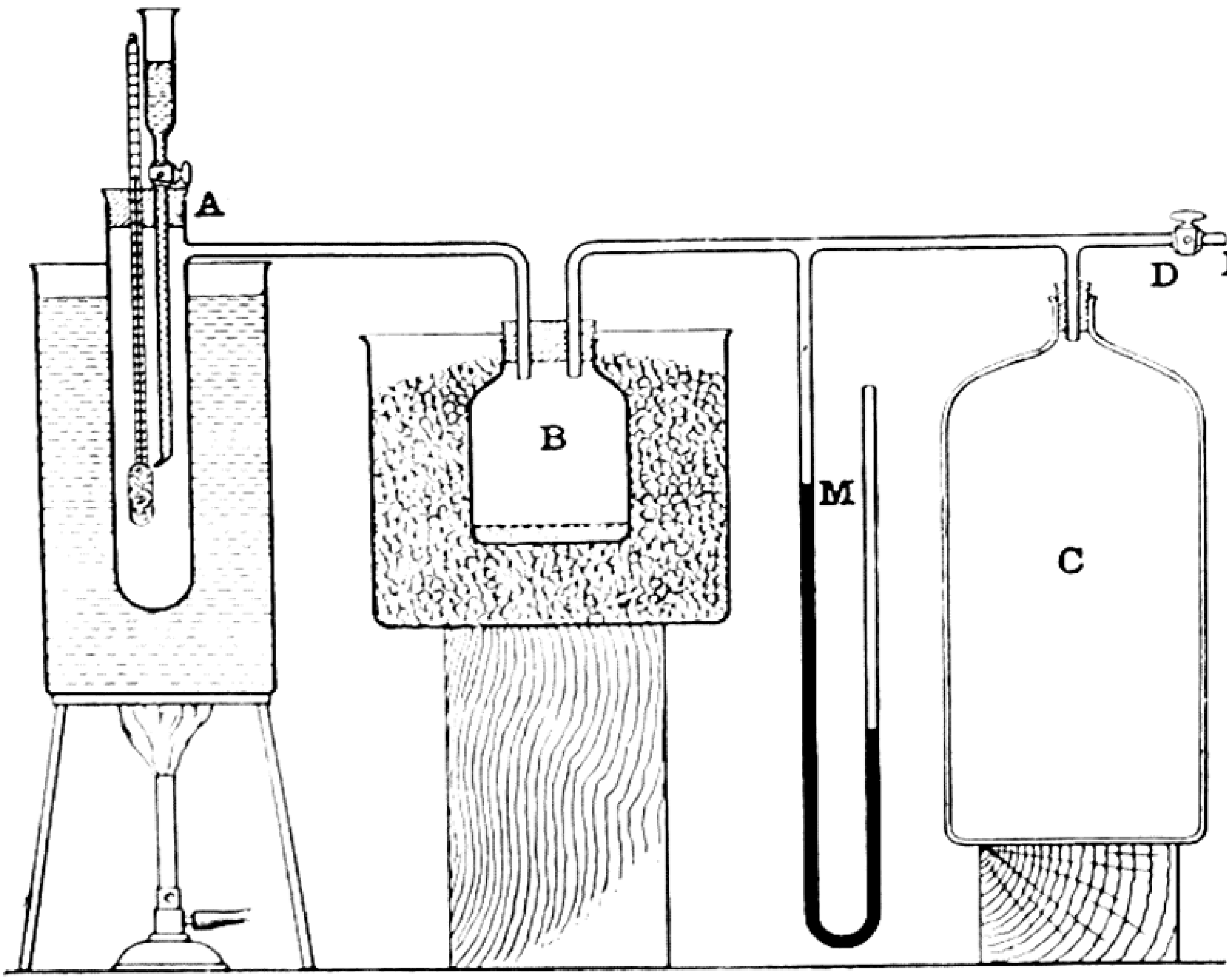


FIG. 48.—Ramsay and Young's Apparatus.

stopper and remove the flame. After it has cooled for several minutes pour cold water on it; the vapour in the flask is condensed, this reduces the pressure on the liquid and causes boiling to begin again quite vigorously.

These experiments show that by varying the pressure the boiling point can be altered within wide limits, but in each case the vapour pressure is equal to the pressure on the surface of the liquid. Hence the maximum vapour pressure can be found by measuring the pressure under which the liquid boils. The best method for doing this is due to Ramsay and Young. A wide boiling tube, A, is tightly closed by a rubber stopper through which project the stem of a thistle

funnel and a thermometer (Fig. 48). The funnel contains the liquid to be experimented on and the lower end of its stem is drawn off to a fine point which is near the thermometer bulb. The latter is wrapped round with cotton wool or asbestos. In communication with A is a bottle B immersed in ice, a large bottle C and a manometer M; the tube F goes to an air or filter pump. A convenient pressure having been established by the pump tap D is closed. The bath surrounding the boiling tube is then heated to a temperature a few degrees higher than the boiling point of the liquid under the given pressure, and liquid is allowed to drip slowly on to the thermometer bulb. As a large surface is exposed boiling takes place quite regularly; when the thermometer reading is steady the temperature and pressure are read. This gives the vapour pressure at the temperature shown by the thermometer. By varying the pressure a series of measurements of the vapour tension at different temperatures can be found. The bottle B is for the purpose of condensing the vapour so that the liquid may be recovered, while C serves to lessen the pressure variations due to accidental causes such as a slight leakage of air into the apparatus. If the vapour pressure is greater than that of the atmosphere air must be compressed into the apparatus which should then be made correspondingly stronger. In this form it may be used for water above 100°. This dynamical method is much more accurate and easier to work than the one given in the last paragraph which is usually called the statical method.

Vapour Pressure of Salt Solutions.—The dynamical method as already described cannot be used to determine the vapour pressure of a salt solution because as the liquid boils off the concentration of the solution is altered. The statical method (p. 87) is, however, available, and another arrangement (p. 103) is also frequently used. If some salt solution is introduced into the space above a barometer column it is found to produce a smaller depression of the mercury than the pure liquid does at the same temperature, hence the vapour pressure of a solution is less than that of the pure solvent. It follows that at 100° the vapour pressure of an aqueous solution will not be equal to the atmospheric pressure and the liquid must be heated to a higher temperature to make it boil. This has already been noted on p. 22. As the vapour leaves the liquid it cools very quickly to 100°, hence to find the boiling point of a solution the thermometer bulb must be placed in the liquid itself and not in the

vapour. The vapour pressure of volatile liquids like ether and alcohol is much greater than that of water and becomes equal to the atmospheric pressure at temperatures below 100°. Such liquids have consequently a low boiling point, *e.g.* ether boils at 34·5° under normal pressure.

Determination of Heights by the Hypsometer.—The pressure at a point in a barometer tube becomes less as the point in question is taken nearer the top of the mercury column. In the same way, during the ascent of a mountain, as the different layers of air are passed through and the limits of the atmosphere are more nearly approached the pressure becomes less and the length of the barometric column is reduced. The height of the mountain can be calculated if the barometric pressure at its summit is measured. Instead of using a barometer for the purpose the temperature at which water boils may be observed, and from a table of maximum vapour pressures the corresponding pressure of the atmosphere can be found. An instrument used for this purpose is called a hypsometer. At great altitudes the boiling point may be lowered to such an extent that it is impossible to cook food. An arrangement for boiling under increased pressure must then be employed.

Dalton's Law for Mixed Vapours.—Let us next investigate how the pressure of a saturated or unsaturated vapour is modified by the presence of a gas or other vapour with which it does not react chemically. According to Dalton the total pressure produced by such a mixture is the sum of the pressures that each component would produce if it alone were present. This is usually known as **Dalton's law.** It is only approximately true in most cases; if it held in every instance it would be possible to produce a pressure as great as we pleased by introducing a sufficient number of different components into the mixture. Regnault tested the law by means of apparatus similar to that in Fig. 46.

EXPERIMENT.—Having set up a barometer in the usual manner introduce a small quantity of air. Suppose the column is depressed h cms. and let the total length of tube occupied by the air be L cms. Next add ether until the space above the mercury is saturated, and suppose the total depression is H cms. Then $(H - h)$ does not measure the vapour pressure of the ether, for the air is now diffused through a larger volume and its pressure is therefore less than h. Call the new air pressure h'. Let L′ cms. of the tube be occupied by air and ether vapour and suppose the sectional area of the tube is S. The volumes of

the air before and after admission of the ether are LS and L'S c.cms. respectively, and the corresponding pressures are h and h', hence by Boyle's law

$$h'L'S = hLS$$

or

$$h' = hL/L'$$

giving the new pressure of the air. The pressure of the mixture being H cms. that of the ether is $H - h' = H - \frac{hL}{L'}$. Working in this manner it will be found that the maximum vapour pressure of ether is the same as it would have been in the absence of air, thus proving Dalton's law.

An easier method of performing the experiment is indicated in Fig. 49. The flask contains air and a small, closed, thin-walled bulb filled with ether. Having noted the air pressure on the manometer the flask is shaken to break the bulb; and after some minutes the new pressure is noted. The section of the manometer tube being small we may suppose the volume of the air constant, the increase in pressure is thus due to the ether alone; this will be found equal to the maximum vapour pressure of the liquid at the temperature of the experiment, if some liquid ether still remains in the flask. It will be noticed that the ether evaporates much more slowly when another gas or vapour is present.

Cooling produced by Evaporation.—During evaporation it is only the more rapidly moving molecules that escape from the liquid surface to form vapour. The average velocity of the remaining molecules is thus reduced, or, in other words, the liquid is cooled. This is merely another method of stating that the latent heat of vaporisation is absorbed when a liquid evaporates. If the space above is crowded with air molecules it will be more difficult, owing to collisions, for molecules to escape from the liquid, this accounts for the relative slowness with which evaporation takes place in presence of a gas. The cooling produced by evaporation may be shown in various ways.

Experiment.—Dip a thermometer in ether and note the temperature, if it is then removed the adherent film of liquid evaporates and the thermometer temperature falls.

Experiment.—The bulb F (Fig. 50) contains ether and is connected with a second bulb N from which the air has been removed before sealing. When N is placed in ice ether vapour is condensed and more evaporates from the surface of the liquid in F, a cooling is thus produced which is readily shown if F is immersed in vessel A (Fig. 17) of the Looser thermoscope. If the bulb contains water instead of ether the fall in temperature may be sufficient to cause it to freeze. In the latter form the apparatus is called Wollaston's cryophorus.

EXPERIMENT.—Pour a little water into a shallow depression in a wooden block and place on it a small copper vessel containing ether. If the ether is evaporated quickly by blowing a current of air through it the cooling produced may be great enough to freeze the water.

In the case of ponds, lakes, etc., evaporation is continuously taking place from the surface, at least in the summer months. From what has been said in the preceding pages it will readily be seen that the conditions favourable to the process are: (1) A high temperature; (2) Little vapour already present in the air; (3) The vapour must be removed by air currents as rapidly as it is formed; (4) A large surface.

Condensation. Liquefaction of Gases.—Since an unsaturated

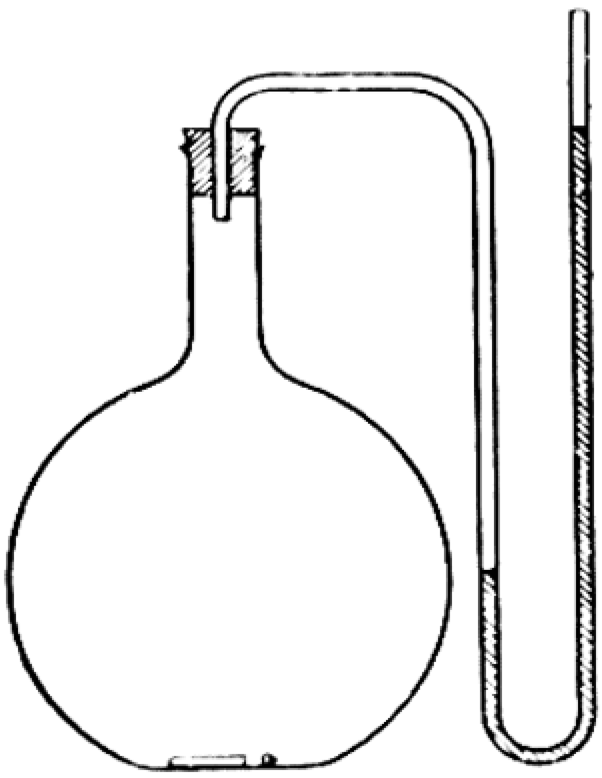


FIG. 49.—Apparatus to prove Dalton's Law.

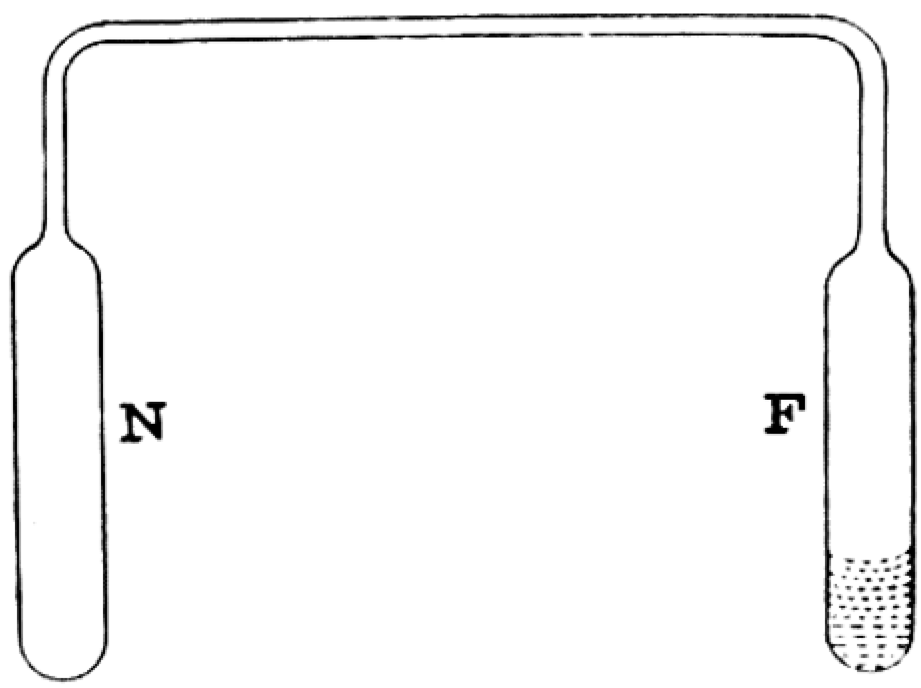


FIG. 50.—Method of showing the Cooling produced by Evaporation.

vapour becomes saturated if the temperature or volume is sufficiently reduced, any vapour may be made to condense into liquid by one or other of these processes or a combination of the two. Steam escaping from a jet is invisible near the orifice where it is purely vapour, but at a greater distance it becomes cooled and condenses into a large number of small particles of water which form a readily visible cloud. Not only vapours but gases also may be liquefied by great pressures if the temperature is low enough. Faraday liquefied a number of gases by such means in 1823. Chlorine may be taken as a typical example. Charcoal absorbs a large amount of chlorine gas; some charcoal saturated with chlorine is placed in one limb of a bent glass tube which is then closed at both ends and the other limb is surrounded by a freezing mixture. Gas is evolved by heating the

charcoal, and, when the pressure reaches about 2 atmospheres, it is condensed in liquid form in the cold limb. While experimenting with carbon dioxide Andrews found that unless it was cooled below 30·9° C. it was impossible to liquefy it, no matter how great the pressure applied. This is called the **critical temperature** of carbon dioxide; the pressure required to produce liquefaction at the critical temperature is called the critical pressure. The behaviour of carbon dioxide is typical of all gases. **It is impossible to liquefy a gas unless it is first cooled below its critical temperature.** This accounts for the difficulty in liquefying helium—a gas whose critical temperature is only a few degrees above the absolute zero. What we have hitherto regarded as vapours behave in a similar way; thus steam cannot be liquefied by any pressure, however large, if its temperature is above 365° C. Except that the pressure necessary to liquefy it is larger there is thus no reason why we should regard carbon dioxide at 30° as a gas and steam at 101° as a vapour, the pressure being one atmosphere in both cases. We may regard gases as merely unsaturated vapours far removed from the temperature of condensation. A test for distinguishing scientifically between a gas and a vapour is, however, provided by the critical temperature. A vapour may be defined as a gaseous substance which can be liquefied by pressure alone, *i.e.* a substance below its critical temperature. On the other hand a gas can be defined as a substance at a higher temperature than its critical temperature.

Wroblewski's apparatus for liquefying oxygen is shown diagrammatically in Fig. 51. The gas was first compressed into a steel cylinder A to a pressure of 120 atmospheres. This cylinder communicated through a metal capillary with a strong glass tube B which was surrounded by a wider tube C. To cool the gas below its critical temperature several steps were necessary. Liquid carbon dioxide was first obtained; this was allowed to evaporate quickly and the cooling produced caused the remainder to solidify. Solid carbon dioxide was next mixed with ether and rapid evaporation reduced the temperature of the mixture to about —80°, which was low enough to liquefy ethylene gas. The liquid ethylene was stored in the reservoir D, from here it flowed through a copper spiral S immersed in solid carbon dioxide and ether into the tube C. Ethylene vapour was quickly pumped off through the small hole O, the rapid evaporation causing the temperature to fall to —150° C. or lower. At this temperature the oxygen in tube B was liquefied. A still lower

temperature can be produced if liquid oxygen is made to evaporate quickly. Such temperatures are measured either by a platinum or a hydrogen thermometer.

Distillation.—A liquid which has been vaporised at one part of an apparatus may be condensed in another part, such a process is called distillation ; it is of great use in freeing a liquid from dissolved impurities either solid or liquid. For example, if an aqueous solution

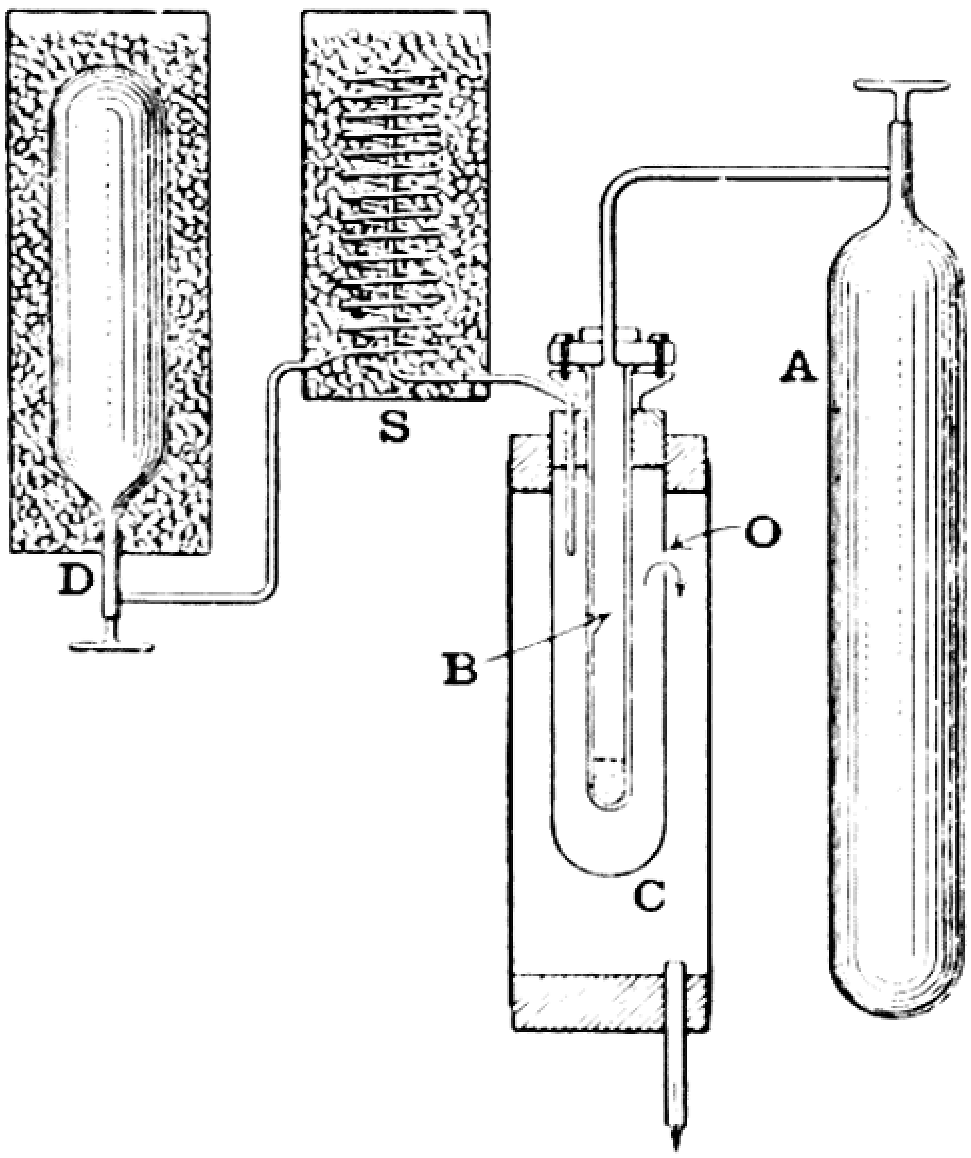


FIG. 51.—Wroblewski's Apparatus for Liquefying Gases.

of salt is boiled the vapour consists of water only which can be condensed in any suitable receiver. Pure water can be obtained from sea-water by this means. Similarly if a mixture of alcohol and water is distilled at a suitable temperature the first portions condensed are relatively rich in the more volatile component alcohol. By repetitions of the process two liquids can be entirely separated if their boiling points are not too close together.

EXAMPLES ON CHAPTER VIII

1. A and B are two barometers. A has a little air above the mercury while B has a little air and a drop of water. The readings of the barometers happen

to be equal at the temperature of the room. Will they still be equal when the temperature is raised or lowered, and if not which will give the higher readings? (L. '93.)

2. A bubble of air is stuck on the side of a vessel in the interior of a mass of liquid. Show that its volume tends to become very great as the boiling point of the liquid is approached.

3. Describe carefully the difference between evaporation and boiling. What effect has the presence of air above the liquid in each case? Why does ether boil at a lower temperature than water? (L. '97.)

4. A barometer tube dipping into a mercury reservoir contains a mixture of air and saturated vapour above a column of mercury which is 70 cms. above the tube in the reservoir, the atmospheric pressure being 76 cms. What is the height of the mercury column when the tube is depressed so as to reduce the volume occupied by the air to half its original value, the pressure of the saturated vapour being 1·5 cms.? (L. '08.)

5. A quantity of air saturated with aqueous vapour occupies a volume of 120 c.cs. at 18° under a pressure of 74 cms.; the pressure is increased to 150 cms., the temperature remaining constant, and the volume is found to be halved. Find the vapour pressure.

CHAPTER IX

HYGROMETRY

Relative Humidity.—In popular language we frequently speak of the atmosphere as dry or moist, but it is easy to see that our sensations may lead us into error concerning its physical state. Thus on a summer morning when there is a slight mist and dew we say the air is moist, while later in the day we call it dry, in spite of the fact that it then contains more water vapour owing to the evaporation of the particles of dew. We are evidently influenced in our judgment by the fact that in the early hours the air is saturated with moisture, but later, owing to the rise in temperature, it is far removed from this condition. To be accurate we must compare the masses of water vapour contained by a given volume of air at the two different times. **The ratio of the mass of water vapour in a given volume of air to the mass required to saturate it at the same temperature is called its relative humidity.** This is usually expressed as a percentage; thus if a certain volume contains 1 gm. of the vapour, while the amount it would contain if it were saturated is 8 gms., the relative humidity is $1/8 \times 100 = 12{\cdot}5$ per cent. Instruments used to determine this ratio are called hygrometers.

The Chemical Hygrometer.—The relative humidity can be found directly by the chemical hygrometer (Fig. 52). The U-tubes are filled with dry calcium chloride, weighed, and connected to an aspirator, which is merely a large bottle full of water with holes closed by stoppers at the top and bottom. When the water is run out air is drawn over the chloride into the bottle; as it passes through the tubes its moisture is abstracted by the drying agent and the amount m absorbed is found by reweighing the tubes. The experiment is then repeated, but the air is made to bubble through water at the temperature of the room, so as to become saturated before it reaches the drying tubes. A further weighing gives the

amount of moisture m' in the saturated air; the relative humidity is $100\, m/m'$. This apparatus is little used, for, although it is capable of very accurate results, the simpler methods given below provide an accuracy which is sufficient for most purposes and require much less time and trouble.

The Dew-point.—Since unsaturated vapours obey Boyle's law very approximately the vapour pressure of the moisture in the atmosphere will be proportional to the amount present. If this holds up to the saturation point the ratio m/m' of the last paragraph can be found by first measuring the pressure f of the vapour in the air, and then, from Regnault's results (p. 88), finding the maximum vapour pressure F at the same temperature; then

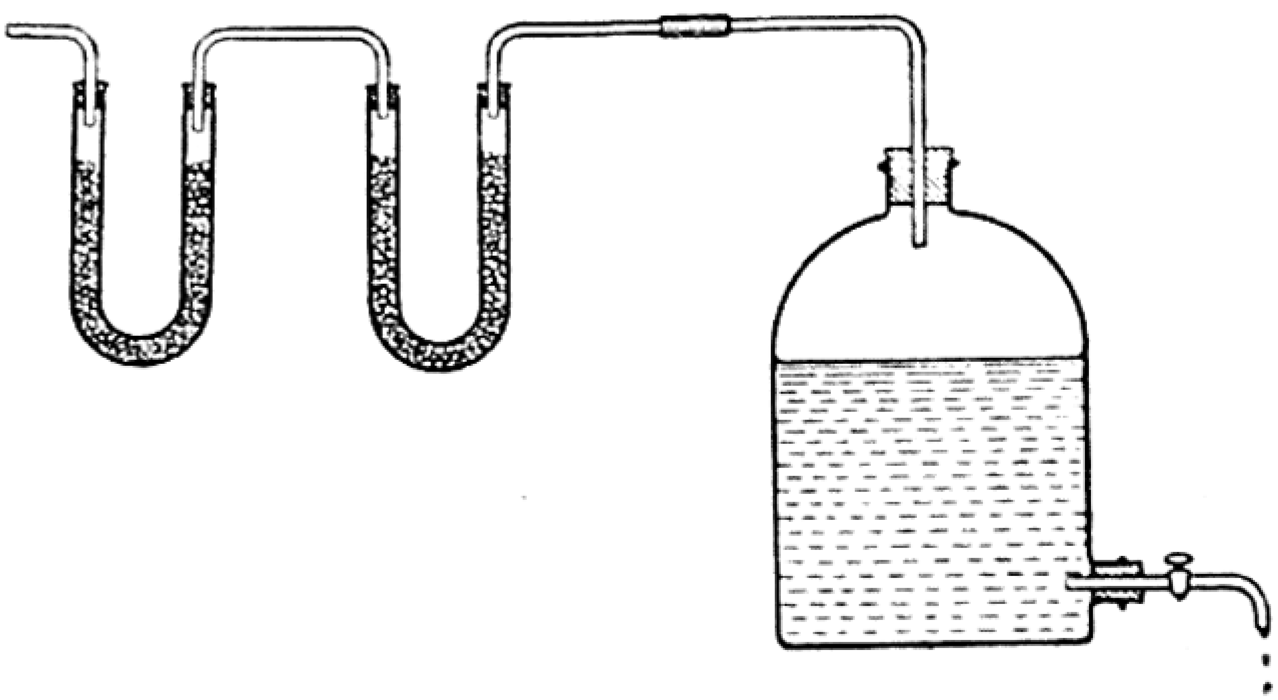


FIG. 52.—Chemical Hygrometer.

$m/m' = f/\mathrm{F}$. Experiments with the chemical hygrometer show that this relation holds with sufficient accuracy for determinations of the relative humidity. When damp air is cooled a temperature is reached at which the moisture it contains is sufficient to produce saturation; any further cooling causes the water vapour to be condensed on surrounding objects in the form of dew. The temperature at which this occurs is called the **dew-point.** The following considerations show that by determining the dew-point we can find the pressure f of the vapour in the air. Let a quantity of air in communication with the rest of the atmosphere be cooled; gas and vapour contract according to the same law, and as the joint pressure is equal to that of the atmosphere the pressure of each is unchanged. Hence the vapour pressure at the dew-point is the same as in the original uncooled air. But

the vapour pressure at the dew-point is known from Regnault's tables, since it is the maximum vapour pressure at that temperature, hence f can be found.

EXAMPLE.—The temperature of the air is 16° and the dew-point is 8°; find the relative humidity.

From Tables we find that the maximum vapour pressures at these temperatures are 13·51 mms. and 7·99 mms. respectively; hence the relative humidity is $\frac{7 \cdot 99 \times 100}{13 \cdot 51} = 59 \cdot 1$ per cent.

Three types of dew-point instrument are described below.

Daniell's Hygrometer.—This is very similar in principle to the cryophorus. The two communicating bulbs (Fig. 53) contain ether

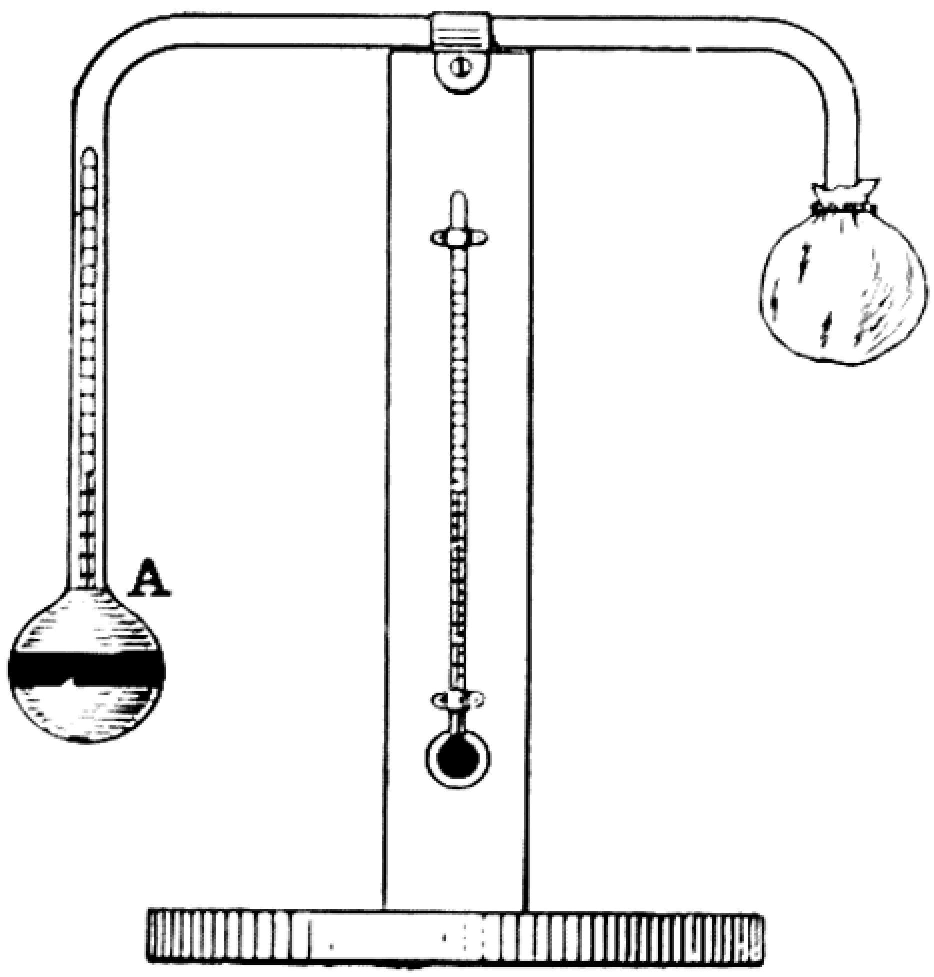


FIG. 53.—Daniell's Hygrometer.

and have been exhausted of air before being sealed. To find the dew-point all the ether is run into one bulb, A, which contains a thermometer, the second bulb is wrapped in muslin and a little ether is poured on it. The rapid evaporation from the wet material cools the bulb and the ether vapour inside it is condensed. As in the cryophorus, this results in a rapid distillation of ether from the other bulb and its temperature falls to the dew-point. In order that the deposit of moisture may be seen easily a bright band of metal is wrapped round the glass. The temperature at which dew begins to

form is read by the enclosed thermometer, the apparatus is then allowed to heat up and the temperature at which the film of moisture disappears is also noted. If the two do not differ by more than a fraction of a degree their mean is taken as the dew-point. The original air temperature is given by a second thermometer on the stand of the instrument. From these observations the relative humidity can be found as in the example given. A Daniell's hygrometer is incapable of giving accurate results for (1) The enclosed thermometer gives the temperature at the middle of the liquid while evaporation takes place at the surface; (2) Since glass is a bad conductor of heat (Chap. XI.) the temperatures inside and outside the bulb may differ appreciably; (3) The ether that evaporates from the muslin may influence the dew-point; (4) Unless the bulb is observed through a telescope water vapour may be deposited from the observer's breath; (5) The rate of cooling cannot be controlled. These chances of error are largely removed in the two instruments described below.

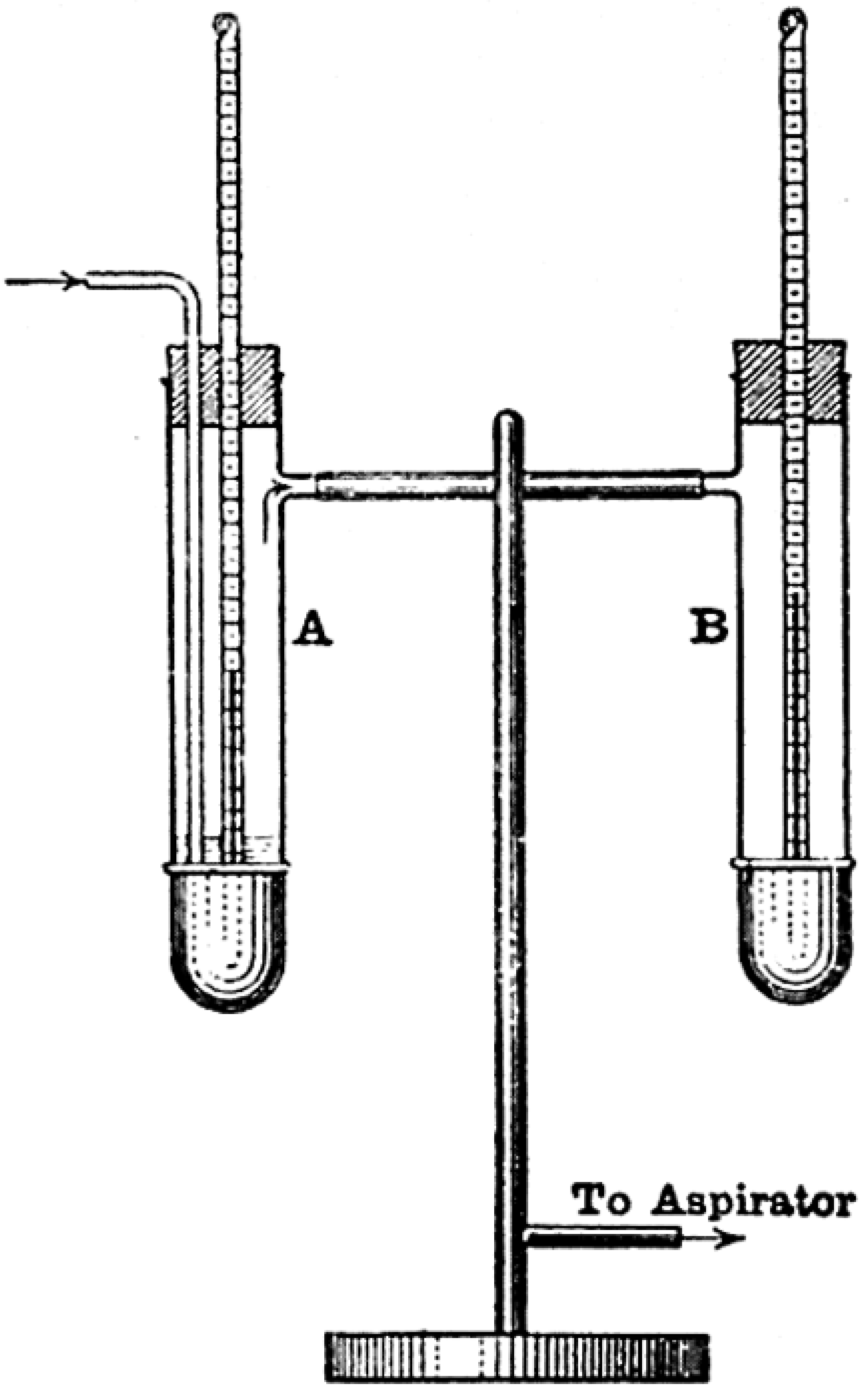


FIG. 54.—Regnault's Hygrometer.

Regnault's Hygrometer.—The lower end of a test-tube, A (Fig. 54), is replaced by a thin silver cap in which is placed some ether surrounding the bulb of a thermometer. A piece of quill tubing passes through a cork at the upper end and extends nearly to the bottom of the tube. A side tube is connected with an aspirator placed at a distance. When water is run out a current of air is drawn through the ether in the direction of the arrows; this serves the double purpose of causing a fall of temperature through rapid evaporation, and at the same time keeps the liquid well stirred. Bubbling is continued until dew is deposited on the silver; the temperature is then noted and the air current stopped. As the temperature rises again the

thermometer is read at the moment when the dew disappears. The observations are made through a telescope, and, in order that the film of moisture may be more easily detected, a second tube, B, is provided similar to the first, so that the two silver surfaces may be compared. A thermometer in the second tube gives the temperature of the air. As silver is a good conductor of heat the temperature of the ether is very little different from that of the outer surface of the cap. An additional advantage of the instrument lies in the ease with which the rate of cooling can be controlled by regulating the outflow of water from the aspirator.

Dines' Hygrometer.—This is a very simple and efficient form of apparatus; a section is shown in Fig. 55. The reservoir A com-

Fig. 55.—Dines' Hygrometer.

municates through a tube with a shallow chamber B which has an upper and lower compartment. Into the upper division the bulb of a thermometer projects, and the chamber is closed above either by a thin piece of black glass or silvered mica on which the deposit of dew may easily be seen. To make an observation the reservoir is filled with a mixture of ice and water and the cold liquid is allowed to flow below the thermometer chamber until dew appears, the temperature is then taken. The flow is stopped at once and the temperature is observed at which the dew disappears. The rate of cooling can be regulated by the tap. In some forms a second reservoir is provided from which tap water is allowed to flow past the thermometer when the disappearance of the dew is being observed.

Wet and Dry Bulb Hygrometer.—For many purposes the dew-point can be obtained with sufficient accuracy by means of a wet and dry bulb hygrometer. This consists of two thermometers (Fig. 56),

round the bulb of one is loosely wrapped some muslin or cotton wool which dips into a small vessel of water placed immediately below; the second thermometer gives the temperature of the air. Owing to evaporation from the large surface exposed by the muslin the temperature of the wet bulb is lower than that of the other thermometer. It is easily seen that this temperature difference is connected with the humidity of the atmosphere, for if the air is dry evaporation will be rapid and the difference of temperature will be large; when no evaporation takes place the two thermometers will read alike. By comparison with one of the instruments described above a table may be constructed from which the dew-point can be found when the temperatures of the two thermometers are known.[1]

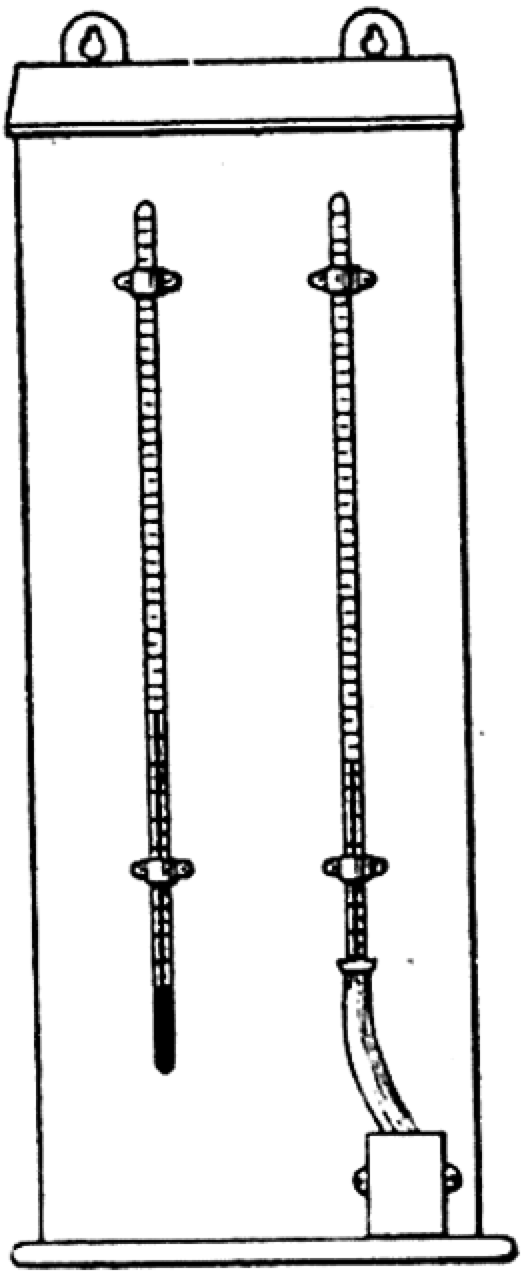


FIG. 56.—Wet and Dry Bulb Hygrometer.

Weight of a given Volume of Moist Air.—Since a body apparently loses weight when immersed in a fluid it will weigh more in vacuo than in air. In very accurate work all weighings must be reduced to vacuo; to do this we must calculate the weight of air displaced. Experiments with Hoffmann's apparatus show that the density of water vapour is 0·62 that of dry air at the same temperature and pressure, and the weight of a litre of dry air at N.T.P. is known to be 1·293 gms. Suppose the vapour pressure obtained from dew-point observations is f cms., the height of the barometer is H cms., the temperature $t°$ C., and that we require the weight of V litres of this moist air. The pressure of the air alone is $(\mathrm{H}-f)$, hence its weight is (see Ex. p. 70)

$$m_1 = 1{\cdot}293\mathrm{V} \times \frac{273}{273+t} \cdot \frac{(\mathrm{H}-f)}{76} \text{ gms.}$$

Also the pressure of the vapour is f, hence its weight alone is

$$m_2 = 0{\cdot}62 \times 1{\cdot}293\mathrm{V} \cdot \frac{273}{273+t} \cdot \frac{f}{76} \text{ gms.}$$

[1] See also Barton and Black, "Practical Physics," pp. 72–76.

The weight of V litres of moist air is $(m_1 + m_2)$, if therefore the volume of a body is known the mass of air it displaces can be calculated and its weight in vacuo found.

Vapour Pressure of Solutions.—The principle of the chemical hygrometer is used in measuring the vapour pressure of solutions. The experiment is conducted in the manner described on p. 98. A certain volume of air is bubbled through pure water and the vapour it contains is absorbed by calcium chloride and weighed. An equal volume is next passed through the solution, which is at the same temperature as the water, and the mass of vapour found as before. From p. 98 these masses are proportional to the vapour pressures, and as this quantity is known for water that of the solution can be calculated.

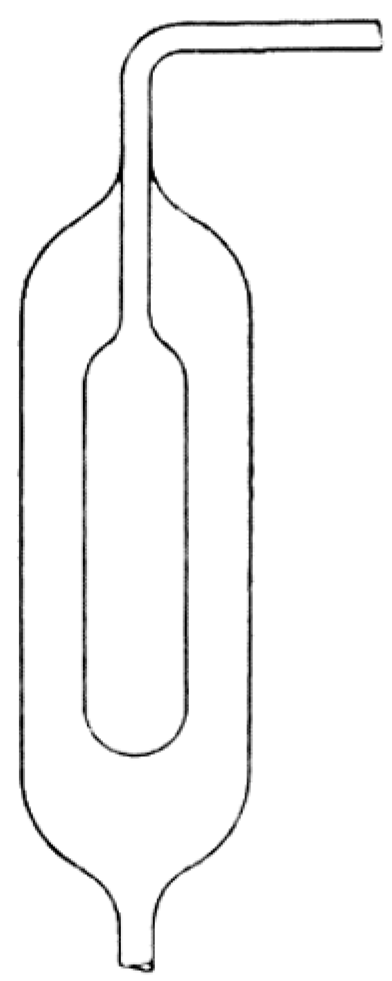


FIG. 57.

Formation of Cloud and Fog.—When the temperature of a moisture-laden atmosphere is sufficiently reduced the aqueous vapour it contains is condensed into small droplets of water forming a mist or fog. If the drops are at a high altitude they form clouds. The necessary cooling may be caused by the air expanding as it gradually rises to the upper layers of the atmosphere; it is found also that dust particles make it easier for a fog to form.

EXPERIMENT.—Replace one of the bulbs of a Looser thermoscope (p. 31) by the glass vessel shown in Fig. 57. The inner tube is connected to the thermoscope and the outer vessel to a bicycle pump. Pump in air, the compression raises the temperature and the index moves. Close the connection to the pump by a pinch-cock; the pump is now removed and the compressed air allowed to attain a steady temperature, when this is reached open the cock quickly, the gas expands and the temperature falls.

EXPERIMENT.—Shake a litre flask containing a little water so as to saturate the air. Pass through the rubber stopper two glass tubes, one connected with a bicycle pump the other closed with a pinch-cock. Compress the air with two strokes of the pump, then, after waiting a few seconds, allow it to expand suddenly by opening the cock. The air is cooled and a fog is formed. If a tube about a foot long tightly plugged with wet cotton wool is introduced between the pump and flask, so as to remove dust particles from the air which enters, the fog is largely reduced. On the other hand it is much denser if some smoke from burning paper is first introduced. It is doubtless due to this effect of dust particles that fogs are so common in large towns,

EXAMPLES ON CHAPTER IX

1. One hundred c.cs. of oxygen, saturated with water, are collected at a pressure of 740 mm. and a temperature 15°. Find the volume of dry oxygen at 0° and 760 mm. pressure, having given that the maximum vapour pressure of aqueous vapour at 15° is 12·7 mm. (L. '87.)

2. Find the weight of 10 litres of laboratory air from the following data: Temperature 15°, pressure of the aqueous vapour present = 8 mm., height of the barometer 770 mm., density of dry air at 0° and 760 mm. = 1·293 gms. per litre, density of aqueous vapour is 0·6 of the density of air under similar conditions.

CHAPTER X

FIRST LAW OF THERMODYNAMICS. MECHANICAL EQUIVALENT OF HEAT

In the preceding pages we have frequently supposed that there is some connection between the heat contained by a mass of gas and the kinetic energy of its molecules. It has also been found necessary in the case of an expanding gas to assume some relation between heat and work to account for the difference between the specific heat at constant pressure and that at constant volume (p. 35). It will be shown in this chapter that heat is a form of energy and that other forms of energy may be converted into heat.

Units.—In the centimetre-gram-second (C.G.S.) system of units the unit of force is the dyne ; it is that force which, acting on 1 gm. for 1 sec., gives to it a velocity of 1 cm. a second. In the English system the unit is that force which, acting for 1 sec., imparts to 1 lb. of matter a velocity of 1 ft. per second ; it is called the poundal. When a force F moves a body through a distance s, measured parallel to the direction in which the force acts, Fs units of work are expended. If F is in dynes and s in cms. the work is given in ergs ; when F is in poundals and s in feet the work is expressed in foot-poundals. The weight of 1 lb. is sometimes used as the unit of force. It is shown in books on mechanics that 1 lb. $= g$ poundals, where g is the acceleration due to gravity. When a force of 1 lb. moves its point of application through 1 ft. in a direction parallel to the force 1 ft.-lb. of work is done. Work is expended against the force when the motion is in the opposite direction to that in which the force acts. Thus if a flywheel of radius R cms. is forced round in opposition to a frictional force of F dynes applied to its rim, during each revolution a point on the rim is moved through a distance 2πR cms., and the work done is F . 2πR ergs. Let two equal and opposite forces F be applied to the ends of a lever of length d and at right-angles to it. During a

revolution each force does $2\pi \cdot \frac{d}{2} \cdot F$ units of work, and the whole work done by the couple is $Fd \cdot 2\pi$, *i.e.* it is equal to the product of the moment of the couple and the angle in radians through which the arm is turned.

The calorie has already been defined, but another unit of heat is sometimes used: it is the amount of heat required to raise 1 lb. of water through 1°, either Fahr. or Centigrade; this is called the lb.-degree unit.

Experiments showing that Heat is a Form of Energy.—Numerous experiments show that heat may be generated by the expenditure of work. Thus a hundred years ago Davy showed that two blocks of ice could be melted by rubbing them together, the heat generated by moving them against the frictional forces was sufficient to cause melting. Similarly Count Rumford observed that during the process of boring a cannon from a solid block of metal sufficient heat was generated to boil a large quantity of water. The amount of heat gained was conditioned entirely by the amount of work expended in driving the drill. The method used by some savage tribes to light a fire is a parallel case, a blunt wooden point is caused to rotate rapidly in a shallow hole cut in a block of wood, enough heat is thus produced to kindle a flame. A block of metal is appreciably warmed by hammering, and the lower end of a bicycle pump is heated on account of the work expended in compressing the air.

EXPERIMENT.—Compress the air in the experiment with a Looser's thermoscope, p. 103; notice that its temperature rises. This is an instance of a nearly adiabatic compression (p. 113).

Mechanical Equivalent of Heat.—The first experiments to show the numerical relation between the work done and the heat produced are due to Joule; the object was to expend a known amount of work in the production of heat and to measure the heat developed. The results showed that **no matter how the work was done, the ratio of work done to the heat generated was constant.** This is the first law of thermodynamics. In symbols, if W is the work expended in the production of H units of heat then $W/H = J$, or $W = HJ$, where J is a constant called the mechanical equivalent of heat. Modern experiments show that $J = 4{\cdot}18 \times 10^7$ if W is measured in ergs and H in calories. The equation therefore tells us that to generate one calorie $(H = 1)$ $4{\cdot}18 \times 10^7$ ergs of work must be done.

Joule's Experiments.—In these experiments work was expended in churning water contained in a calorimeter and the resulting temperature rise θ was measured. If M was the total water equivalent in grams of the calorimeter and its contents the heat generated was $M\theta$ calories. The apparatus shown in Fig. 58 was used to measure the work expended. The water was churned by a paddle carrying a number of vanes, these passed between a system of fixed vanes attached to the walls of the calorimeter (see figure below). To prevent conduction of heat from the vessel as its temperature

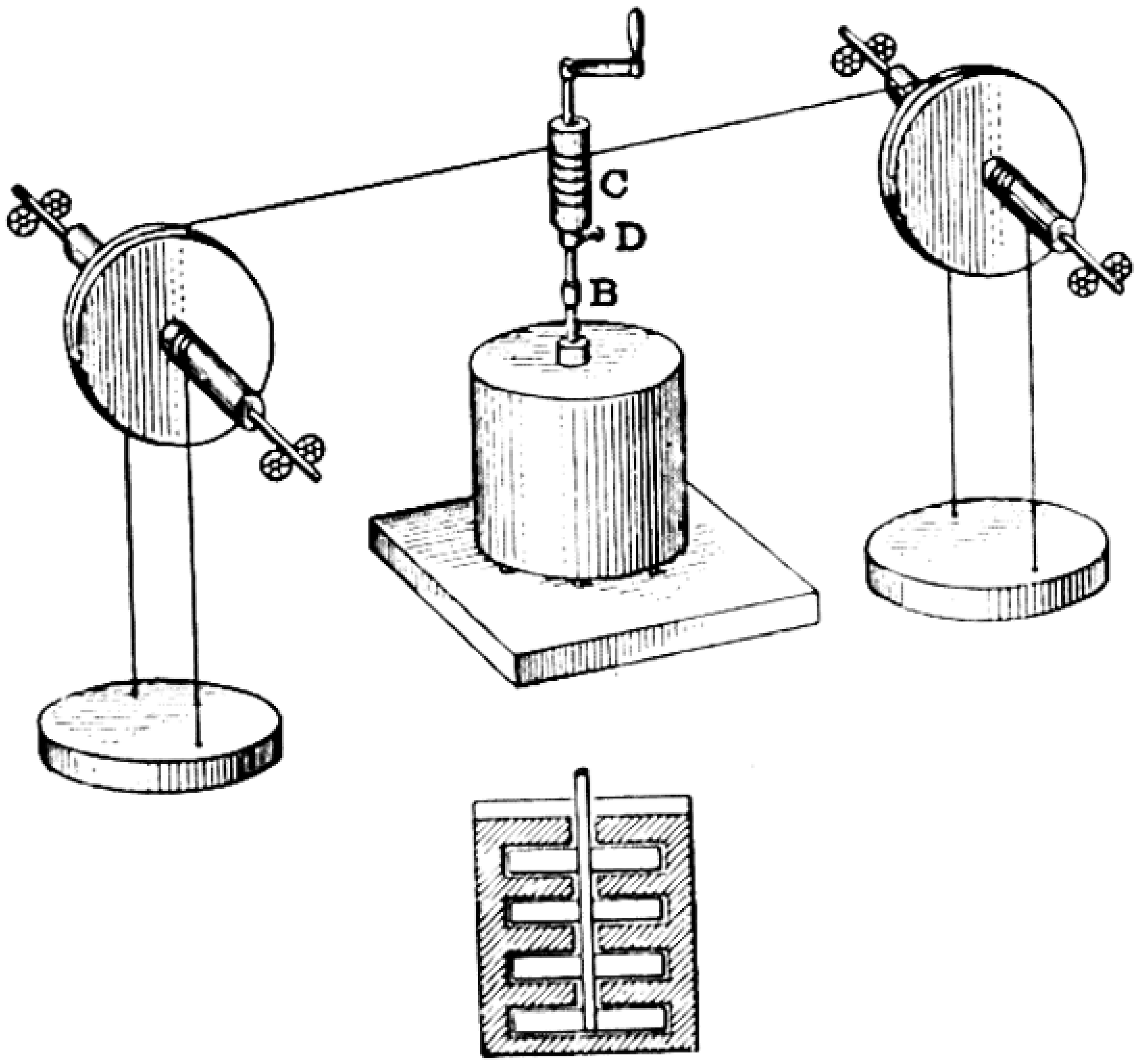


FIG. 58.—Joule's first Apparatus for determining the Mechanical Equivalent.

rose, the metal axis of the paddle was interrupted at B by a boxwood cylinder. A flexible cord passed round the wooden drum C and its ends were wound on to two large pulleys supported on friction wheels. The pulleys carried equal weights, which were supported by strings wound round the axles, their height from the ground could be read off vertical scales. When the weights were allowed to fall they made the pulleys revolve and the paddle was put in motion. The pin D was then quickly removed, when the weights could be wound up again by a handle without turning the paddle. The fall experiment was repeated a large number of times and the temperature of the water was read at frequent intervals. Let m be the mass

in grams of each of the weights, h the height in cms. through which they fell, then their joint potential energy in their highest position was $2mgh$ ergs. If all this was expended in churning the water the total work done in n falls was $W = 2nmgh$ ergs, hence

$$\frac{W}{H} = J = \frac{2nmgh}{M\theta} \text{ ergs per calorie.}$$

To obtain an accurate result several corrections must be applied of which the following are the chief :—

(1) As the calorimeter is at a higher temperature than its surroundings it will lose heat by radiation and conduction during the half-hour or so that the experiment lasts. The observed rise in temperature will therefore be too small. The necessary correction can be found by noting the rate of cooling at the beginning and end of the other observations. Conduction losses are reduced by standing the vessel on a badly conducting base.

(2) The weights are moving with a velocity v cms./sec. when they reach the ground, and each has kinetic energy $\frac{1}{2}mv^2$ ergs ; this must be subtracted from their original potential energy to get the work done in turning the pulleys and paddle. Hence the work done in n falls is $2n(mgh - \frac{1}{2}mv^2)$. As it was found that the weights moved with a uniform velocity before they reached the ground, v could be observed by noting the time taken to move over a measured distance near the end of their path.

(3) A certain amount of work is spent in overcoming friction in the moving parts *outside* the calorimeter. To determine the frictional force the drum C was disconnected from the paddle at D and the cord from the pulleys was passed round it in such a manner that when one weight fell it caused the other to rise. A mass m_1 gms. was placed on one weight to make it fall, this additional mass being so chosen that the motion was uniform. The frictional resistance was therefore m_1g dynes and the total work done against it was $nh \cdot m_1g$ ergs. Hence the total work expended in churning the water was

$$[2n(mgh - \tfrac{1}{2}mv^2) - nm_1gh] \text{ ergs.}$$

Actually Joule took as the unit of work the ft.-lb. and for the heat unit the quantity of heat required to change the temperature of 1 lb. of water by 1° Fahr. With these units J was found to be 772. In other experiments he used an iron paddle to stir mercury in an iron vessel ; he also measured the work spent in compressing

air into a reservoir immersed in a calorimeter. The value found for the mechanical equivalent was practically the same in every case.

Rowland's Experiments.—Considering the small rise of temperature obtained, which was about half a degree, Joule's results are surprisingly consistent, but the most accurate experiments by the method of churning water are those of Prof. Rowland. In these the temperature rose at the rate of $0{\cdot}5^\circ$ per minute. The calorimeter and stirrer were similar to Joule's except that the paddle projected through the base and was turned by a steam engine (Fig. 59). The

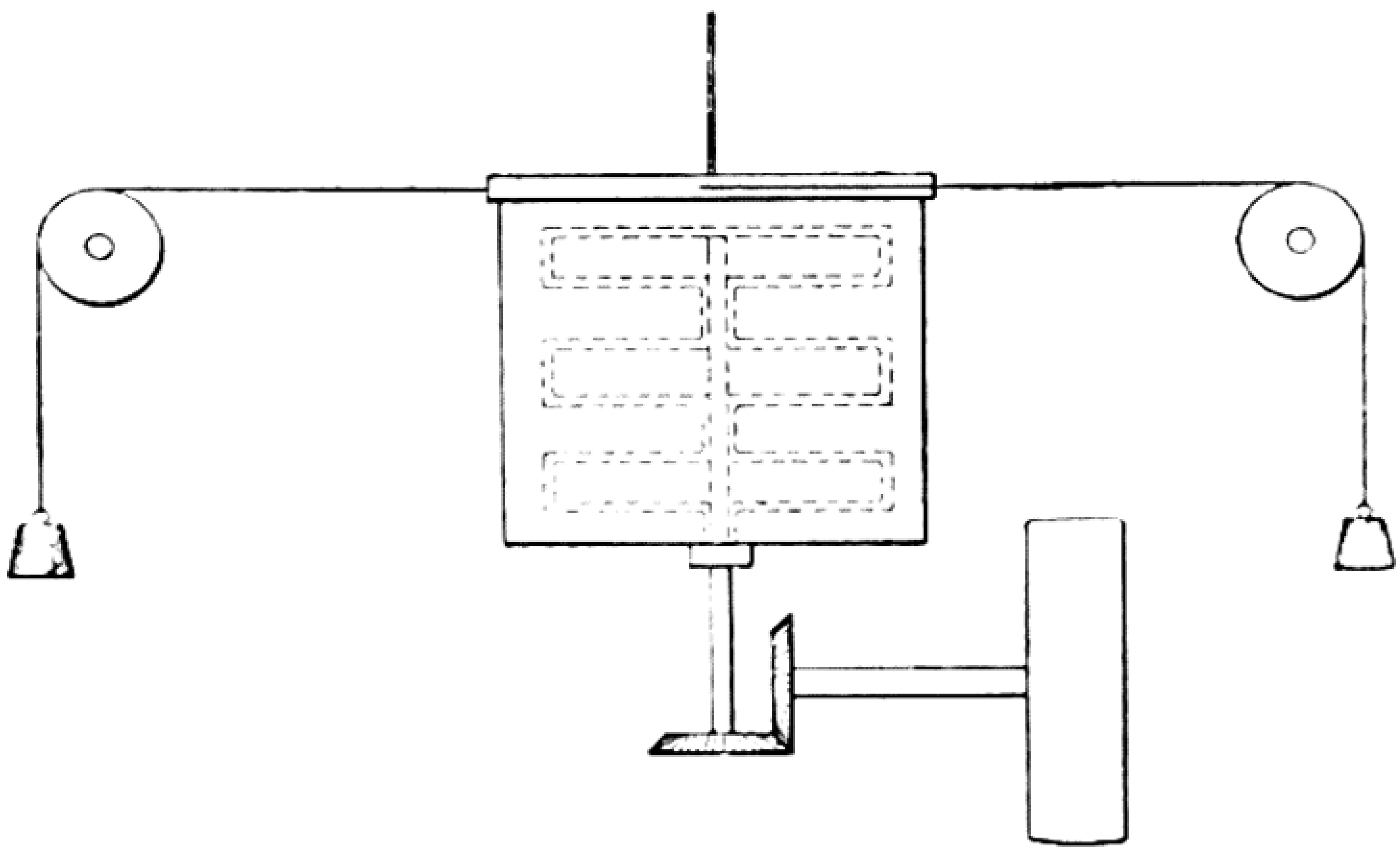


FIG. 59.—Diagrammatic Sketch of Rowland's Apparatus.

top of the calorimeter was fastened to a circular wooden disc which hung from the end of a thin wire. When the paddle turned the friction of the water tended to move the calorimeter in the same direction, but its motion was prevented by passing a string round the disc and hanging equal weights from the ends. If d is the diameter of the disc and m the mass of one weight, the moment of the couple which stops the motion is mgd. Now the water exerts equal and opposite couples on the calorimeter and the paddle, hence the moment against which the latter is forced round is $mg \,.\, d$, and the work done in one revolution is $2\pi \,.\, mgd$ (p. 106). The work expended during n revolutions is therefore known, and as the heat developed can be measured, J can be found as before. The radiation losses are relatively much smaller than in Joule's experiments.

A Laboratory Method of determining the Mechanical Equivalent.— A simple apparatus can be used for this purpose. An outer brass cone, A, shown in section in Fig. 60, is fixed in ebonite to the base of a brass cylinder and is held in position by a ring of ebonite near the top (this substance is a bad conductor). A second brass cone, B, fits smoothly in the first and is attached at its upper end to a circular wooden disc of diameter d. The inner cone contains a known mass of water, a stirrer, and thermometer. By means of an endless band going to a small motor the outer cone is rapidly rotated; the friction

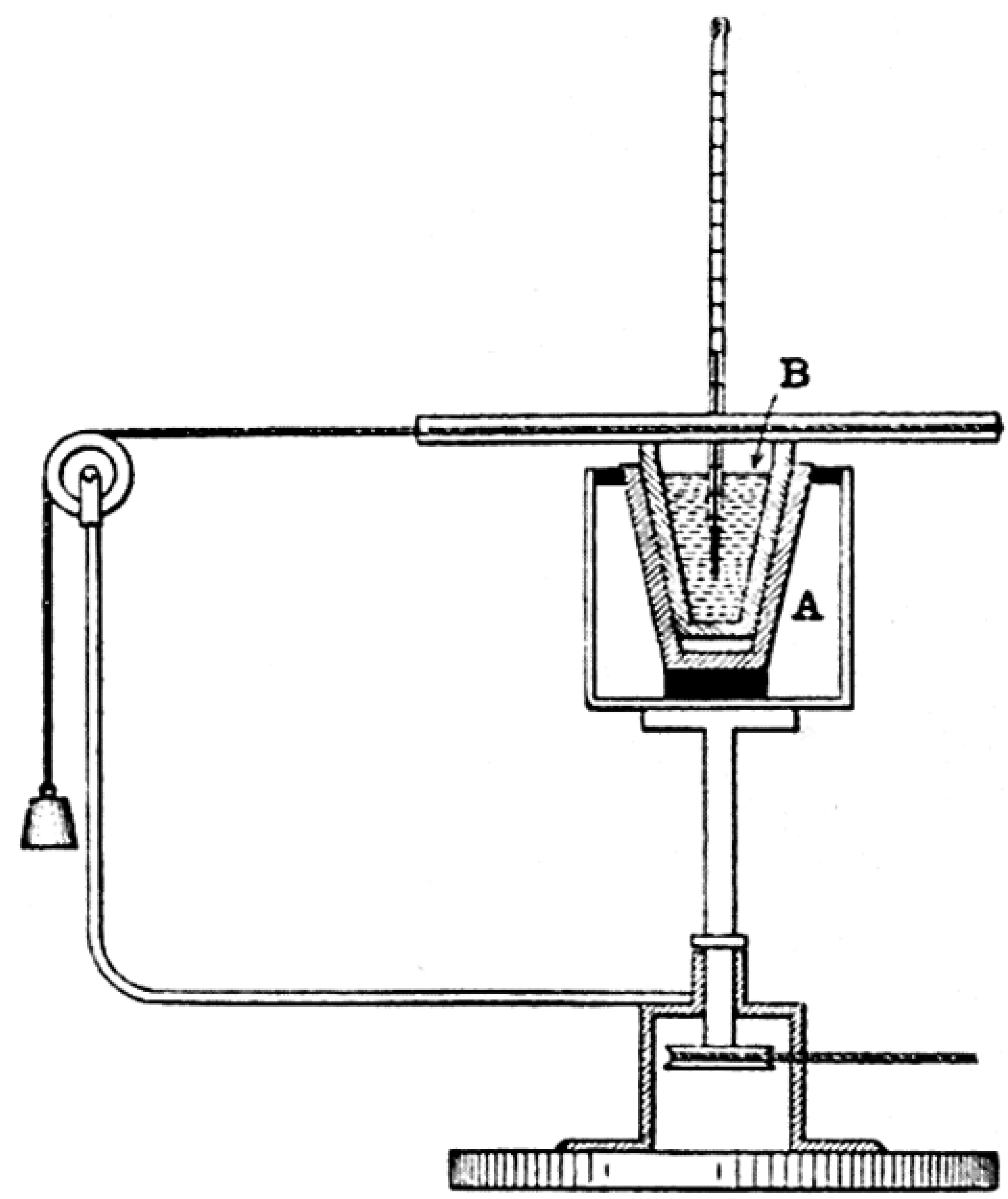


Fig. 60.—Laboratory Method of finding the Mechanical Equivalent.

between the two tends to make the inner cone follow in the same direction, but this is prevented by a weight of m gms. fastened to a string passing round the wooden disc. By suitably adjusting the speed and the weight the latter can be kept floating in the air, and as the couples on the cones are equal and opposite the motion of the inner cone is opposed by a couple whose moment is $mg \cdot d$, and the work done in n revolutions is $mgd \cdot 2\pi n$ ergs. Knowing the total water equivalent of the two cones and their contents, the heat developed by the friction in n revolutions can readily be found and J calculated.

Work done by a Gas expanding against a Uniform Pressure.—Let a quantity of gas be confined in a cylinder which is closed by a light piston of area S, and suppose the gas to expand, pushing the piston out a distance x cms. against the atmospheric pressure of p dynes. The increase in volume of the gas is xS cms.^{3} Also the total external pressure on the piston is pS dynes, and the work done during the expansion is $pS \cdot x$ ergs (force $\times$ displacement); *i.e.* the work $= p \cdot \delta v$, where δv is the increase in volume. Let us make use of this result to calculate the work done against the atmospheric pressure when a gram of water at 100° is converted into steam. If the barometer stands at 76 cms. it is known that the increase of volume is 1690 cms.^{3} approximately, and this expansion takes place against the atmospheric pressure. The density of mercury being 13·6, the atmospheric pressure in dynes/cm.^{2} is $13{\cdot}6 \times 76 \times 980 = 1{,}013{,}000$, hence the work done $= 1{,}013{,}000 \times 1690$ ergs. The equivalent of this in calories, taking $J = 42 \times 10^6$, is $\dfrac{1{,}013{,}000 \times 1690}{42 \times 10^6} = 40{\cdot}7$ cals. This accounts for part of the latent heat of vaporisation, the remainder is spent in pulling the molecules of water apart against their mutual attraction.

Calculation of J from the Two Specific Heats of Air.—Let a gram of air at a pressure p_1 dynes and absolute temperature T_1° occupy v_1 cms.^{3}. To raise its temperature 1° requires C_v cals. if the volume is kept constant, C_v being the specific heat at constant volume. On the other hand, if the pressure is constant the gas expands to a new volume v_2, and work equal to $p_1(v_2 - v_1)$ ergs is done on account of this expansion; an amount of heat C_p cals. must be supplied in this case, C_p being the specific heat at constant pressure. The difference $(C_p - C_v)$ cals. is used to provide the work done in expanding; multiplying by J to bring this to ergs we have two expressions for the work done, and these must be equal, hence

$$p_1(v_2 - v_1) = J(C_p - C_v)$$

But from the gas equation (p. 69)

$$p_1 v_1 = RT_1 \quad \text{and} \quad p_1 v_2 = R(T_1 + 1)$$

hence $\quad p_1(v_2 - v_1) = R \;$ and $\; J(C_p - C_v) = R$

But $R = p_1 v_1/T_1 = p_0 v_0/273$, where p_0, v_0, are the pressure and volume of 1 gm. of air at 0° C. or 273° absolute. From measurements of the

density of air it is known that 1 cm.^{3} at N.T.P. weighs 0·00129 grms., hence the volume of 1 gm. under these conditions is $= 1/0·00129$ cms.$^3 = v_0$. Also $p_0 = 1{,}013{,}000$ dynes,

$$\therefore R = \frac{1{,}013{,}000}{273 \times 0·00129}$$

Now $C_p = 0·2375$ cals. and $C_v = 0·169$ cals.; substituting these three values in the equation $R = J(C_p - C_v)$ we get $J = 42·0 \times 10^6$

In making this calculation we have assumed that all the work done by the gas in expanding is spent against the external pressure. It is, however, possible that some work is done in pulling the molecules apart against their mutual attractions, just as work has to be done when a weight is raised from the ground against the attraction of the earth. Joule was the first to show that this effect was negligible; his apparatus is shown in Fig. 61. A metal reservoir, A, was filled with dry air at a pressure of 22 atmospheres; another reservoir, B, was exhausted and joined to the first through a tube furnished with a stop-cock. The two were placed in a calorimeter containing water and when the temperature had become steady the stop-cock was opened. No work was done against the external pressure since B was exhausted, but if the molecules exercised an appreciable attraction on each other work would be spent in increasing their distance apart. This internal work would use up some of the heat energy of the gas and the temperature would fall. As it was found that the temperature did not change appreciably Joule concluded that no internal work was done; the calculation just given is therefore justifiable. Later experiments by Joule and Thomson, in which a more delicate method was used, have shown that this conclusion is not strictly true. These experiments we shall not attempt to describe.

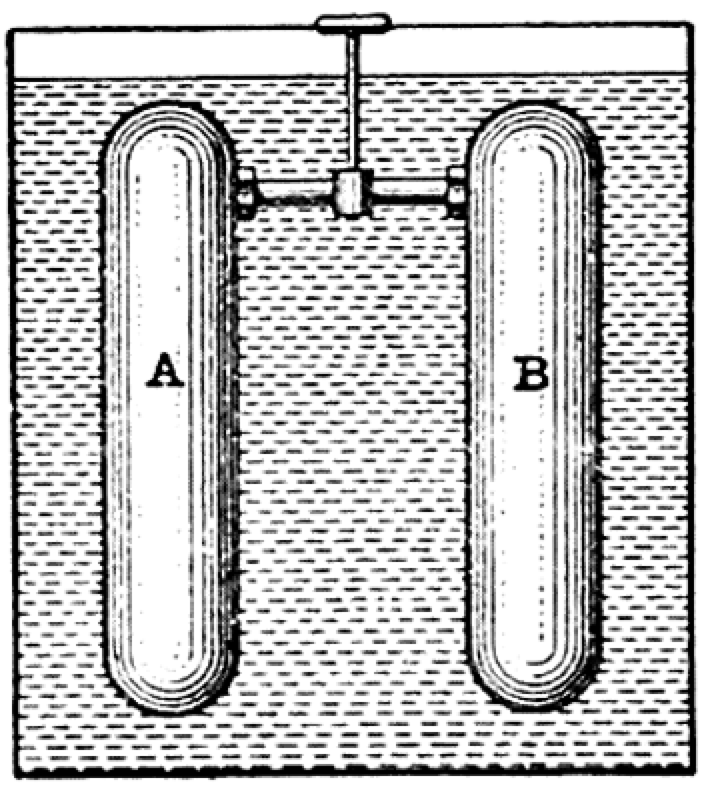


FIG. 61.—Joule's Apparatus to show that the Internal Work done by an Expanding Gas is Zero.

Other Methods of finding J. Conservation of Energy.—Several methods other than those already given have been used to determine

the mechanical equivalent. Thus when an electric current passes along a wire heat is generated, the energy in ergs can be measured electrically, and hence J can be found; this method is described on p. 406. The nett result of all such experiments is to show that energy in all its forms, chemical, mechanical, potential, or electrical, may be converted into heat, and to generate one calorie requires the expenditure of $4{\cdot}18 \times 10^7$ ergs. We may regard heat as a kind of common denominator to which all other forms of energy can be reduced. Also experiment shows that the various forms of energy are interchangeable; thus the potential energy of water at the top of the Zambesi Falls is convertible into the kinetic energy of a water turbine, and this is made to drive a dynamo which generates electrical energy; the chemical potential energy in coal is transformed into mechanical energy in the steam engine and so on. In all cases, as far as experiment goes, it is found that no energy is lost, it merely changes its form. This statement is called the law of **conservation of energy**; it is one of the most important discoveries of modern science.

Isothermal and Adiabatic Changes.—Any variation in the state of a system which takes place at constant temperature is called an isothermal change. The fusion of a solid at its melting point and the vaporisation of a liquid at its boiling point are instances of such. Similarly Boyle's law gives us the isothermal relation between the pressure and volume of a perfect gas. An adiabatic change is one that takes place without heat entering or leaving the system in question. Thus the expansion of a gas as in the experiment on p. 103 is an adiabatic expansion because it takes place so quickly that heat cannot flow into the gas from the surroundings while the change in volume is proceeding. As work is done in this expansion and no heat is supplied, a portion of the heat energy of the gas is converted into work and the temperature falls. The converse happens when air is suddenly compressed in a bicycle pump, work is done on the gas and, as no heat leaves it, its energy is increased; this appears as a rise in temperature. It can be shown that if the pressure and volume of a gas are changed adiabatically from p_1, v_1 to p_2, v_2, respectively, these quantities are connected by the equation $p_1v_1^\gamma = p_2v_2^\gamma$, where $\gamma = C_p/C_v$, the ratio of the specific heats. This is the adiabatic relation corresponding to the isothermal one given by the Boyle's law equation. We shall meet with adiabatic changes in the section on sound.

EXAMPLE.—A quantity of air at 76 cms. pressure is suddenly compressed to half its volume, calculate the new pressure.

Here $$v_1/v_2 = 2 \quad \text{and} \quad \gamma = 1{\cdot}4.$$

$$p_2 = p_1\left(\frac{v_1}{v_2}\right)^{\gamma} = 76\,(2)^{1\cdot 4}$$

$$\therefore \log p_2 = \log 76 + 1{\cdot}4 \log 2$$

From a table of logarithms we find $\log p_2 = 2{\cdot}3022$ and $p_2 = 200{\cdot}5$ cms.

Had the compression taken place isothermally the final pressure would have been 152 cms.; we see then that the resistance to an adiabatic change is greater than it is to one which takes place at constant temperature.

EXAMPLES ON CHAPTER X

1. An engine consumes 40 lbs. of coal of such calorific power that the heat developed by the combustion of 1 lb. is capable of converting 16 lbs. of water at 100° into steam at the same temperature, and during the process the engine performs 16×10^6 ft.-lbs. of work. What percentage of the heat produced is wasted? [Lat. ht. of steam = 536.] (L. '81.)

2. Describe Joule's method of determining the mechanical equivalent of heat from the expansion of compressed air. Explain what happens when air is allowed to expand into a vacuum. (L. '82.)

3. Distinguish between the specific heat of air at constant pressure and at constant volume, and show how to determine the latter when the former, together with the mechanical equivalent of heat, are known. (L. '83.)

4. One gm. of air is heated under constant pressure from 0° to 10°, determine the work in ergs and in gms.-cms. due to the expansion. [Coefficient of expansion = 1/273, 1 c.c. of air at N.T.P. = 0·001293 gms., 1 c.c. of mercury at 0° = 13·596 gm., $g = 981$ cm. secs. (L. '84.)

5. When temperatures are expressed on the Centigrade scale the latent heat of fusion of ice is represented by 80, and the mechanical equivalent of heat by 423·9 (metre-gms.). Express the same quantities on the Fahr. scale and explain why one is represented by a larger and the other by a smaller number. (L. '85.)

6. Water oozes slowly from under a pressure of 20 atmospheres and is collected in a vessel. How much hotter is this water than it was inside? (L. '90.)

7. A quantity of damp air under pressure is suddenly allowed to expand. Describe what happens, and show what has become of the energy of the compressed air. (L. '91.)

8. A lb. of coal in burning can raise 8000 lbs. of water 1°. Used in an engine the coal supplies 1,400,000 ft.-lbs. of work per lb. burnt. What fraction of the heat is transformed to work? [Mech. equiv. = 1400 ft.-lbs.-deg. Cent.] (L. '94.)

9. Water at 15° C. and 1000 atmospheres pressure is passed through a porous plug and escapes at 1 atmosphere pressure. Calculate the temperature of the escaping water, given 1 atmosphere = 10^6 dynes per cm.^{2}, and the mechanical equivalent of heat = $4{\cdot}2 \times 10^7$ ergs.

CHAPTER XI

PROPAGATION OF HEAT. CONDUCTION AND CONVECTION

Conduction, Convection, Radiation.—Heat travels from one point to another by three processes named respectively (1) Conduction, (2) Convection, (3) Radiation. As a typical instance of the first we may take the case of an iron bar heated at one end. According to the kinetic theory of matter the molecules of a substance are supposed to be oscillating to and fro, the motion becoming more vigorous as the temperature rises. Owing to collisions the molecules at the hot end share their energy with their slower moving neighbours, these in turn carry energy to the next layer, and so a rise of temperature travels down the bar, although the molecules themselves do not move from their mean positions. In the process of convection the heated particles wander through the substance carrying their heat with them, and by frequent collisions the rise in temperature is diffused throughout the whole mass. Convection currents can occur only in liquids and gases.

Experiment.—Fill a large beaker with cold water and drop down the middle of it a single crystal of potassium permanganate. Heat the beaker immediately under the crystal by a small flame, convection currents can be clearly seen rising up the central portions and returning by the colder sides.

Experiment.—Make a complete rectangle out of glass quill-tubing, fill it with water and drop in a crystal of potassium permanganate. Hold it with its plane vertical and gently heat one side; a convection current rises from the heated part and travels round the tube.

In each of these processes heat is propagated through the intervention of particles of matter; in the third process, radiation, it travels through space from which all matter is removed. For example, at a height of a few hundred miles the density of the atmosphere must be practically nil, yet heat reaches us from the sun through millions of miles of this vacuous space. It is known that the particles of a hot body emit certain waves which carry off its heat

energy; when these fall on matter part of the energy they carry is absorbed causing the molecules of the cold body to oscillate more vigorously, *i.e.* the temperature rises. As we cannot conceive waves travelling through empty space it is supposed that the universe is filled with some medium called the ether; concerning the properties of this medium, beyond its capacity for transmitting waves, we know very little. It is known that the distance between successive waves, the wave-length as it is called, is very small, only a fraction of a mm. (see Chap. XXIII. for an exact definition of wave-length). The effects produced by the waves are dependent on the wave-length, the longer ones produce heating effects, others produce the sensation of light when they fall on the retina of the eye, while the shortest are chiefly notable for the chemical changes they promote. Heat propagated in this manner is sometimes called "radiant heat," but the term is not a good one, for the energy does not appear as heat except when the waves fall on matter; in addition light waves produce heat, and the only difference between the two is in their wave-length. A better term is radiant energy, or merely radiation. Owing to the similarities just mentioned the study of radiation will be deferred for the most part until we come to the section on light.

Experiment.—Cover the bulb of a thermometer with soot by holding it near a smoky flame. Place it at the centre of a small glass flask closed by a rubber stopper, and pump out as far as possible all the air. No matter how far the exhaustion is pushed the thermometer rises in temperature when held near a hot body, showing that radiation can travel across from the flask walls without the aid of matter.

Examples of Heat Conduction.—Silver, copper, and metals generally are good conductors of heat; powders, liquids, and gases are poor conductors. Thus on a frosty morning a metal door-knob feels colder than the wood because it rapidly conducts heat away from the hand, but if the door is exposed to a hot sun the metal part feels hotter than the wood because it conducts more heat to the body. If a piece of thin paper is wrapped round a brass rod it may be held in a flame for a few seconds without being scorched; if the brass is replaced by wood it is scorched at once because the latter substance does not conduct heat away with sufficient rapidity. A sphere of solder can be melted in a small paper bag and water can be boiled in a similar receptacle; in the last instance the heat is carried away by convection currents. It is owing to bad conduction that a lighted match can be held in the hand and glass tube worked in a blow-pipe

flame; for the same reason the lower end of a lighted candle is not melted.

EXPERIMENT.—Replace the bulbs of a Looser thermoscope (Fig. 17) by flat-bottomed flasks with the flat parts uppermost. On one flask place a disc of copper, on the other an equal disc of iron. When a flask of boiling water is placed on each the thermoscope shows that the most heat passes through the copper. If a disc is replaced by a shallow, hollow vessel filled with water very little heat passes through; liquids are very bad conductors. This is strikingly shown by the next experiment.

EXPERIMENT.—Attach a small lump of ice to a sinker and drop it to the bottom of a test-tube nearly filled with water. The tube may now be held in an inclined position and heated near its upper end until the water boils, but sufficient heat is not conducted downwards to melt the ice. If the tube had been heated from below convection currents would have equalised the temperature throughout the mass.

Gases also are bad conductors; woollen clothing is warmer than cotton largely because of the air it entangles, convection currents are set up with difficulty among the fibres of the material, hence heat cannot get through except by conduction. The feathers of birds and down quilts owe their efficacy to a similar cause.

EXPERIMENT.—*Leidenfrost's phenomenon.* Heat a clean sheet of metal to redness and let a few drops of water fall on it. They run to and fro over the surface like mercury on clean glass, but do not boil away furiously as we might expect. At the first contact with the plate a cushion of vapour is formed which prevents heat reaching the liquid except by conduction through this layer or by radiation. With care it is possible to pass a beam of light between the liquid and the plate. Remove the flame; as the plate cools the cushion of vapour becomes unable to support the drop, contact with the plate follows and the liquid boils away rapidly. Owing to the bad conduction of a layer of vapour it is possible to lift a piece of red-hot coal with the fingers without injury provided the hand is first thoroughly wetted.

EXPERIMENT.—Lower a piece of fine copper gauze into a Bunsen flame; the flame appears to be pushed down and does not get to the upper side of the gauze unless it is very hot.

EXPERIMENT.—Fix the gauze in position a couple of inches above the top of the burner before the tap is turned on. If the gas is now lighted above the gauze the flame is unable to penetrate below.

In order that gas may be ignited it must be raised to a certain minimum temperature, but the metal conducts heat away so rapidly that this temperature is not reached above the gauze in the one experiment or below it in the other. This principle is applied in the Davy lamp used by miners. It sometimes happens that mines contain an explosive mixture of gases, if these came in contact

with a naked flame an explosion would follow. To hinder this the flame is entirely surrounded by a mantle of copper gauze, then, owing to conduction, the temperature outside never becomes high enough to ignite the mixture except in extreme cases.

Instances of Heat Convection.—When a building is heated by hot water a boiler is placed in the basement and pipes slightly inclined to the horizontal go from this to a cistern in the top storey. The hot water in the boiler has a smaller density than that in the pipes, it therefore rises, carrying heat with it. As it passes through the various rooms its heat is radiated from the surfaces of the pipes, and by the time it reaches the cistern it is cool. From here it sinks through vertical pipes to the boiler again and a continuous circulation is thus brought about by convection. The draught of a chimney is due to convection currents of hot air. On a hot summer's day land near the sea is at a higher temperature than the water, owing to the larger specific heat of the latter. An upward current of hot air is produced over the land which is replaced by a colder one coming off the sea, thus causing a sea-breeze. In the evening the land cools more quickly and the conditions are reversed; the prevailing breeze is then from land to sea. Joule's apparatus (p. 55) depends for its action on convection currents.

Temperature at any Point in a Bar.—When a cylindrical bar is heated at one end some time elapses, it may be several hours, before the temperature at every point becomes steady. Let us consider what conditions influence the temperature of a small slice of the bar not far away from the hot end before and after this steady state is established. If the substance is a good conductor much heat will travel to the slice; of this a part flows away across the colder end, a further amount is lost by radiation from the curved surface, and part is retained, in the earlier stages, to raise its temperature. The change in temperature will be great in proportion as the thermal capacity (mass $\times$ specific heat) is small. After some time the temperature becomes steady at every point. When this state is reached thermal capacity has no further influence, the heat a slice receives at its hot end now either flows away by conduction or is lost to the surroundings from the curved surface. The latter losses are said to be due to surface emission. If we have two equal bars of copper and iron for which we may regard the surface losses as negligible, the ratio of the temperatures at two corresponding sections will

depend partly on the conducting powers and partly on the thermal capacities until the steady state is reached; afterwards it will depend on conduction alone, the better the conductor the higher the temperature. If heat is lost from the surface the temperature will be reduced at every point because there is less heat to be transmitted.

EXPERIMENT.—Take equal wires of copper and bismuth, coat them with paraffin wax and put one end of each in a Bunsen flame. The wax melts more quickly along the bismuth in the early stages, but in the end more is melted on the copper. The latter metal is therefore the better conductor, but the small thermal capacity of bismuth more than compensates for this while the temperature is rising. It can be shown that the ratio of the thermal conductivities, as defined in the next paragraph, is equal to the ratio of the squares of the lengths along which the wax is melted.

These remarks show that in comparing conducting powers we must heat the bars long enough for the steady state to be reached, otherwise the results depend on the thermal capacity. It simplifies matters also if the surface emission can be neglected. Now, if a thick bar is split up into two others of half the section more surface is exposed, hence the surface losses are of less importance in thick bars.

Thermal Conductivity. Searle's Apparatus.—We must now define more exactly the conducting power, or, as it will be called in future, the thermal conductivity of a substance. Consider a plate of the substance of thickness l cms., whose opposite faces are kept at temperatures θ_1 and θ_2. Heat will flow from the hotter to the colder side, and if we consider an area S some distance away from the edges the lines of flow will be perpendicular to the faces. The quantity of heat that flows across this area in t seconds can be shown to be—

(1) Proportional to the area S.

(2) Proportional to the time t.

(3) Proportional to the difference of temperature $(\theta_1 - \theta_2)$ between the faces.

(4) Inversely proportional to the thickness l.

If we denote the quantity of heat in calories by Q,

then
$$Q \propto S\frac{(\theta_1 - \theta_2)}{l} \cdot t$$

or
$$Q = kS\frac{(\theta_1 - \theta_2)}{l} \cdot t \text{ cals.}$$

where k is a constant called the **thermal conductivity of the material**

The physical meaning of k is easily seen; for if the plate is of unit thickness and the difference of temperature between its faces is 1°, the heat flowing across an area of 1 sq. cm. in a second is

$$Q = k \cdot \frac{1 \times 1}{1} \times 1 = k \text{ cals.}$$

The thermal conductivity is therefore the quantity of heat that flows in 1 second through 1 sq. cm. of a plate of unit thickness when there

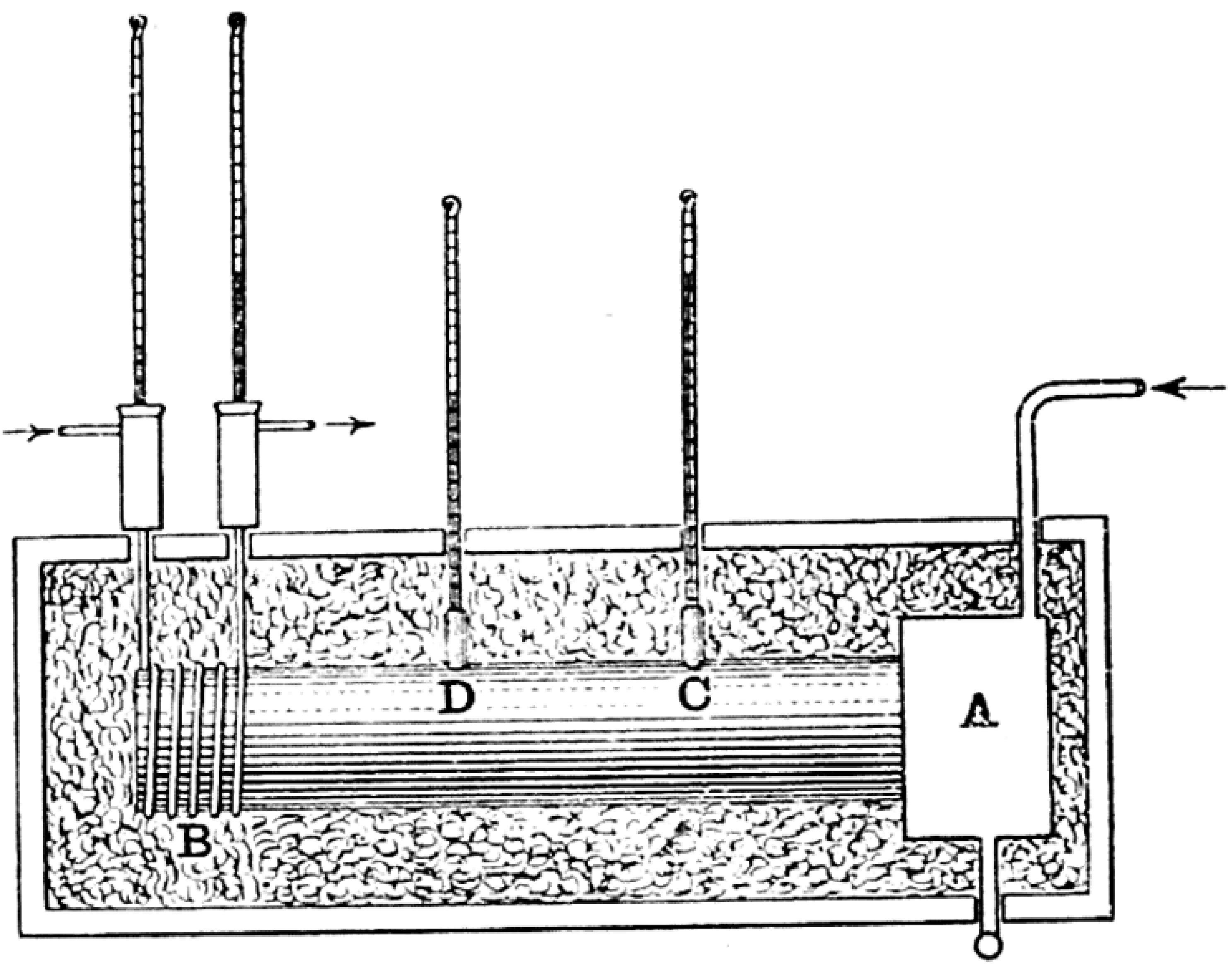


FIG. 62.—Searle's Apparatus for measuring Thermal Conductivity.

is 1° difference of temperature between its faces. The quantity $(\theta_1 - \theta_2)/l$ is called the temperature gradient; it shows by how much the temperature falls when we pass over a distance of 1 cm. in the direction in which the heat is flowing. Thus if a plate is 5 cms. thick and the temperature difference between its faces is 20°, for every cm. we pass into the plate the temperature falls 4°.

Fig. 62 shows an apparatus due to Searle by means of which we may prove that the flow of heat is proportional to the temperature gradient and then measure the conductivity by using the equation given above. A thick metal bar projects at one end into a steam chamber A where it is heated. The farther end, B, is wrapped

round with several turns of thin copper tubing through which a steady stream of cold water flows. The bar is thickly lagged with felt so that the surfaces losses can be neglected. When the steady state is reached all the heat which enters at the hot end flows along the bar, and running into the water at B raises its temperature as it passes through the copper tubes. Let m gms. of water flow through the tubes per second, θ_3 be its temperature at entrance, and θ_4 at exit, then the heat absorbed by it in this time is $m(\theta_4 - \theta_3)$ cals. ; this is the quantity of heat Q that passes any point of the bar in one second. As the necessary temperatures can be measured by the thermometers shown in the figure, Q is determined. At C and D two thick copper pieces are let in, they are bored to carry thermometers whose bulbs reach to the level of the bar. Let the temperatures at C and D be θ_1 and θ_2 respectively, and let CD be l cms. These quantities are easily found, hence we know also the temperature gradient $(\theta_1 - \theta_2)/l$ By using different vapours, or steam at different pressures, to heat the bar we can readily prove that $Q \propto (\theta_1 - \theta_2)/l$ when the steady state is reached. If, in addition, we measure the section of the bar, the conductivity can be calculated from the equation already given.

Measurements of the conductivity of liquids and gases are complicated by the presence of convection currents ; these can be eliminated, at least in part, by applying heat to the substance at its upper boundary. The details of such experiments are too complicated to be included in an elementary book.

It might be supposed that the conductivity of a solid could be determined by some such means as the following : Make a calorimeter with the substance in question forming the base of the vessel, and fill it with cold water. Blow steam at the base for some minutes ; from the rate at which the temperature rises inside the calorimeter the amount of heat flowing through in one second can be calculated if the weight of the contained water is known. As the temperatures above and below can be measured, also the thickness and area of the base, the conductivity can be calculated. Experiments of this type only give good results in the case of poor conductors owing to the difficulty of finding the exact temperature of the faces. The following modification can be used to find the conductivity of a thin glass tube.

EXPERIMENT.—AB is the tube in question (Fig. 63). Its thickness d and its external and internal radii are found. Let R be the mean of the two radii. The tube is connected to a vessel of water in which the head is kept constant,

it is further surrounded by a steam jacket C. Water enters at the end A at a temperature θ_1 and leaves at B at a temperature θ_2. If m is the mass in gms. that flows through in 1 sec. the number of calories that pass through the walls from the steam to the water is $m(\theta_2 - \theta_1)$. The temperature of the external wall is, say, 100°, the mean temperature inside is $(\theta_1 + \theta_2)/2$, hence the temperature gradient is $\frac{100 - (\theta_1 + \theta_2)/2}{d}$. The mean area through which the heat flows is $2\pi R . l$, where l is the length in the steam-jacket. Hence substituting in the formula on p. 119 we have

$$m(\theta_2 - \theta_1) = k . \frac{100 - (\theta_1 + \theta_2)/2}{d} . 2\pi Rl$$

All the quantities in this equation are known except k. In order that all the

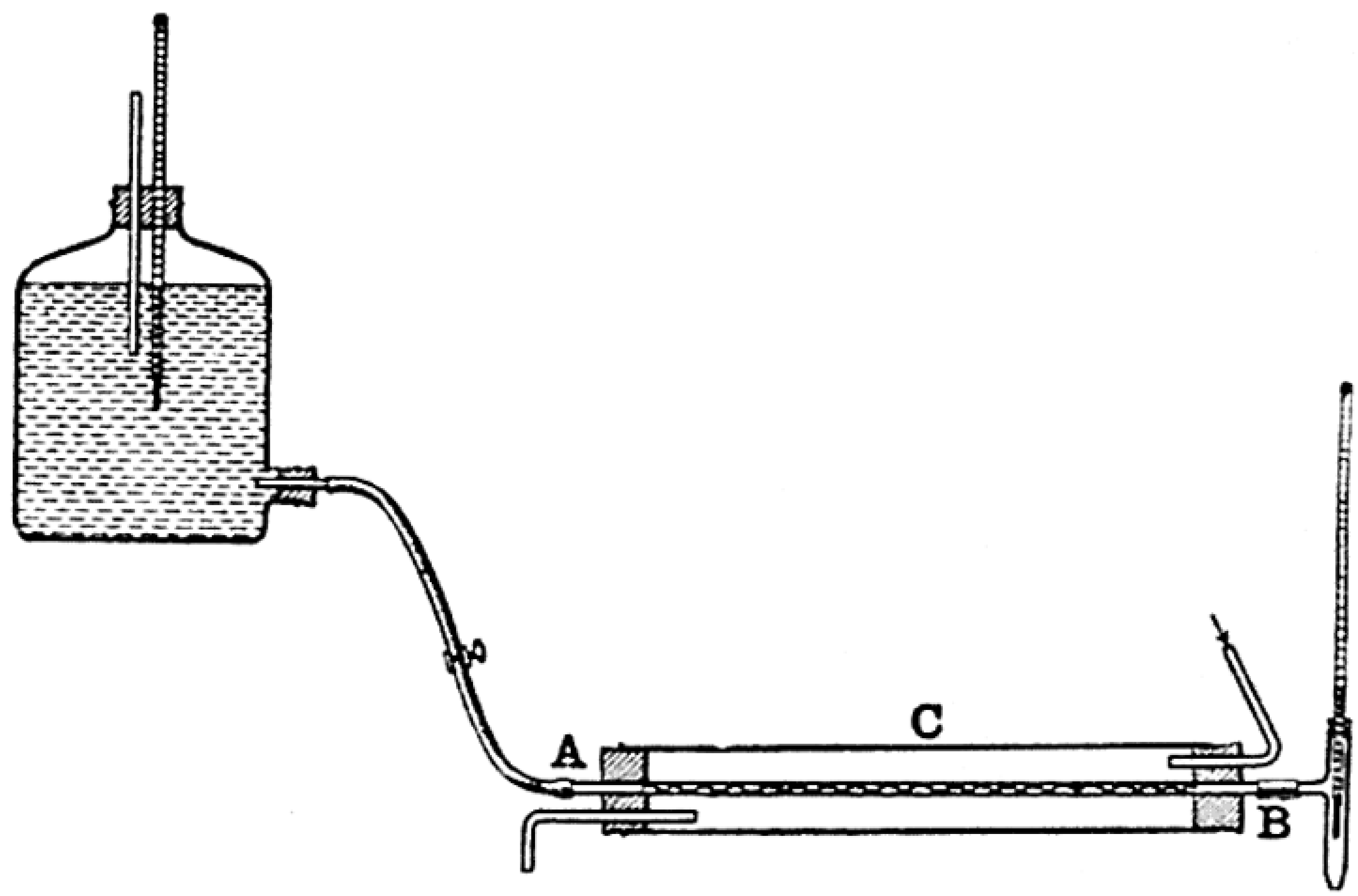


Fig. 63.—Simple Conductivity Apparatus.

water may be brought into contact with the heated wall a bent piece of wire is fixed along the axis of AB; this throws the liquid into eddies and keeps it well mixed.

EXAMPLES ON CHAPTER XI

1. Define conductivity for heat and show how the fundamental units of length, mass, and time enter into its numerical specification. Taking the conductivity of iron as 0·17 C.G.S. units, what difference of temperature would exist between the surfaces of an iron wall, 3 cms. thick, through every square metre of which heat is streaming, from a furnace on one side to boiling water on the other, at the rate of 30,000 C.G.S. units per minute? (L. '91.)

2. Radiation has long been falling on a slab with a blackened surface, each

sq. decimetre of which absorbs 10,000 ergs per second; and the energy is transmitted to a back surface, 0·5 cm. distant, where it is removed by water. What steady difference of temperature must exist between the two surfaces of the slab if its conductivity is 0·02 C.G.S. units? (L. '92.)

3. A metal vessel, 1 sq. metre in area, and whose sides are 0·5 cm. thick, is filled with melting ice, and is kept surrounded by water at 100°. How much ice will be melted in an hour? The conductivity of the metal is 0·02 and the latent heat of fusion of ice is 80. (L. '95.)

4. Suppose 10 cms. of ice to have already formed on a pond, and that the air is at −5°. How long approximately will it take for the next mm. to form? [Conductivity of ice = 0·005, latent heat = 80.] (L. '04.)

CHAPTER XII

PROPAGATION OF HEAT. RADIATION

Instruments used.—It has been seen that when radiation falls on a body part of it is absorbed, causing a rise in temperature. Any apparatus whose condition is appreciably changed by the reception of a small quantity of heat can therefore be used as a detector of radiation. No substance is known which absorbs all the radiation which falls upon it, but lamp-black, or soot, absorbs more than 90 per cent., and, what is more important, it absorbs all radiations equally no matter what their source. A differential air thermometer with one bulb covered with lamp-black was used as a detector by the early experimenters, but it is now superseded by electrical methods. The principles on which these are based will not be fully understood without some knowledge of electricity, but it may be briefly stated that when two dissimilar metal rods are joined together at their ends and one junction is heated, an electrical current flows through them. This can be measured by a galvanometer. A set of such antimony-bismuth junctions are covered with lamp-black and arranged so that the effects of the separate junctions are added, such an arrangement is called a thermopile. It is usually placed inside a metal cone to screen it from all radiations except those coming in a definite direction, when these fall on the junctions they are heated and a current passes through the galvanometer (see Chap. XL.). Another detector consists of a thin strip of platinum covered with lamp-black. When its temperature rises, owing to incident radiation, its electrical resistance is increased; this is measured by suitable means such as a Wheatstone's bridge (p. 382). An apparatus of this type is called a bolometer.

Emissive Power.—The rate at which a body loses heat by radiation may depend (1) On the nature of the surface; (2) On the temperatures of the body and of its surroundings; (3) On the

material of which the body is composed. When radiation coming through air falls on a surface some of the waves may merely be turned back or reflected ; this is especially the case when the surface is bright. A good reflector is therefore a bad absorber. But reflexion takes place equally when the waves are travelling from the interior of the substance towards the air ; hence, if the surface is a good reflector, most of the heat is returned to the interior. Thus good reflectors emit very little radiation.

EXAMPLE.—A bright kettle takes longer to heat but retains its heat better than a black one.

A substance which absorbs all the radiation which falls upon it is called a "**perfectly black**" body. Practically we may treat lamp-black as such. The ratio of the quantity of radiation emitted per sec.

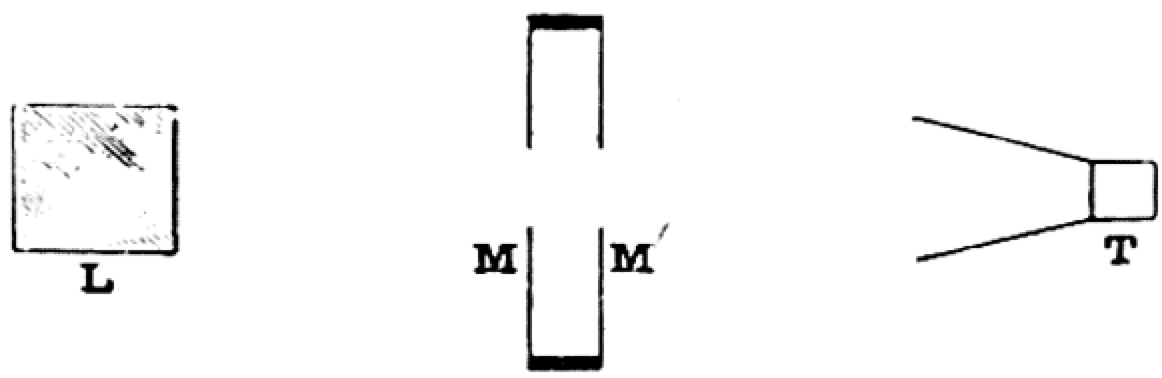


FIG. 64.—Apparatus for the Comparison of Emissive Powers.

by a cm.^{2} of a surface to the quantity emitted by a cm.^{2} of a perfectly black body under equal conditions is called the **emissive power** of the surface. Emissive powers can be compared by the method of la Provostaye and Desains. A metal cube, L (Fig. 64), usually called a Leslie's cube, is filled with boiling water or other liquid and its vertical faces are covered with the substances to be compared. About 50 cms. away is a thermopile T (the galvanometer used with this is not shown), and between this and the cube is a double metal screen MM′. The sides of the screen facing the cube and thermopile are covered with lamp-black while the inner faces are bright. If the left face of M were bright it would be possible for radiation falling on it to be reflected back to the cube and from thence to the thermopile ; the bright face behind hinders direct radiation from M to the thermopile, while M′ acts as an additional check to this and also prevents the reflection of radiation coming from the right. The emissive powers are proportional to the currents produced. It is found that a lamp-black surface is the best radiator, but bright

metal surfaces emit very little radiation; the relative emissive powers vary also with the temperature of the source L.

Newton's Law of Cooling.—Next let us investigate how the temperature of a body and its surroundings influences the rate at which heat is emitted. Unless the body is placed in a vacuum part of the heat losses will arise from conduction and convection through the surrounding air, but these are less important if the radiation losses are large. This is arranged for by covering the radiating surface with lamp-black.

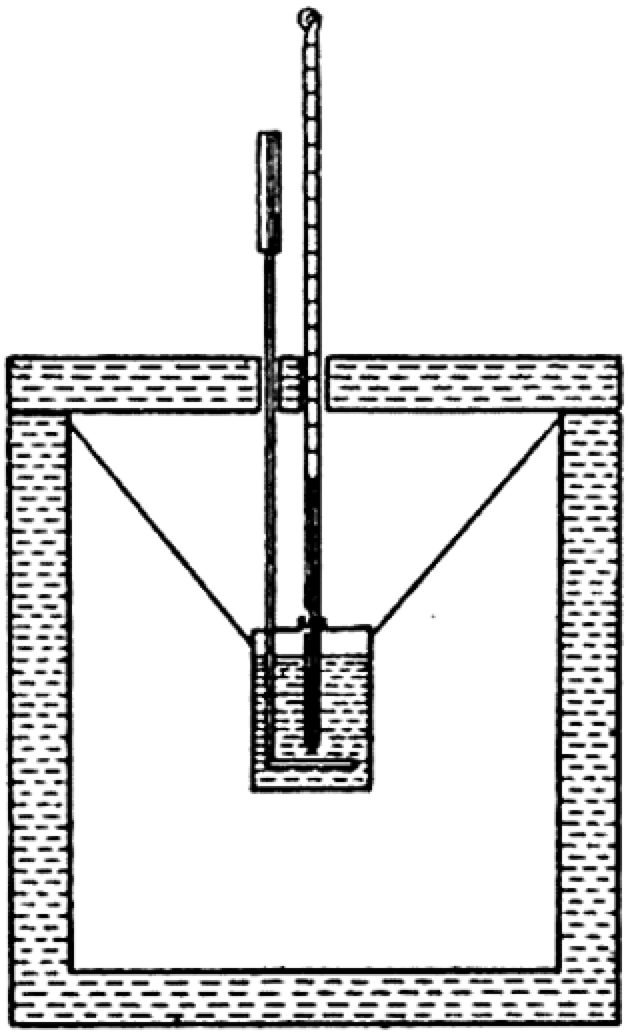


FIG. 65.—Apparatus to test Newton's Law of Cooling.

EXPERIMENT.—Take a small, thin-walled, calorimeter which can be closed with a metal lid provided with holes for a stirrer and thermometer; cover it with lamp-black by holding it in a smoky flame, then fill with water at a temperature about 50°. Hang it in a double-walled vessel (Fig. 65), the space between the double walls being filled with water in which a thermometer is placed. (Instead of this we may put a heavy weight in a calorimeter and sink it in a large beaker of water.) Keep the warm water well stirred and note its temperature every half-minute. Plot a curve showing the difference of temperature between the inner and outer vessels at different times. From this read off the fall of temperature during any minute, *i.e.* the rate of cooling, and also the temperature at the middle of each minute. Tabulate these results and plot a new curve showing rate of cooling and the excess of temperature of the hot water over that of its surroundings. This will be found to be a straight line if the temperature excess is not more than a few degrees, hence within these limits the rate of cooling of a body is proportional to its excess of temperature over that of its surroundings. This is called **Newton's law of cooling** although Newton's experiments were carried out under very different conditions.

For large temperature differences the law does not hold, as the curve shows. It holds sufficiently well, even when convection currents are present, to enable us to calculate the radiation losses from the vessels used in calorimetry; in these cases the excess of temperature is usually small. Within the limits in which Newton's law is true the rate of cooling is independent of the actual temperatures of the hot body and its surroundings, it depends only on the *difference* of temperature between the two. Thus

the rate of cooling when the small calorimeter is at 50° and the outer vessel at 45° is the same as if the respective temperatures were 20° and 15°.

Effect of the Nature of the Liquid on the Rate of Cooling.—If the small calorimeter of the last experiment is filled with turpentine instead of water it is found that the rate of cooling is faster although the blackened surface has remained unaltered. The results take a very simple form if instead of comparing the rates of cooling we compare the amounts of heat lost. To make the comparison the specific heat of turpentine must be known. Suppose with water in the calorimeter it takes t_1 seconds for the temperature to fall from 25° to 20°. If m is the mass of the calorimeter, s its specific heat, and m_1 the mass of water contained, the heat lost is $5(m_1 + ms)$ cals., and the heat lost per second is $5(m_1 + ms)/t_1$. Repeat the experiment through the same interval of temperature when the water is replaced by turpentine. Let m_2 be the mass of turpentine, s_2 its specific heat, and t_2 the time required. Then the heat lost per second is $5(m_2s_2 + ms)/t_2$ cals. It will be found that *the heat lost per second is the same in each case*, hence

$$\frac{m_1 + ms}{t_1} = \frac{m_2s_2 + ms}{t_2}$$

If the surface is unaltered the heat lost per second is independent of the nature of the liquid.

Specific Heat from the Rate of Cooling.—The principle just given can be used to find the specific heat of a liquid. Using the apparatus shown in Fig. 65 the time taken to cool from, say, 35° to 30° is observed, first with water in the small calorimeter, next when it contains an equal volume of the liquid. (Equality of volume ensures that the cooling surfaces will be equal in the two experiments.) The specific heat is calculated from the equation just given, where m_2 and s_2 refer to the liquid. Regnault found that the method was useless for powders or solid bodies. The reason for this is obvious; the rate of cooling depends on the rapidity with which heat is conducted from the interior to the surface, *i.e.* upon the thermal conductivity. For liquids it is very convenient, especially when but a small quantity of the substance is available; thermal conductivity does not enter in this case since the liquid is well stirred. In order that the surface may not be altered it is well to heat the liquids to

a suitable temperature in a beaker before placing them in the calorimeter.

EXPERIMENT.—Compare the emissive powers of bright and black surfaces by noting the rate of cooling of a bright calorimeter containing water, then smoke its surface and observe the rate of cooling over the same temperature interval as before. The emissive powers are inversely proportional to the times of cooling.

EXPERIMENT.—*Calculate the heat emitted in 1 sec. from a sq. cm. of surface when its mean temperature is 25°.* The time taken for the calorimeter in the last experiment to cool from 27° to 23° is found, the heat emitted per sec. is then $4(m_1 + ms)/t_1$ cals. This must be divided by the area of the calorimeter surface. The heat emitted in 1 sec. per cm.^{2} when the body is 1° hotter than its surroundings is sometimes called the surface emissivity. Calculate this for the calorimeter surface.

Absorption of Radiation.—Let a quantity of energy equal to Q ergs fall on a surface in a second and let Q′ be the amount absorbed; the ratio Q′/Q is called the **absorptive power,** or coefficient of absorption, of the surface. It is difficult to measure absorptive powers directly, but they may easily be compared by a method due to la Provostaye and Desains. A thermometer bulb coated with one of the substances in question is placed in a closed box and the radiation allowed to fall on it through a suitable lens. The temperature rises until the heat lost by radiation is equal to that gained by absorption; let the steady temperature be $t_1°$. A cooling curve is now plotted for the thermometer starting at a temperature slightly higher than $t_1°$; from this we can determine as on p. 126 the rate of cooling when the mean temperature is $t_1°$, let it be $\theta_1°$ per second. Then if M is the thermal capacity of the bulb the heat lost per second is $M\theta_1$. But if Q is the radiation falling on it per second when exposed to the source and A_1 the absorbing power, the heat absorbed in a second is A_1Q, hence

$$A_1Q = M\theta_1$$

Similarly for a second substance exposed to the same source

$$A_2Q = M\theta_2$$

hence
$$\frac{A_1}{A_2} = \frac{\theta_1}{\theta_2}$$

The results show that substances with large emissive powers have also large absorptive powers. By means of Ritchie's apparatus

it may in fact be proved that the emissive power of a surface, as defined on p. 125, is equal to its absorptive power. The bulbs of a differential air thermometer are formed from cylindrical metal boxes (Fig. 66), the surface P is covered with lamp-black, T is made of bright metal. A box containing hot water is placed between and equidistant from them. If the surface R is bright but S is lamp-blacked it is found that there is no movement of the index when the box is placed in position. If Q is the heat emitted from S and A the absorptive power of T, then the heat absorbed by the right-hand bulb is AQ. Also if E is the emissive power of R the heat it emits is Q′ where, from the definition of emissive power (p. 125), Q′/Q = E or Q′ = EQ. The whole of this is absorbed by P, hence, as the index does not move, EQ = AQ or E = A.

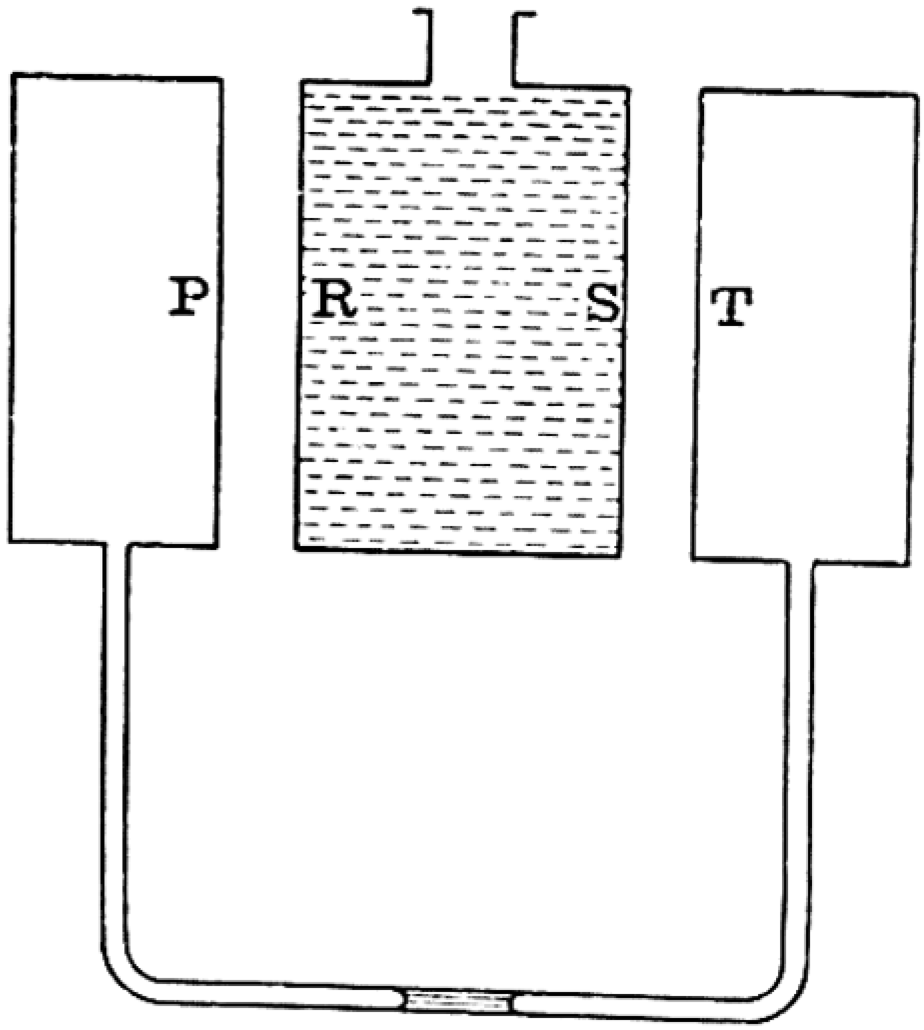


FIG. 66.—Ritchie's Apparatus.

Prevost's Theory of Exchanges. Stefan's Law.—The experiments described in the last paragraph show that a thermometer has a steady temperature when its losses arising from emission are just balanced by the radiation it absorbs from surrounding bodies. According to Prevost's theory of exchanges this is true for every case of temperature equilibrium. When a hot body is brought near a thermometer each is emitting radiation, and the temperature of the thermometer rises because it receives more than it loses. Similarly when a block of ice is brought near, the temperature of the thermometer falls because it does not gain as much radiation from the ice as it sends to it. Stefan has shown that the **amount of radiation emitted by a body is proportional to the fourth power of its absolute temperature.** For a perfectly black body this law has been proved to hold over a very wide range. Taking the case of a black body at an absolute temperature T_1 placed in an enclosure whose walls, also black, are at an absolute temperature T_2, the heat the body loses by radiation in a given time is cT_1^4, where c is a constant depending on the nature of

the surface. During the same time it gains radiation from the walls equal to cT_2^4; the rate of cooling is therefore $= c(T_1^4 - T_2^4)$. If T_1 is much greater than T_2 this is practically equal to cT_1^4. Suppose, on the other hand, T_1 and T_2 are nearly equal so that $T_1 = T_2 + t$, where t is very small compared with either. The rate of cooling is then

$$c[(T_2 + t)^4 - T_2^4]$$

But $(T_2 + t)^4 = T_2^4\left(1 + \frac{t}{T_2}\right)^4$

$$= T_2^4\left(1 + 4 \cdot \frac{t}{T_1} + \text{terms containing higher powers of } \frac{t}{T_2}\right)$$

by the binomial theorem.

As t/T_2 is small these higher powers may be neglected, and the rate of cooling is

$$cT_2^4\left(1 + 4\frac{t}{T_2}\right) - cT_2^4 = 4ctT_2^3$$

This shows that the rate of cooling is proportional to the temperature excess t in accordance with Newton's law. The calculation shows clearly that Newton's law of cooling can hold only when the temperature of the hot body is slightly above that of its surroundings. To calculate the rate of cooling in other cases Stefan's law must be used.

EXAMPLES ON CHAPTER XII

1. In what respects does radiant heat differ from light? Why are rock salt lenses employed for experiments with radiant heat coming from a source at a low temperature, while glass lenses suffice when the sun or an electric lamp is employed as a source of heat? (L. '80.)

2. How are the radiating and absorbing powers of a surface connected? Describe experiments to verify the connection. (L. '88.)

3. A piece of ice is placed in front of a thermopile and the needle of its galvanometer is seen to move. Describe as fully as you can all that is going on. (L. '90.)

4. How do you account for the fact that on a frosty night it is often colder at the bottom of a valley than on the neighbouring hill sides? (L. '02.)

5. How would you show that a large amount of the energy radiated by a gas flame consists of non-luminous heat rays, and how would you measure the percentage stopped by a sheet of glass? (L. '09.)

CHAPTER XIII

RECTILINEAR PROPAGATION OF LIGHT

THE word "light" is used in two senses; we speak of the sensation of light, and the same term is used to denote the physical cause of this sensation. It is in the second sense that the word is used in the following pages.

Geometrical and Physical Optics.—Light may be studied from two points of view; in the first method certain simple laws are first established by experiment, and from them by mathematical and physical reasoning we proceed to deduce other, probably more complicated, results. From this standpoint we are not concerned with the physical nature of light, nor with the reason why the fundamental laws are obeyed: this branch of the subject is called **Geometrical Optics.** In the second method an attempt is made to go further and to form some hypothesis as to the nature of light; from this certain consequences are deduced which can again be subjected to the test of experiment. This is the province of **Physical Optics.** The two methods cannot be kept separate without falling into error. In the following pages light is studied by the first, or geometrical, method, but it is advantageous to assume one of the chief results of the physical optical theory, viz. that light consists of extremely short waves. Any substance through which light travels is called a medium; this term also includes the non-material ether which is supposed to fill all space (p. 116). Bodies which emit light are called self-luminous bodies. They are known to contain certain particles which vibrate rapidly to and fro, thus setting up disturbances in surrounding media which, for want of a better name, are called waves. The distance which a wave travels while the particle makes one complete vibration is called the **wave-length.** These wave-lengths are extremely small; if they lie between 4×10^{-5} and 8×10^{-5} cms. the waves produce the sensation of sight when they enter the eye.

Other waves of longer and shorter wave-length are also given out which do not produce this sensation; as the laws governing their propagation are the same as for light waves they may conveniently be studied together.

The three laws on which the study of Geometrical Optics is based are—

(1) The rectilinear propagation of light, *i.e.* the fact that light travels in straight lines.

(2) The laws of reflexion.

(3) The laws of refraction.

Rectilinear Propagation of Light.—Some of our commonest notions are based on the assumption that light travels in straight lines. Thus in sighting a gun we point it in the direction in which the light reaches us from the object mark, and we assume that the moon is actually situated in the direction in which it is seen.

Experiment.—Place three pieces of cardboard in vertical positions behind each other and 6 ins. apart. Make a small hole in each and arrange these in the same straight line by threading the cards on a knitting needle. If a lamp is placed behind the first hole light travels through each of the others and may be received on a screen placed behind the last. If one screen is slightly displaced sideways light no longer gets through.

In a darkened room which has a small hole in the shutter the path of the light is shown by particles of dust floating about; it is seen to be a straight line.

The pin-hole camera provides a simple illustration of the same law. A candle is placed a short distance behind a screen of cardboard in which a small pin-hole has been made; light travels in straight lines from the different points of the flame, passes through the hole, and falling on a screen behind produces a series of illuminated patches. Fig. 67 shows how this results in an inverted picture of the flame being formed on the screen. The size of this picture will evidently depend on the relative distances of screen and candle from the hole. If a second pin-hole is made, not far from the first, another candle flame will be seen on the screen; if the two overlap the resultant picture will be blurred. Hence we see that a large hole, which we may regard as a number of small holes near together, will not produce a clear picture. If three pin-holes are made close together so as to form a small triangle, the three pictures produced coincide so nearly on a distant screen that there is little loss of

clearness, *i.e.* the shape of the small hole does not affect the form of the picture; but if the screen is close to the hole the picture produced by each part is very small, and, as there is very little overlapping, an illuminated spot is seen of the same shape as the hole itself.

A **homogeneous medium** is one whose properties do not vary from point to point. It will be seen later that when light travels from one medium to another its path is usually bent at the surface of separation; bearing this in mind the first law of Geometrical

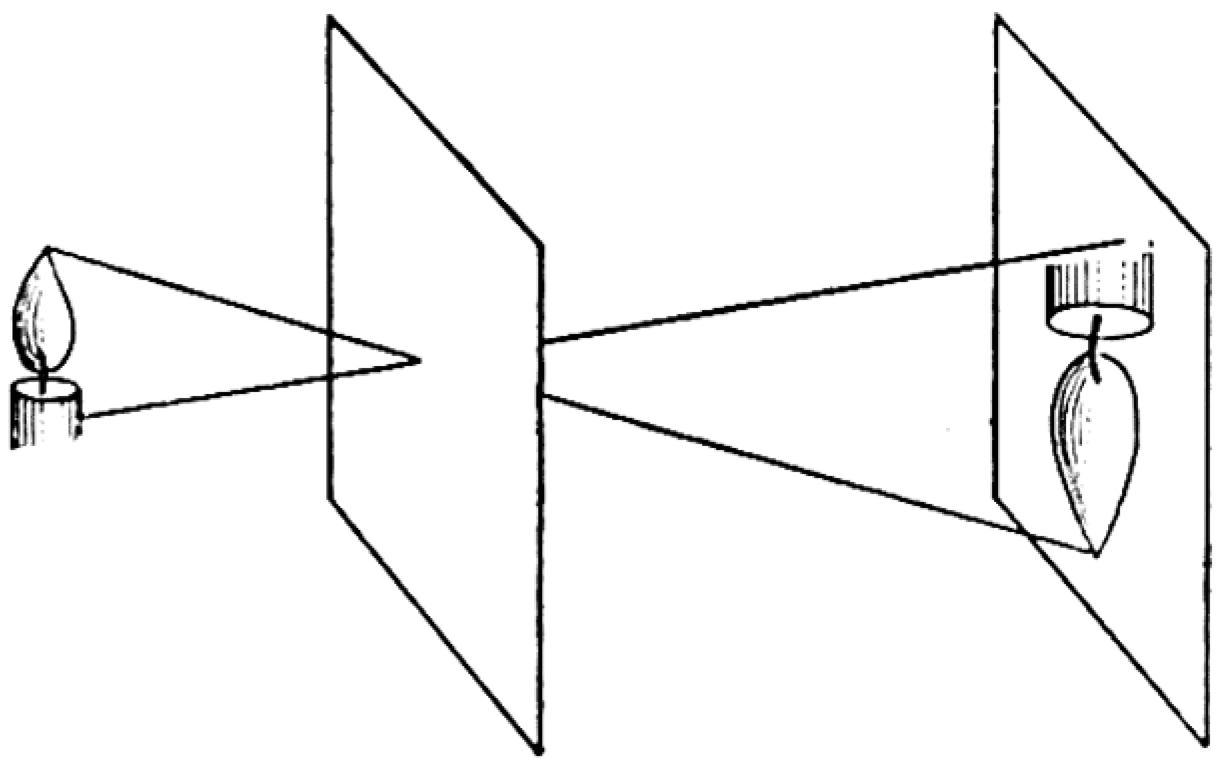


FIG. 67.—The Pin-hole Camera.

Optics can be stated in the following terms: **Light travels in straight lines in a homogeneous medium.**

Definition of Terms.—The straight lines along which light travels are called **rays**. A collection of rays forms a beam or pencil of light. If the rays converge to or diverge from a point the beam is said to be convergent or divergent respectively; when the rays are parallel we have a parallel beam. Rays diverge in all directions from any point of a luminous body, but when it is very distant the rays with which we deal are inclined at such a small angle that they may be regarded as parallel. Thus the rays coming from a star to the eye form a parallel beam.

Shadows.—The formation of shadows is a direct consequence of the fact that light travels in straight lines.

EXPERIMENT.—Take a small source of light, such as the arc-light, fix it some distance away from a vertical piece of brass tube several inches in diameter, and notice that a well-defined shadow is thrown on a screen held a few feet away.

Fig. 68, A, shows how this is produced. The opaque tube prevents any light from the arc reaching the part BC of the screen, an eye placed between B and C would not see the arc. Evidently if light travels in straight lines we shall have from the triangles OBC, OPQ.

Diameter of tube : width of shadow
= distance of tube from arc : distance of screen from arc.

Measure these distances and verify the relation.

EXPERIMENT.—Replace the arc-light by the broad gas flame of a bat's-wing

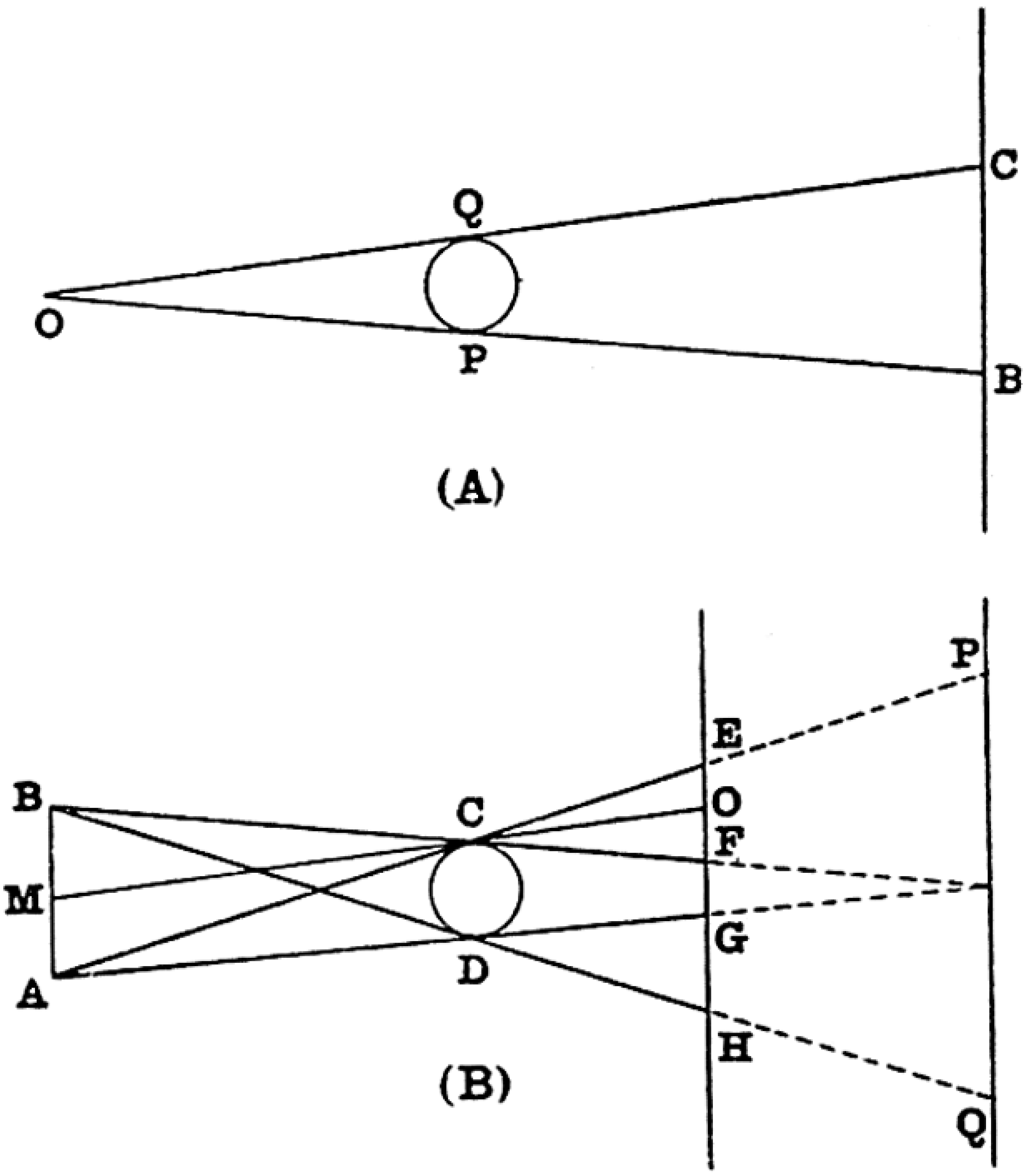


FIG. 68, A and B.—Illustration of how Shadows are formed.

burner. The shadow is now clear at the centre but becomes indistinct towards the edges.

Fig. 68, B, shows how the shadow is formed when such an extended source is used, AB represents the flame, CD the brass tube and EH the screen. Any straight line drawn from the flame to a point on the screen between F, G, must pass through the tube. Hence the part FG receives no light, it is the region of complete shadow or **umbra.** Outside this there are regions EF, GH, which receive light from some parts of the flame but not from the whole; they are the regions of half-shadow or **penumbra.** Thus the part OF receives no

light from any part of the flame nearer to A than M, the rays starting from AM in the direction of OF are stopped by the tube. If an observer looks towards the flame through a hole at O he will see only the part BM, if the hole is between F and G no part of the source will be seen. Beyond E and H the screen is fully illuminated. The difference between this and the preceding case is seen to be due to the extended source of light that is used. The relative and actual sizes of umbra and penumbra depend on the relative positions of source, tube and screen. If the screen is placed near the tube the penumbra is small, while if it is placed at PQ there is scarcely any umbra.

Eclipses are results of the formation of shadows by the moon or the earth. It happens at certain times that the moon moves into a position between the sun and some portion of the earth's surface, the sunlight is intercepted and the sun is said to be eclipsed. At points on the earth which are in the umbra the eclipse is total, where only the penumbra occurs the eclipse is partial. Fig. 68, B, illustrates what may happen if AB is taken to represent the sun, CD the moon, and EH the earth. Lunar eclipses are caused by the earth getting into a position between the sun and moon. Fig. 68 B illustrates this case if CD now represents the earth, and part of the screen EH the moon. The moon is not self-luminous, the light we receive from it is reflected sunlight, hence if it is in the shadow cast by the earth no light can be sent back and it is eclipsed. If it is in the umbra, FG in the figure, the eclipse is total, if in the penumbra a partial eclipse takes place.

EXAMPLES ON CHAPTER XIII

1. On a clear, sunny day a flag-staff casts a shadow on the ground and it is found that the portion due to the lower end is the best defined; explain this. How would white clouds affect the shadow?

2. Why are well-defined circular shadows sometimes seen on the ground beneath an arc lamp?

3. A strip of wood 1 cm. in width is held in a vertical position between a gas flame and a screen, its distance from the former is 50 cms. and from the latter 30 cms. If the flame is 2 cms. wide find the diameter of the umbra and the width of the penumbra on one side of the shadow.

CHAPTER XIV

REFLEXION OF LIGHT FROM PLANE SURFACES

Diffusion and Reflexion.—When light falls on the surface of separation of two media which differ in their optical properties the rays are divided into two portions: one part is returned into the first medium—this is said to be reflected; another part enters the second medium and is there absorbed if the medium is opaque, or is transmitted if it is transparent. We will confine our attention at present to the reflected rays. The direction in which these rays travel is governed by definite laws—the laws of reflexion. When the surface of the second medium is not highly polished each of the small irregularities on it will reflect light, and, as these irregular surfaces may be inclined at all angles, rays will be reflected in all directions. The light is then said to be diffused or scattered. It is owing to diffused light that the surfaces of bodies are visible; thus it is difficult to see the surface of a brightly polished mirror because of the absence of diffused light, it is much easier to see if we breathe upon it. Taking advantage of diffusion we can easily make apparent the path of rays in a transparent medium.

EXPERIMENT.—Fill a flask with distilled water which has been filtered, and focus on it a beam of light from a lantern. The path of the rays is seen with difficulty, but the addition of a few drops of milk renders it quite easy to follow because the milk particles diffuse light in all directions. This effect is one of the most delicate tests for the presence of suspended particles in what may appear at first sight to be a homogeneous liquid. Note also that the path of the beam from the lantern is made visible in the air by the dust particles floating about in it; a cloud of smoke makes it still clearer owing to the increased number of diffusing particles.

In what follows we are concerned with the rays which are reflected from a brightly polished surface or mirror. Well-polished silver is the best reflector of light-waves; ordinary looking-glasses are backed

by an amalgam of mercury and tin to improve their reflecting qualities.

The Laws of Reflexion.—Some of the terms used must be first explained. Let AB (Fig. 69) represent a plane mirror, EO a ray of light travelling towards it, and OF the path of the reflected light. EO is the incident and OF the reflected ray. Draw OP perpendicular to the mirror at the point where the incident ray meets it; OP is the normal. The plane containing the incident ray and the normal is called the plane of incidence; that containing the reflected ray and the normal the plane of reflexion. Also the ∠EOP between the incident ray and the normal is called the angle of incidence, ∠FOP is the angle of reflexion. The laws of reflexion then state—

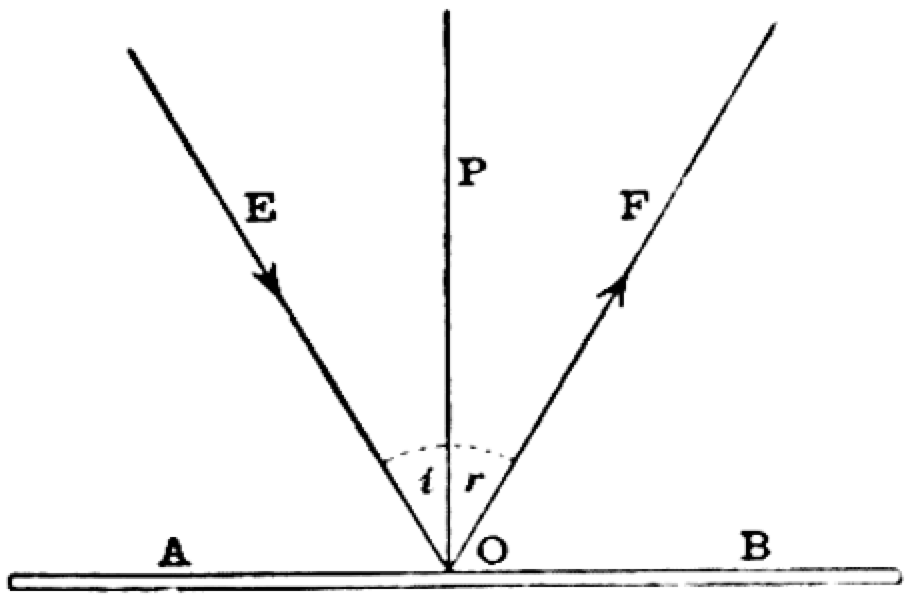


Fig. 69.

(1) The incident ray, the reflected ray, and the normal lie in the same plane, or the planes of incidence and reflexion coincide.

(2) The angle of incidence is equal to the angle of reflexion.

The truth of these laws may be proved by the two following experiments, a more accurate proof is given later when certain optical apparatus has been described. Each experiment illustrates an apparatus or principle that will frequently be used in later sections.

Experiment.—*To prove the laws of reflexion by Hartl's optical disc.*—This apparatus is a very convenient one for showing the path of the rays in various experiments. It consists of a white cardboard, circular, disc graduated in degrees, to the centre of which mirrors, lenses, or prisms may be fixed by screws (Fig. 70). The disc is held in a vertical plane and can be rotated round a horizontal axis. To the right is a circular screen pierced with one or more slits to admit light from an arc-lamp or distant window; the path of the rays is made evident by the bright lines they trace on the cardboard. Fix a strip of plane glass mirror, AB, at the centre of the disc with its length along a marked diameter; a line perpendicular to this shows the direction of the normal and coincides with the zero mark of the graduations. Pass a beam of light through a single slit so that it falls on the mirror at the centre; the paths of the incident and reflected rays can be seen and the angles they make with the normal read off. The angle of incidence is varied by rotating the graduated disc. It is found in all cases that the angles of incidence and reflexion are equal; the

first law also is true since the incident ray, the normal, and the reflected ray lie in the plane of the cardboard.

Experiment.—Fix a large sheet of white paper to a drawing-board and place on it a strip of looking-glass, AB (Fig. 71), with its plane vertical. Stick two pins vertically in the board in the positions indicated by P and Q. The straight line through these points may be taken to represent the path of an incident ray; we require the path of the corresponding reflected ray. Since the ray is bent back at reflexion it will appear to come from two pins P′, Q′, apparently behind the mirror; looking into the mirror fix two pins at R, S so that they appear to be in a straight line with P′, Q′. RS is then the reflected ray. Rule in the outline of the mirror and draw the normal ON with the help of a set-square, the angles of incidence and reflexion can then be measured with a protractor. A number of rays should be traced in this manner and the corresponding angles found. Since the points of the four pins are in the plane of the paper the planes of incidence and reflexion coincide.

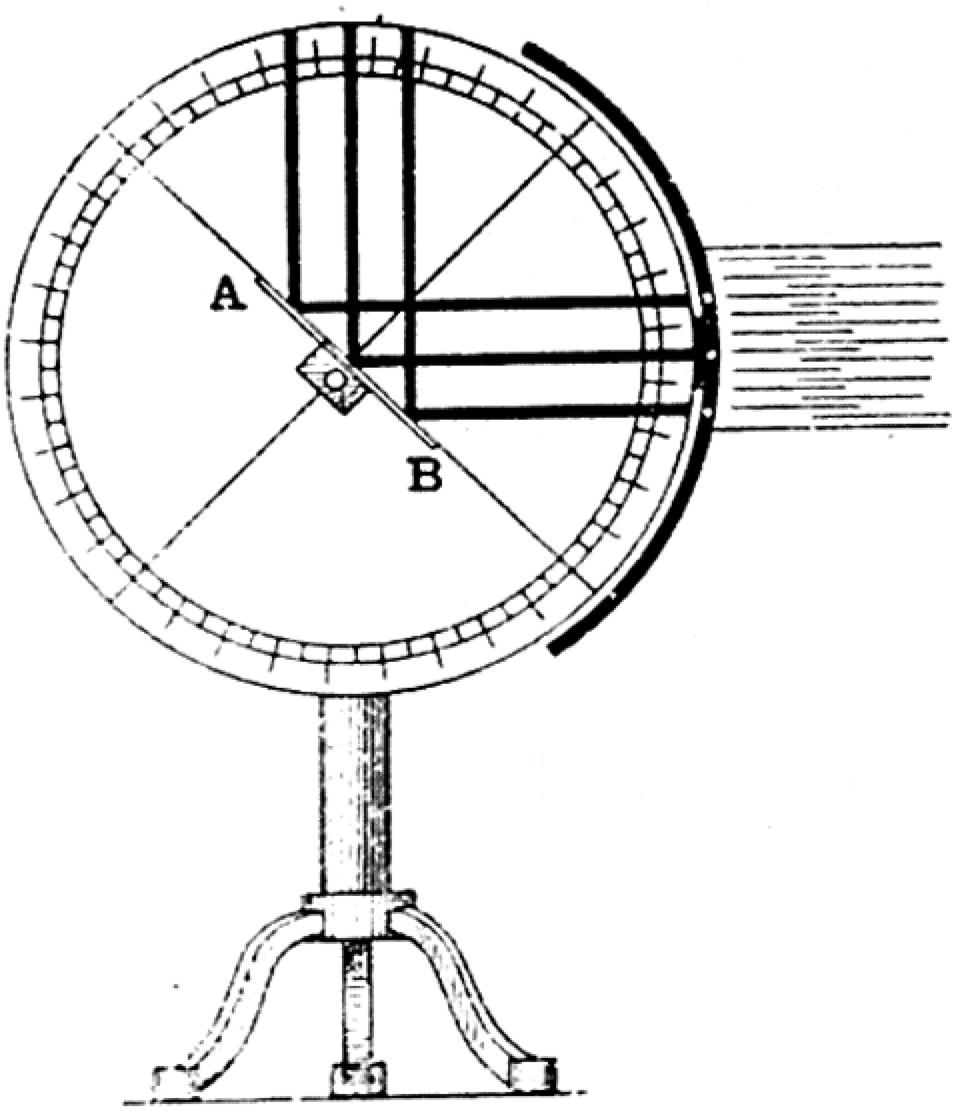


Fig. 70.—Hartl's Optical Disc.

Images.—When a pin is held in front of a plane mirror we see a picture of it which appears to be behind the reflecting surface; this picture is called the **image** of the pin. If rays of light starting from one point afterwards appear to pass through another point the second point is called the image of the first; if the rays actually pass through the second point it is a **real** image, if they only appear to pass through it the image is called **virtual**. In the example just given the image of the pin is behind the mirror, as the rays do not actually come from this place, but only appear to do so on account of reflexion, the image is virtual. The images formed by a pin-hole camera are real. We shall meet with other instances of each kind.

Experiment.—*To find the position of a virtual image formed by a plane mirror.* Use the apparatus of the last experiment. Fix a pin at P a few inches in front of the mirror (Fig. 71); it sends out rays in all directions and it is required to find the point from which these rays appear to diverge after reflexion. Fix a second pin, R, just in front of the mirror and place a third pin at S in

such a position that S, R, and the image of P all appear to be in the same straight line. RS is one reflected ray. Move the pin R to the right or left and find other reflected rays in a similar manner. Rule in the position of the mirror, then remove it and produce the reflected rays backwards; they will meet approximately at a point which is the position of the image. Measurements will show that the image P′ lies on that normal to the mirror which passes through P, and is as far behind the mirror as the object is in front. Since the light is reflected chiefly from the back surface the measurements must be made from this edge.

The result obtained from this experiment should be remembered.

Method of Parallax.—This is a method of finding the position of an image when it is impossible or inconvenient to trace the paths of

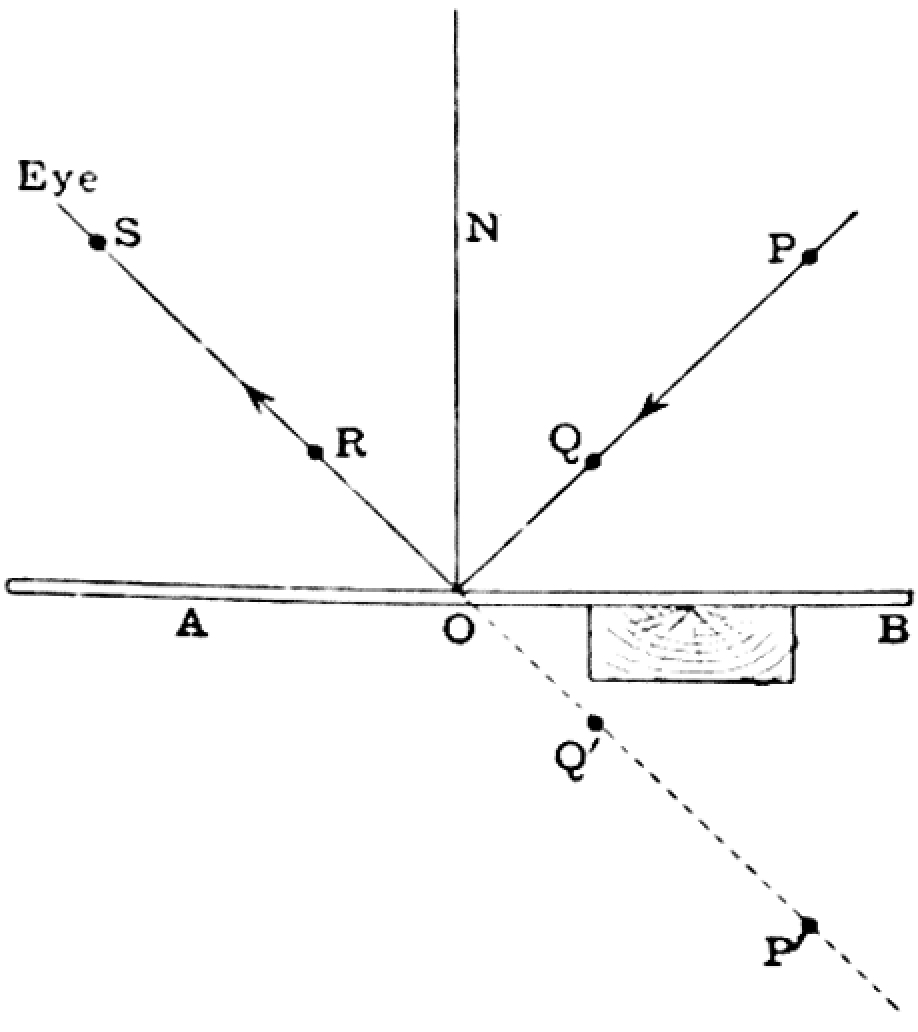


FIG. 71.—Apparatus to prove the Laws of Reflexion.

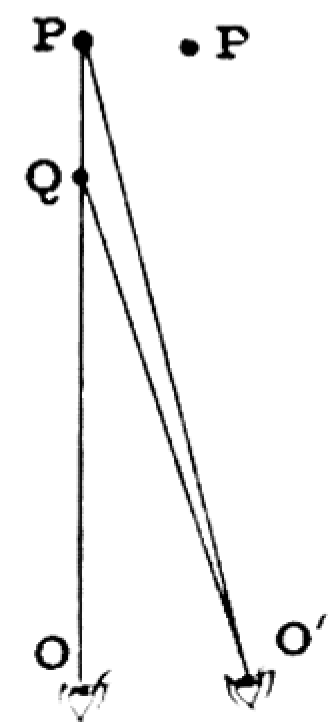


FIG. 72.—To illustrate Parallax.

the rays. It consists in placing a pointer so that it does not appear to shift relatively to the image when the observer changes his position, image and pointer then coincide.

EXPERIMENT.—Stick a pin vertically in a drawing-board, and about 5 mms. behind it fix a larger pin. Shut one eye and get into such a position that the pins appear in the same straight line. Now move the head to the right; the pins shift relatively to each other and appear no longer to be in line. Fig. 72 shows the new appearance; P, Q, are the pins, O the first and O′ the second position of the eye. Exactly the same relative motion would have been produced if the head had remained fixed at O and the farther pin had been

moved to P′. That is, when the head is moved to one side, the farther pin appears to move relatively to the nearer one in the same direction as the head.

This principle is made use of to place a pointer so as to coincide with an image; the two can only be coincident when no relative motion takes place; until this position is obtained we can determine as above which is the more distant.

EXPERIMENT.—Place a mirror strip vertically on a drawing-board and fix a pin 6 ins. in front of it. Look into the mirror in such a direction that the pin covers its own image, fix behind the mirror a second larger pin in this straight line so that its upper part can be seen over the edge. Find by the parallax method which is the further away, the image or the second pin, hence arrange finally that the two coincide. Confirm the results of the last paragraph.

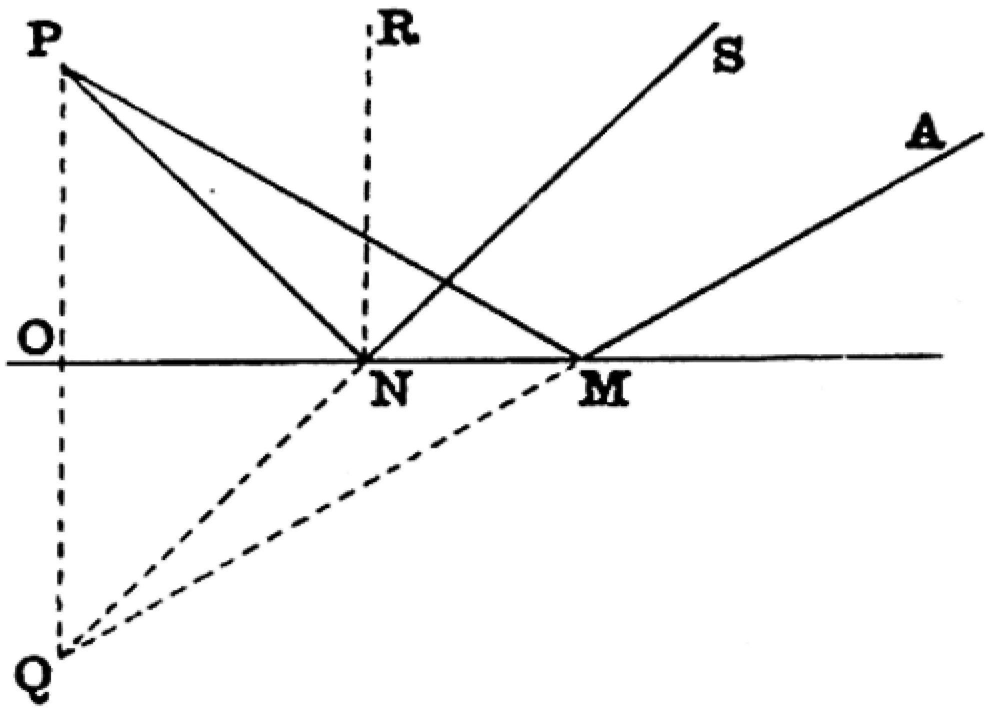


FIG. 73.

The same results can be deduced theoretically from the laws of reflexion. Let OM (Fig. 73) be the mirror and P the object pin. Draw PO perpendicular to OM and produce it to Q, where OP = OQ. Join P, Q, to any point N on OM, and produce QN to S. Draw NR normal to the mirror. Then

$$\triangle\, QON = \triangle\, PON$$
$$\therefore\ \angle QNO = \angle PNO$$
hence
$$\angle PNO = \angle MNS$$
$$\therefore\ \angle PNR = \angle SNR$$

Hence if PN is any ray starting from P, NS is the corresponding reflected ray, since the angles of incidence and reflexion are equal. But NS passes through Q, and the same is true of any other reflected ray MA. Hence the image is at Q, where PQ is normal to the mirror and OP = OQ.

Path of the Rays by which an Image is seen.—Let a small object P be placed in front of a mirror and let us trace the rays by means of which the image P′ is seen. Of the rays starting from P (Fig. 74), a narrow pencil enters the eye after being reflected at the mirror, this

appears to come from P′. Join P′ to the edges of the eye-pupil. The part P′S of the pencil has no actual existence. Join the points where the rays cut the mirror to P, the pencil PSE shows the path of the rays. In the same manner we can construct the path of the rays by which the image of an extended object is seen; the rays from each point must be found separately. If the object P is placed between two parallel mirrors A, B (Fig. 75), a succession of images is seen. Thus in mirror A an image is formed at P_1, where $PA = P_1A$; the reflected rays now appear to come from this image and when they fall on the second mirror they will form a further image at P_2 as if they actually came from P_1; hence $P_1B = P_2B$. Similarly P_2 gives rise to an image P_3, where $P_2A = P_3A$, and so on. Another series is formed starting from the mirror B; the first is at Q_1 where

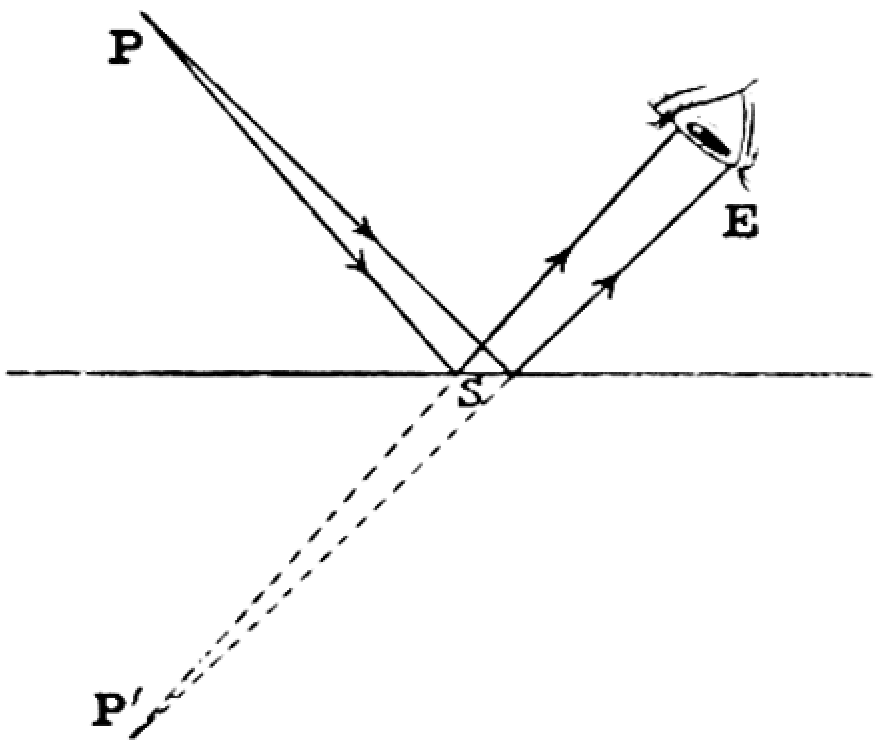


FIG. 74.—Showing the Path of the Rays to the Eye.

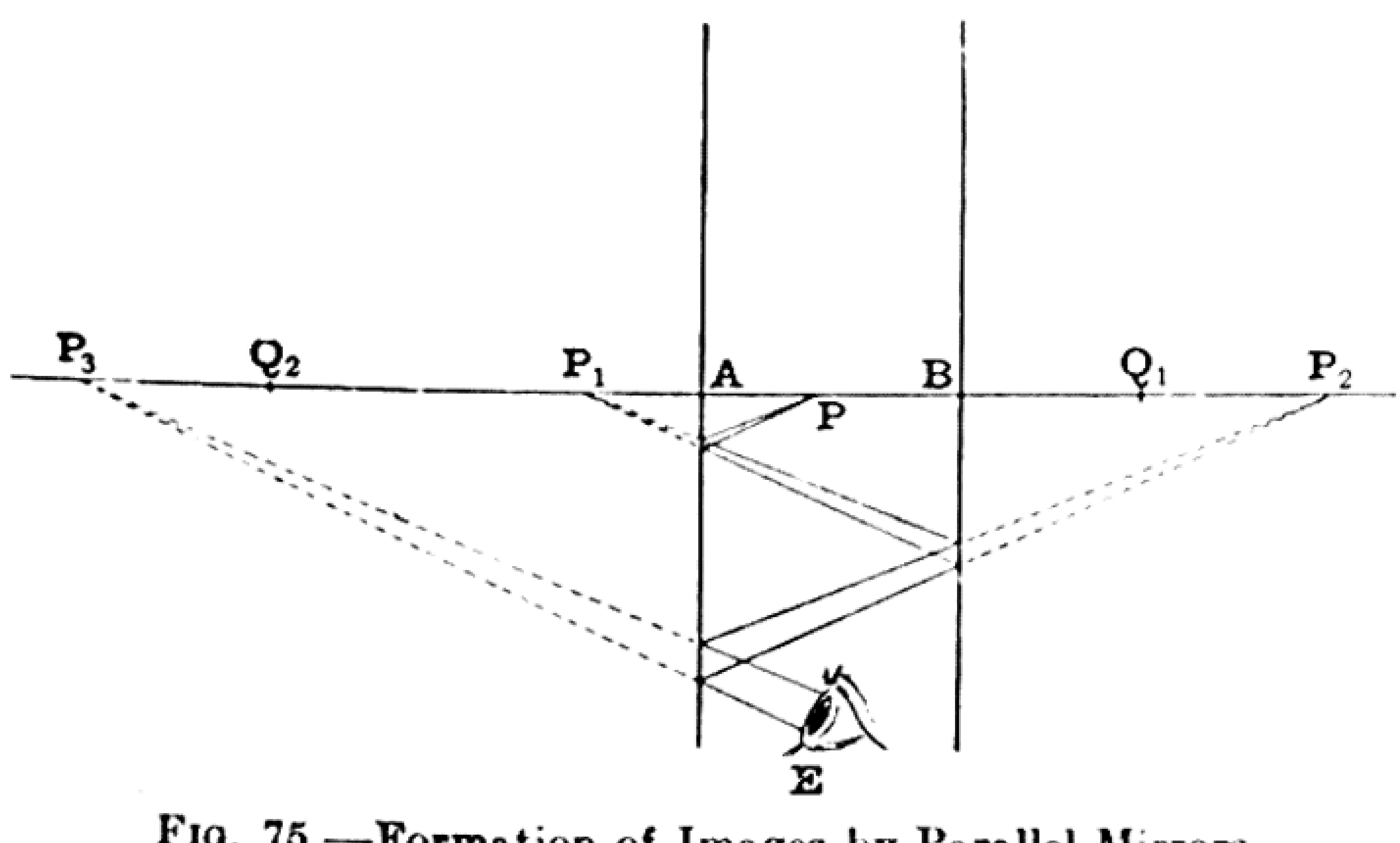


FIG. 75.—Formation of Images by Parallel Mirrors.

$PB = Q_1B$, this is imaged in A at Q_2, hence $Q_1A = Q_2A$, etc. Let us draw the rays by which a given image, say P_3, is seen. Draw from P_3 a divergent pencil entering the eye at E; before their final

reflexion at mirror A these rays apparently came from P_2, hence join the points where they cut the left-hand mirror to P_2. But P_2 is the image of P_1, thus P_1 must be joined to the points where the rays from P_2 cut mirror B. Finally P_1 is the image of P, hence P must be joined to the points where the pencil from P_1 intersects mirror A. The figure shows the complete path of the rays.

EXPERIMENT.—Find by the parallax method the position of P_3 using a pin as object and verify that $P_3A = P_2A$.

The mirrors at the opposite sides of a barber's shop show a large

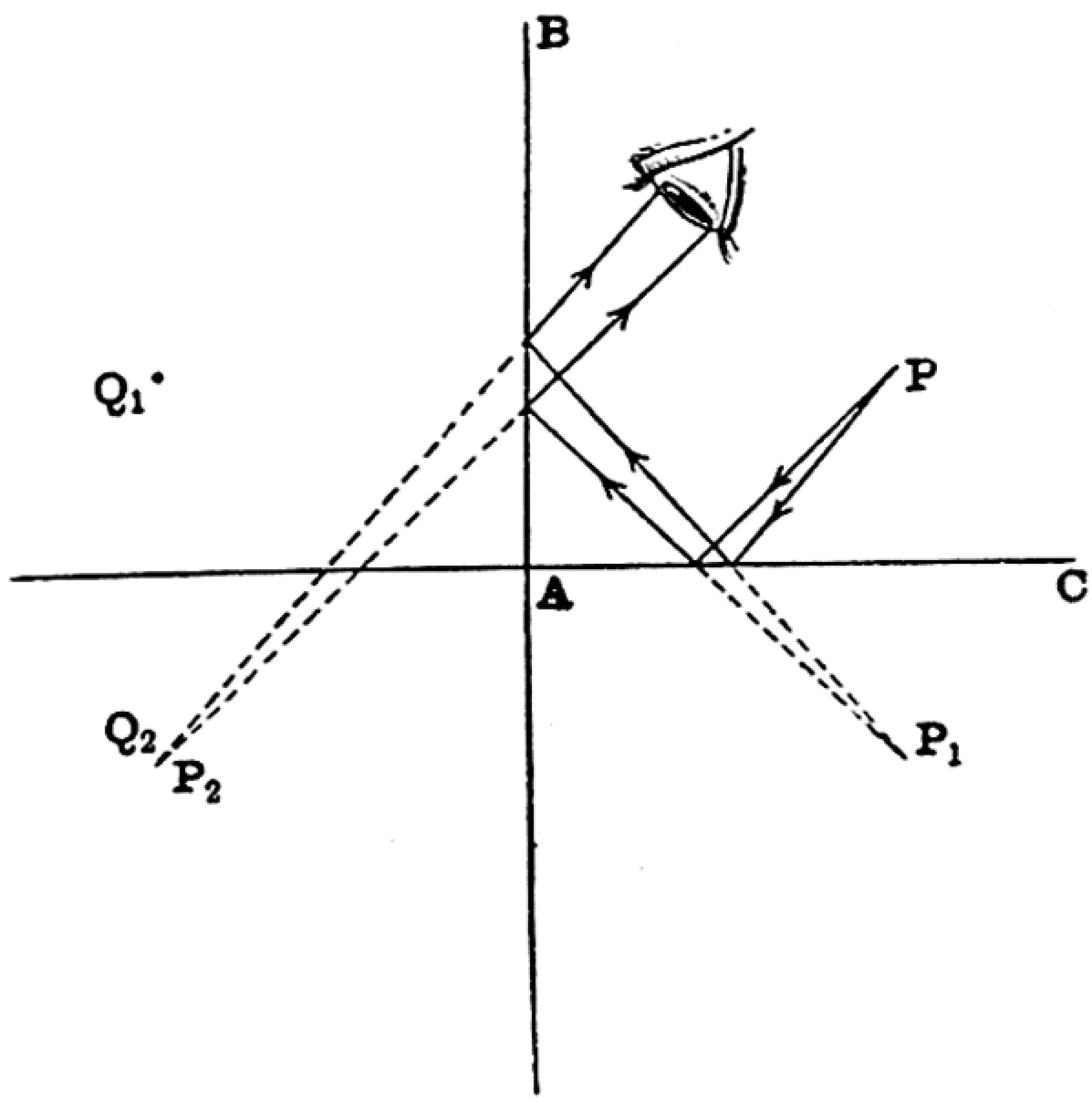


FIG. 76.—Formation of Images by Inclined Mirrors.

number of images formed in this manner. When a person stands before a looking-glass it is found that his right side corresponds to the left side of the image. This is called lateral inversion; it results from the fact that the image of a point is on the normal drawn from the object point to the mirror.

Inclined Mirrors.—Let AB, AC (Fig. 76) represent two plane mirrors fixed at right angles, and let P represent a small object placed in the angle between them. As for the parallel mirrors there will be two series of images, but the number in this case is

limited. Thus AC forms an image at P_1 and this in turn gives rise to an image at P_2 which is formed by the mirror AB. As the rays apparently coming from P_2 fall on the back of each mirror, no further image of this series can be formed. Similarly the mirror AB forms an image of P at Q_1, and, as this is in front of AC, an image is formed in the latter at Q_2; this coincides with P_2. It also is the last image of the series. Only one of the images P_2, Q_2, can be seen at the same time. For example, let the observer's eye be placed anywhere in the angle PAB; join Q_2 to the eye by a narrow pencil. From where this cuts a mirror draw rays to Q_1 and P_1. The figure

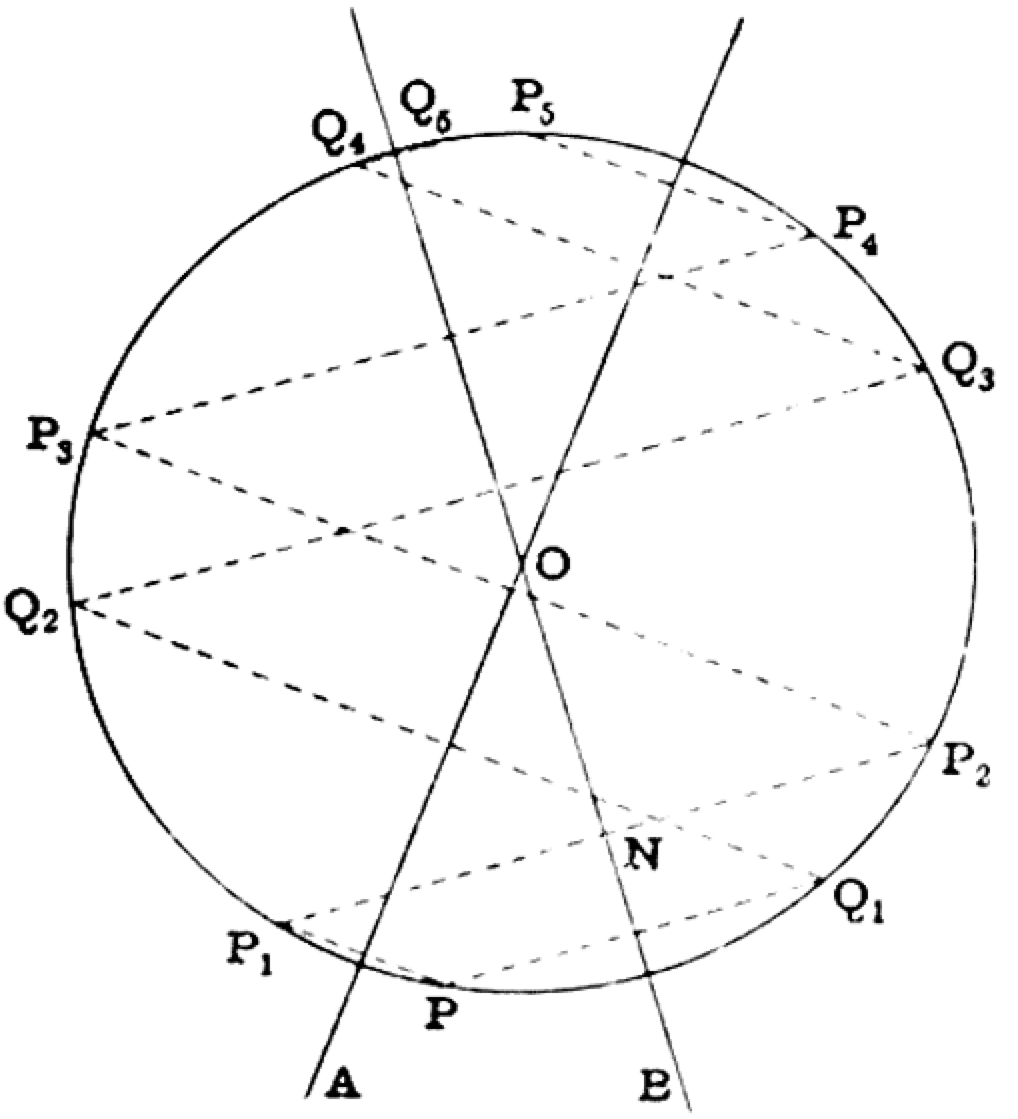


FIG. 77.—Mirrors inclined at any Angle.

shows that the eye will not receive the rays apparently coming from Q_1; hence it is the image P_2 that is seen in this case. To see Q_2 the eye must be placed in the angle PAC.

Similar but more complicated results are obtained when the mirrors are inclined at any angle (Fig. 77). Thus mirror OA forms an image of P at P_1, and since OA bisects PP_1 at right angles $OP = OP_1$. Also P_1 gives rise to an image P_2 in OB, and $P_1N = P_2N$ where P_1P_2 is perpendicular to OB. Hence $OP_2 = OP_1 = OP$, showing that all the images lie on the circumference of a circle whose centre is O and radius OP. As in the previous case, the final image of the series is

that which is formed behind each mirror, *i.e.* which lies in the angle vertically opposite to the angle AOB. There is, of course, a second series formed by P giving rise to an image Q_1 in OB, etc. The number of images formed in any instance, and whether the last of each series coincide, is best determined by a figure drawn to scale or by calculating in succession the angles AOP_1, AOP_2, etc.

EXPERIMENT.—Trace the rays by which the image P_4 is seen.

The child's toy called the Kaleidoscope is an application of the principle of inclined mirrors. Three strips of mirror inclined to each other at angles of 60° are placed in a tube, and a glass box at one end encloses some bits of coloured glass. Light from the sky passes through the box, and, after a series of reflexions, enters the eye which is placed at the other end of the tube. Beautiful patterns arising from reflexions in two or more mirrors are formed in this way.

Rotating Mirror.—If the mirror of Fig. 69 is turned through an angle θ the reflected ray is turned through an angle twice as large if the direction of the incident ray remains the same. For the angle between the incident and reflected rays is $2i$, if i is the angle of incidence; turn the mirror through an angle θ, the normal turns through the same angle, hence the angles of incidence and reflexion are now $(i + \theta)$, and the angle between the incident and reflected rays is $2(i + \theta)$. As the original angle between the rays was $2i$, the reflected ray has been deflected through an angle 2θ.

EXPERIMENT.—Verify this result experimentally by the pin method.

This principle is employed in the measurement of small angles, *e.g.* the angular deflexion of a galvanometer needle (p. 369). In order to measure such a deflexion accurately a long pointer must move over a graduated scale, by using rays of light we get the advantage of a pointer as long as we please without adding to the weight of the needle. Fig. 78 illustrates one method of use. The needle is attached by a suitable suspension to a small piece of plane mirror, AB, which faces a well-illuminated mm. scale placed horizontally at a distance of one metre or more. Above the scale is a telescope focussed on the image of the divisions seen in the mirror. Suppose that initially the rays from some point P on the scale strike the mirror at nearly normal incidence and are reflected into the telescope. An image of the division P will be seen on looking into the instrument; this can, by slight adjustment, be made to coincide

with a vertical wire in the eye-piece. When the mirror turns through an angle θ into the position A′B′ rays from some other point P′ enter the telescope and form an image on the wire. If we suppose the path of the light reversed, so that the rays start from the wire and strike the scale at P or P′, the case is identical with that given above, and the angle between the rays OP and OP′ in the two positions of the mirror is 2θ. The distance PP′ is known from the readings in the telescope and OP can be measured, hence

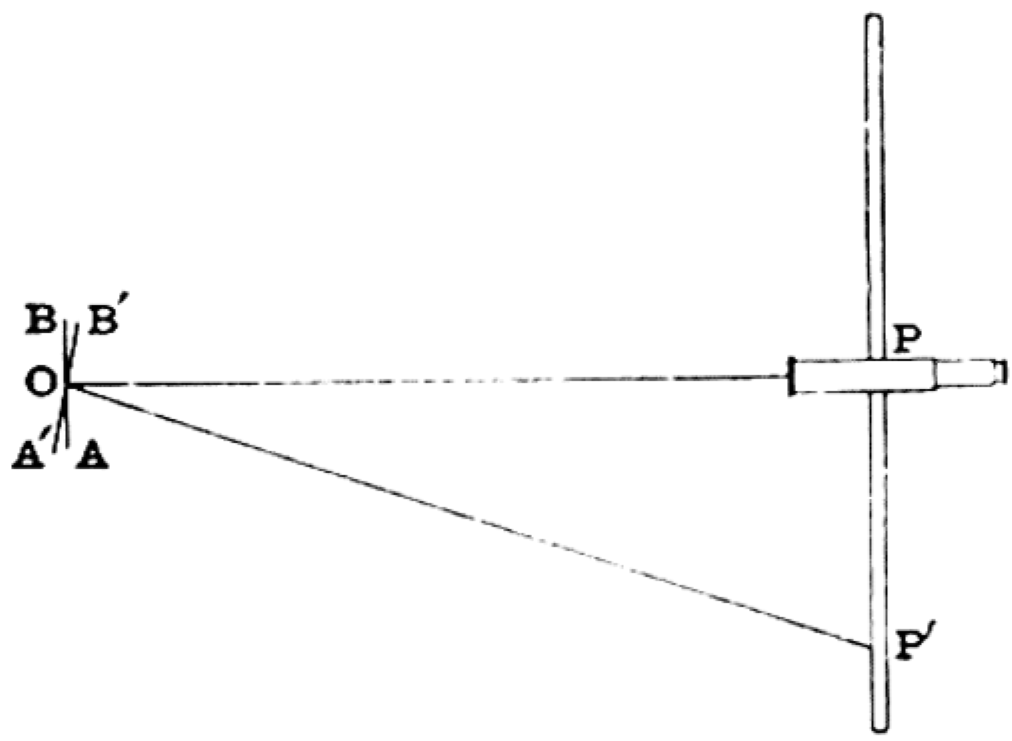


FIG. 78.—Telescope and Scale Method of measuring an Angular Deflexion.

$\tan POP' = \tan 2\theta$ can be calculated and θ found from tables. If the deflexions are small, as is often the case, they may be taken as being proportional to distances such as PP′, they can then be compared without a book of tables. Other applications of this principle will appear in later chapters.

EXAMPLES ON CHAPTER XIV

1. Straight lines are drawn from a luminous point to a plane mirror and from thence to the eye. Of the possible paths from the point to the eye by way of the mirror show that that taken by the rays of light is the shortest.

2. Two mirrors are inclined at an angle of 60° and a small object is placed between them so that it is twice as far from one mirror as from the other. Find the total number of images in each series and whether the last of each coincide.

3. In Fig. 76 prove that a ray which has been reflected from each mirror is parallel to its original direction.

4. Two parallel mirrors are 19 ft. apart and a candle is placed between

them 7 ft. from one mirror. Find the distance between the 3rd and 4th images seen in each mirror.

5. Find the inclination of two mirrors when a ray incident parallel to one is, after two reflexions, made parallel to the other.

6. Two mirrors are inclined at an angle B and a ray falls on one of them at an angle of incidence $\left(\frac{\pi}{2} - A\right)$; prove that the deviation of the ray after the 1st, 2nd, and 4th reflexions is 2A, 2B, and 4B respectively. Hence find the angle between the mirrors when a ray returns along its path after four reflexions.

7. In the telescope and scale method of measuring deflexions the distance from scale to mirror in a certain instance was 2 metres. If the deflexion along the scale was 50 mms. find, in degrees, the angle through which the mirror turned. Supposing that the scale can be read to 0·5 mm. calculate the least angular deflexion that can be measured.

8. How would you arrange two mirrors so as to be able to see the side of your head when looking straight forward? Give a drawing showing the complete course of the ray. (L. '02.)

CHAPTER XV

REFLEXION FROM SPHERICAL MIRRORS

In dealing with the subject matter of this chapter the following mathematical results will be found useful.

(1) Let ABC (Fig. 79) be any triangle, then it is shown in books on trigonometry that

$$\frac{AB}{BC}=\frac{\sin C}{\sin A}$$

(2) Let the line bisecting the angle B divide the side AC into the segments AD, CD,

then $$\frac{AB}{BC}=\frac{AD}{CD}$$

For, from △ ABD, $$\frac{AB}{AD}=\frac{\sin D}{\sin \frac{B}{2}}$$

and from △ CBD $$\frac{BC}{CD}=\frac{\sin D}{\sin \frac{B}{2}}$$

$$\therefore \frac{AB}{AD}=\frac{BC}{CD}$$

and $$\frac{AB}{BC}=\frac{AD}{CD}$$

(3) Similarly if BE is the bisector of the external angle at B

$$\frac{AB}{BC}=\frac{EA}{EC}$$

For $\angle$ EBD is a right angle and in $\triangle$ EAB

$$\frac{AB}{EA}=\frac{\sin E}{\sin EBA}=\frac{\sin E}{\cos ABD}=\frac{\sin E}{\cos \frac{B}{2}}$$

And in $\triangle$ EBC $\dfrac{BC}{EC}=\dfrac{\sin E}{\sin EBC}=\dfrac{\sin E}{\sin\left(\frac{\pi}{2}+\frac{B}{2}\right)}=\dfrac{\sin E}{\cos\frac{B}{2}}$

$$\therefore \frac{AB}{EA}=\frac{BC}{EC}$$

or
$$\frac{AB}{BC}=\frac{EA}{EC}$$

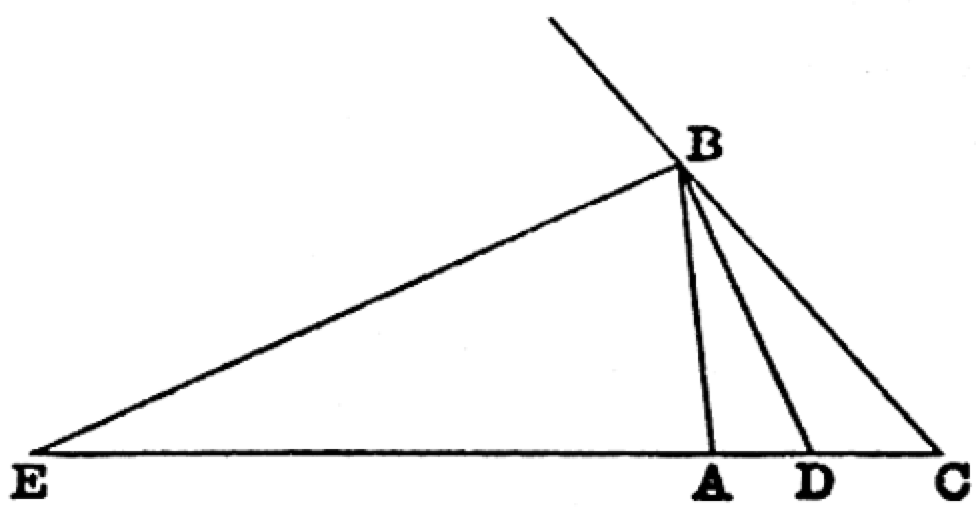
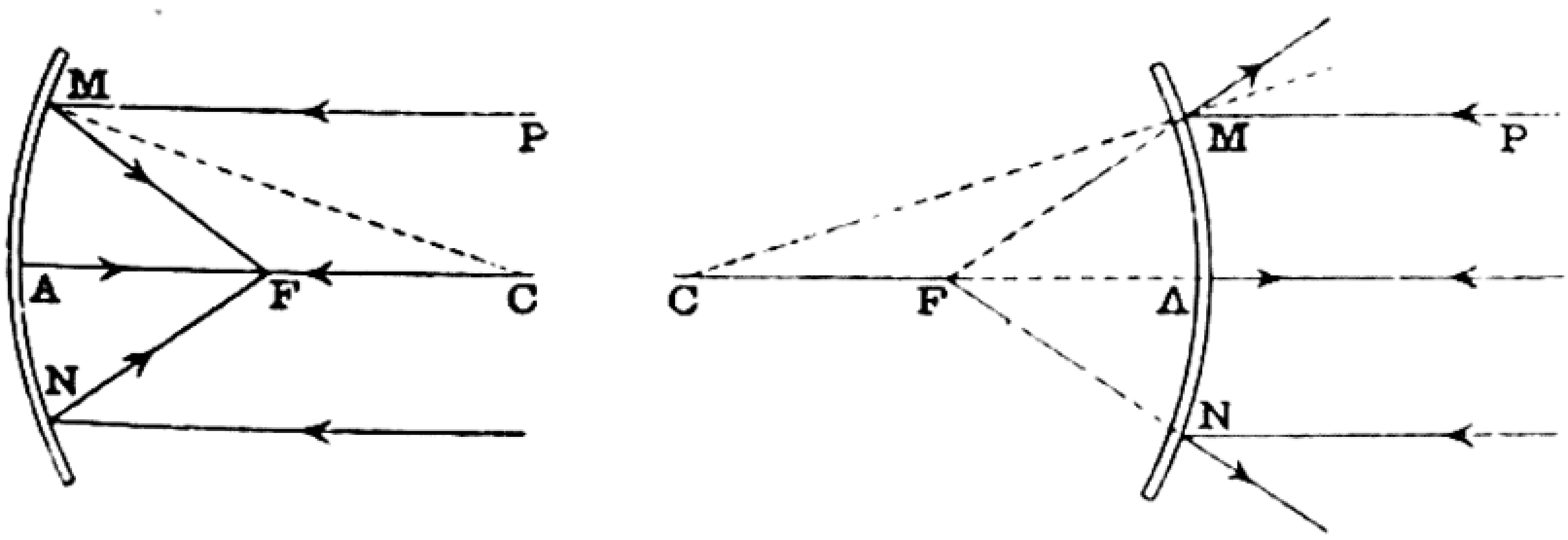


FIG. 79.

Principal Focus.—Let MAN (Fig. 80, *A* and *B*) represent a spherical mirror cut from a sphere whose centre is C. C is called the centre of curvature, or briefly the centre, of the mirror. CA is the radius of curvature, or simply the radius. The middle point, A, of the mirror is called the pole (not to be confused with the centre defined above), and the straight line passing through C and A is the principal axis. Any line drawn from C to the mirror is a radius and therefore cuts it normally. If we are working only in one plane, as in pin experiments, cylindrical mirrors may be used in place of spherical ones.

EXPERIMENT.—Fasten a concave mirror on the Hartl's optical disc (p. 138), and allow a beam of parallel rays coming through the slits to fall on it parallel to the principal axis. Provided the rays strike the mirror not far from the pole it will be seen that after reflexion they meet at a point F as in Fig. 80 (*A*). If a convex mirror is used the path of the light is that shown in Fig. 80 (*B*); the rays are made to diverge by reflexion, but if produced backwards they pass through a point F as in the previous case.

Confining our attention to the small part of the mirror situated round the pole, it is seen that if a beam of rays falls on it in the direction of the axis they all pass through a point F after reflexion. This point is called the **principal focus,** or focal point, and the distance FA is called the **focal length** of the mirror. The figures show that the focal point is real for a concave and virtual for a convex mirror. Measurements show that F is midway between A and C, *i.e.* the **focal length is half the radius of curvature.** This result can be established theoretically from the laws of reflexion, for, take any ray PM in the figure parallel to the axis and join C to M. Then CM (Fig. *A*) is the normal at the point M and $\angle$ CMP is the angle of incidence. Since PM is parallel to CA, $\angle$ PMC $=$ $\angle$ ACM, but $\angle$ PMC $=$ $\angle$ FMC, from the laws of reflexion, hence $\angle$ ACM $=$ $\angle$ FMC



(A) *(B)*

FIG. 80.—Principal Focus of Spherical Mirror.

and FM $=$ FC. But if M is near A, FM $=$ FA very nearly, and FC $=$ FM $=$ FA. The student should notice that it is only when the incident rays are limited to the part of the mirror near A that this result is true. It is left as an exercise for the reader to prove the corresponding proposition for a convex mirror.

EXPERIMENT.—Place a strip of cylindrical mirror on a drawing board and fix a pin vertically just in front of it. Fix a second pin some distance away so that the two pins and their images all appear in the same straight line. The line joining the pins represents an incident ray, and since the reflected ray returns along this path it is a normal to the mirror and therefore passes through C. Find two other normals in a similar way; the point where they meet is the centre of curvature. Next draw parallel lines to represent parallel rays falling on the mirror; if two pins are fixed in these in succession the corresponding reflected rays can be found as on p. 138, the point where they meet is the focal point. Rule in the outline of the mirror and measure CA and FA.

Image of any Point on the Axis. Conjugate Points.—In Fig. 81 let rays be supposed to start from some point P on the axis of a concave mirror whose centre is C and focal point F. We require to find where these rays meet after reflexion; this point will be the image of P. Let PM be one incident ray, draw the normal CM and make $\angle$ QMC = $\angle$ PMC; then MQ is the reflected ray. In the $\triangle$ PMQ, CM bisects the angle at M,

$$\therefore \frac{\mathrm{PC}}{\mathrm{QC}} = \frac{\mathrm{PM}}{\mathrm{QM}}$$

If, as we shall suppose in all that follows, M is near A, then PM = PA and QM = QA approximately.

Hence $$\frac{\mathrm{PC}}{\mathrm{QC}} = \frac{\mathrm{PA}}{\mathrm{QA}}$$

Putting PA = u, QA = v, CA = r, FA = f, and measuring all distances from A as origin, we get

$$\frac{\mathrm{PA} - \mathrm{CA}}{\mathrm{CA} - \mathrm{QA}} = \frac{\mathrm{PA}}{\mathrm{QA}}$$

or $$\frac{u - r}{r - v} = \frac{u}{v}$$

i.e. $$ur + vr = 2uv$$

Dividing throughout by uvr,

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$$

But $$f = \frac{r}{2}$$

hence $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

The same formula is true for any ray from P provided its point of incidence is near A, hence all the rays from P meet at Q after reflexion, and Q is the image of P. Evidently if the rays start from Q they will meet at P after reflexion and P will be the image of Q. Two points P and Q such that the rays starting from one pass after reflexion through the other are said to be **conjugate**. The formula

just given should be remembered ; v and u are the distances of image and object respectively from the pole of the mirror.

The image is virtual when a convex mirror is used, but its position is given by the same formula if a suitable convention is made with

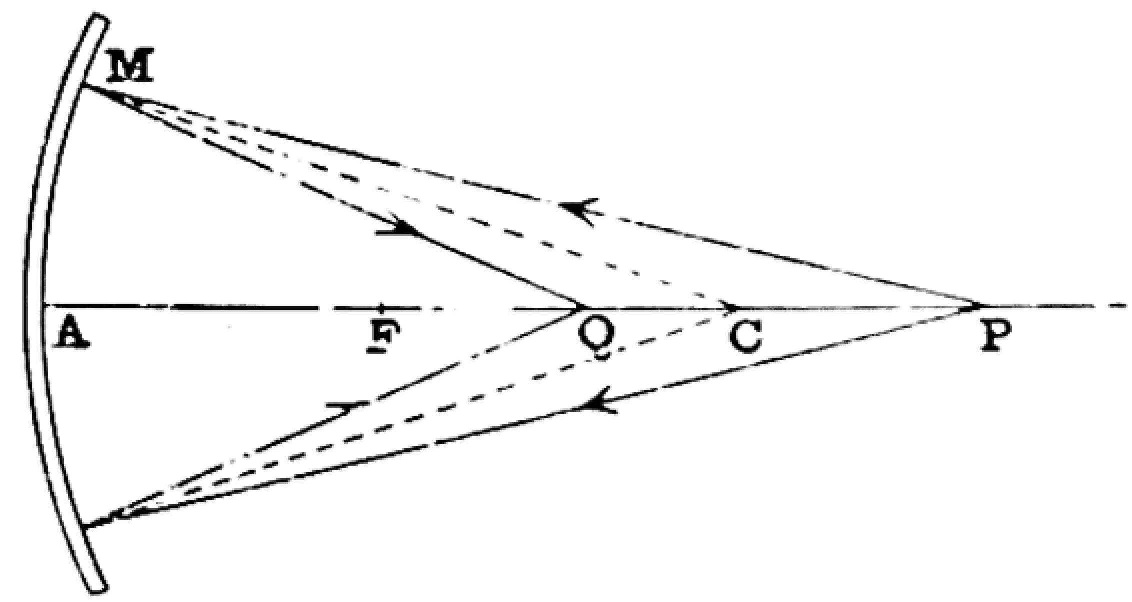


FIG. 81.—Showing the Position of Conjugate Points for a Concave Mirror.

regard to the signs of the different lengths. If distances are measured from the pole as origin, lines drawn in opposite directions must be looked upon as being positive or negative. We shall adopt the usual rule and call all distances positive when they are measured

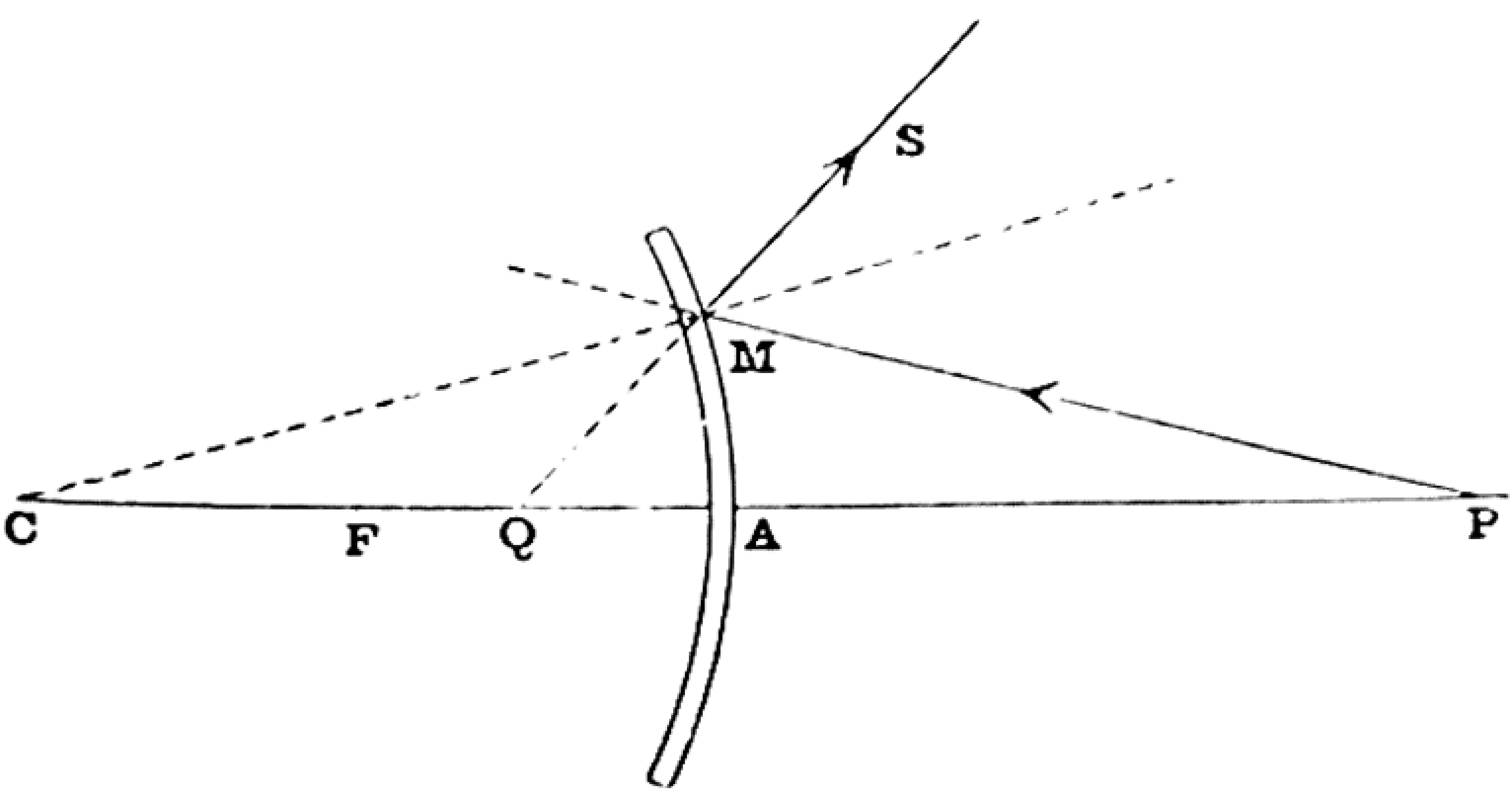


FIG. 82.—Showing the Position of Conjugate Points for a Convex Mirror.

from the mirror in a direction opposite to that in which the incident light is travelling; distances measured in the same direction as the incident light is going are to be taken as negative. Hence r and f are negative for a convex mirror. In Fig. 82, PM is the incident and MS the reflected ray; draw the normal CM and produce SM to cut

the axis at Q. Since CM bisects the $\angle$ PMS of the $\triangle$ PMQ, we have (p. 147)

$$\frac{PC}{QC}=\frac{PM}{QM}=\frac{PA}{QA} \text{ approximately.}$$

And
$$PC = AP + AC = u + (-r) = u - r$$
$$QC = AC - AQ = -r - (-v) = v - r$$

with the rule as to sign.

Hence
$$\frac{u-r}{v-r}=\frac{u}{-v}$$

and, as before,
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Example.—A gas flame is placed 35 cms. in front of a convex mirror whose radius is 24 cms.: find the position of the image.

Here $u = 35$
and $f = -12$

$$\therefore \frac{1}{v}+\frac{1}{35}=-\frac{1}{12}$$

and $v = -9{\cdot}07$ cms.

That is the image is 9·07 cms. *behind* the mirror.

Experiment.—Fix a pin in a drawing board 6 in. in front of a strip of cylindrical mirror and find as on p. 138 the direction of the reflected rays. The point where these meet is the image; show that the above formula is true.

Experiment.—Find also the position of the image by the parallax method.

When a small object is placed at the centre of curvature of a concave mirror all the rays are normals; after reflexion they will retrace their paths and form an image coinciding in position with the object. This follows also from the formula when $u = r$.

Experiment.—Support a concave mirror on a stand, use a bit of white card as object and arrange it by the parallax method to be at the same distance from the mirror as its real image; this distance is the radius of the mirror.

Graphical Construction of Images.—According to our approximation *all* the rays coming from a point on the axis meet after reflexion at the conjugate point; hence the point of intersection of *two* reflected rays is the position of the image. Now the directions of three reflected rays are known for—

(1) Those rays which before incidence are parallel with the axis pass after reflexion through the focus F.

(2) By supposing the path of the light to be reversed it follows that those incident rays which pass through the focus will be parallel with the axis after reflexion.

(3) Incident rays which pass through the centre of curvature strike the mirror normally and retrace their path.

Let us make use of this to find the image of a small object PQ (Fig. 83). Draw from Q a ray parallel to the axis; after reflexion this passes through F. Draw a second ray through C; after reflexion it retraces its path and the point Q′ where the two reflected

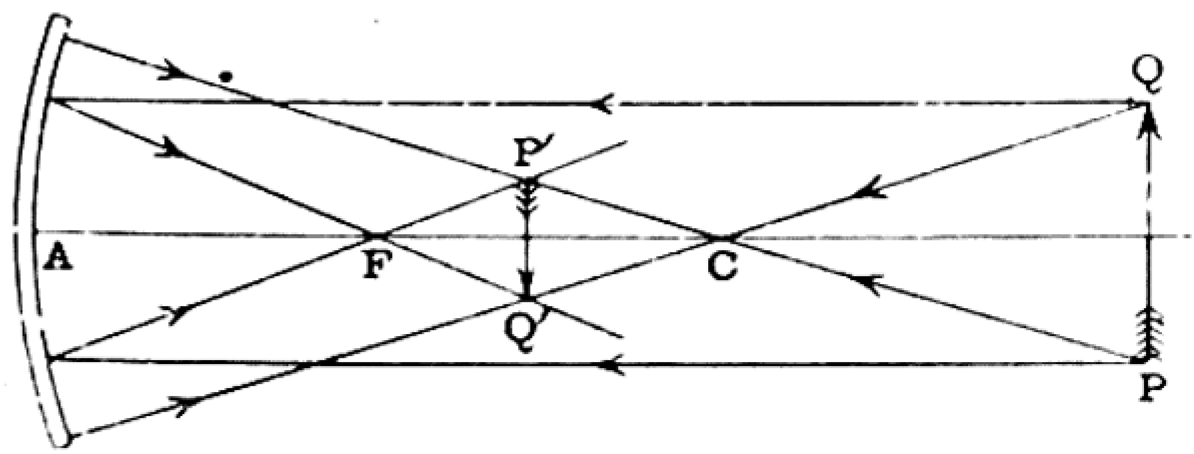


FIG. 83.—Formation of an Image by a Concave Mirror.

rays meet is the image of Q. The image of P is found in a similar manner, and P′Q′ is the complete image. The figure shows that it is real, inverted, and smaller than the object. If the object is placed at P′Q′ evidently the same drawing will give the image PQ; in this instance the rays which pass through F are parallel with the axis after reflexion and the second ray is found by joining Q′ to C and producing it to meet the mirror, whence it retraces its path. Fig. 84 (*A*) shows the image of an object placed between the principal focus and the concave mirror. In this case the reflected rays have to be produced backwards in order to cut each other and the image is virtual, erect, and larger than the object. It illustrates the use of a concave shaving mirror. Fig. 84 (*B*) shows how an image is formed when a convex mirror is used.

Relative Positions of Image and Object.—The equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ enables us to calculate the position of the image for any given position of the object, hence we can find how the image moves when the object approaches the mirror starting from a great distance

away. For this purpose, however, a simpler formula, first given by Newton, is preferable. In Fig. 85 the image of a small object has been found by drawing two incident rays, one parallel with the axis, the other through the principal focus. The arcs AB′, AB may be regarded as practically straight lines. Let all distances be measured

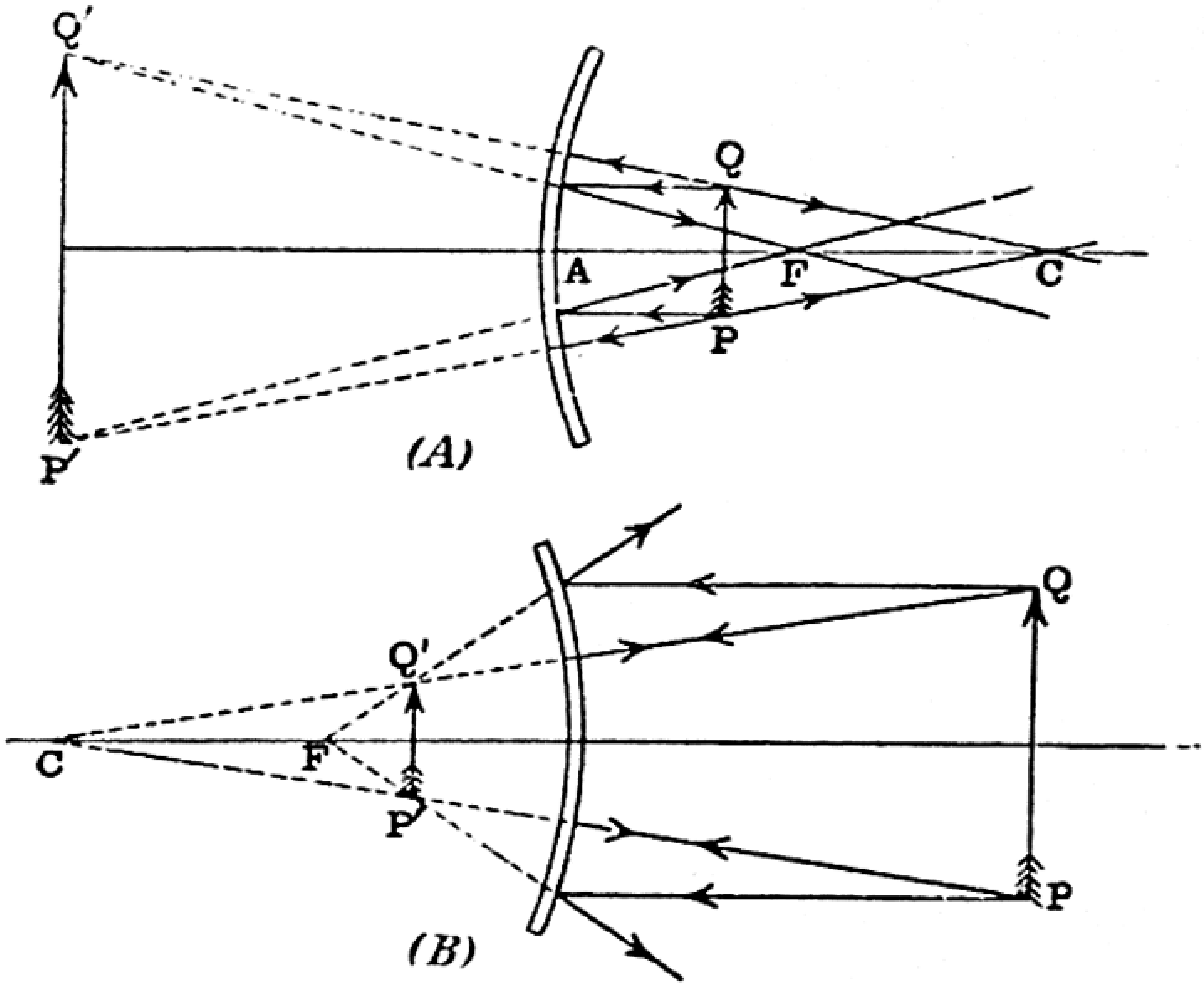


FIG. 84.—Construction of Virtual Images.

from F instead of from A, using the same sign convention as before, and let PF $= x$, P′F $= x'$, then from △'s P′Q′F, AB′F we have

$$\frac{\text{linear size of image}}{\text{linear size of object}} = \frac{P'Q'}{PQ} = \frac{P'Q'}{AB'} = \frac{FP'}{FA} = \frac{x'}{-f}$$

FA being negative since it is measured from F in the same direction as the incident light.

Similarly from △'s PQF, ABF, we have

$$\frac{\text{linear size of image}}{\text{linear size of object}} = \frac{AB}{PQ} = \frac{FA}{FP} = \frac{-f}{x}$$

Hence
$$\frac{x'}{f} = \frac{f}{x}$$

and
$$xx' = f^2$$

The same result can be deduced from the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, for in the figure $u=f+x$, $v=f+x'$, if these values of u and v are substituted we get $xx'=f^2$. The student should show by each method that this equation is true for a convex mirror. Since f^2 is positive no matter what the sign of f we conclude that x and x' have the same sign as each other, *i.e.* image and object are either both to the right or both to the left of F. The following conclusions can now be drawn for a concave mirror :—

(1) When x is very large x' is very small, *i.e.* when the object is very distant the image is at the principal focus.

(2) As x decreases x' increases ; as the object moves towards the mirror the image moves to meet it. While x is greater than f, x' is smaller than f, if the object is further from the mirror than C the image is between F and C.

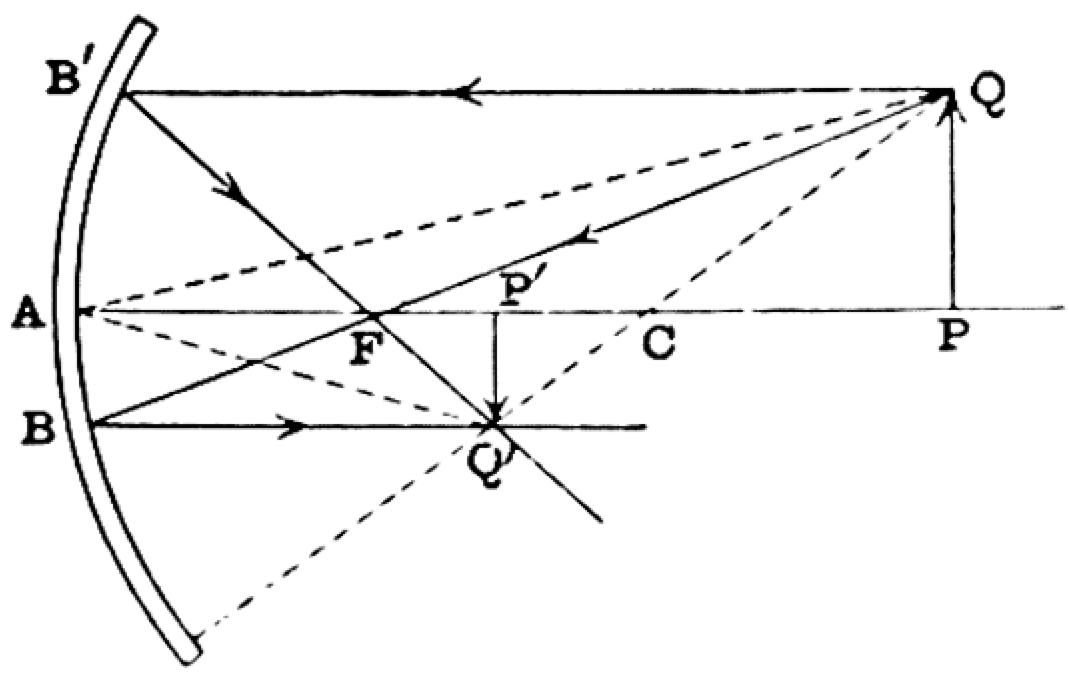


FIG. 85.

(3) When $x=f$ then $x'=f$, *i.e.* the object and image meet at the centre of curvature as we have already found.

(4) If $x<f$ then $x'>f$; if the object is situated between C and F the image is further from the mirror than the centre of curvature.

(5) When x is very small, x' is very large, *i.e.* if the object is close to F the object is a very great distance away.

(6) When $x<f$ and is negative, meaning that the object is between F and A, $x'>f$ and is also negative ; the image is therefore behind the mirror.

(7) If $x=-f$ then $x'=-f$, *i.e.* image and object meet at A.

In ordinary circumstances only the first three cases can be realised for a convex mirror, in (3) when $x=f$ the object is at the

surface of the mirror and image and object coincide. If a beam of convergent light is allowed to fall on the mirror we may regard the point to which the rays converge as a virtual object and the position of the image can be found from the equation. The student should work out the results for a convex mirror corresponding to cases 4–7.

Magnification.—The ratio of the length of a straight line in the image to the length of the corresponding line in the object is called **the linear magnification.** In Fig. 85 join Q to C and produce this line to meet the mirror; the ray QC returns along its path and therefore passes through Q′ the image of Q. Distances measured in opposite directions must be taken to be of opposite sign; thus P′Q′ is negative if PQ is positive. From △'s CPQ, CP′Q′ we have

$$\frac{-\text{Image}}{\text{Object}}=\frac{-P'Q'}{PQ}=\frac{-CP'}{CP}$$

or

$$\text{magnification}=\frac{\text{distance of image from centre}}{\text{distance of object from centre}}$$

Join Q to A; the ray QA gives rise to a reflected ray Q′A and the $\angle QAP = \angle Q'AP'$, hence △'s QAP, Q′AP′ are similar, the angles P, P′ being right angles, and

$$\frac{-\text{Image}}{\text{Object}}=\frac{-P'Q'}{PQ}=\frac{P'A}{PA}=\frac{v}{u}$$

$$\therefore\ \text{magnification}=-\frac{v}{u}\quad \text{. . . . .} \quad (1)$$

Also from △'s FP′Q′, FAB′

$$\frac{-\text{Image}}{\text{Object}}=\frac{-P'Q'}{AB'}=\frac{FP'}{FA}=\frac{v-f}{f}$$

or

$$\text{magnification}=-\frac{v-f}{f}\quad \text{. . . . .} \quad (2)$$

Finally from △'s FAB and FPQ

$$\frac{-\text{Image}}{\text{Object}}=\frac{-AB}{PQ}=\frac{FA}{FP}=\frac{f}{u-f}$$

and

$$\text{magnification}=-\frac{f}{u-f}\quad \text{. . . . . . .} \quad (3)$$

It is left as an exercise for the student to show that the same

expressions hold when the mirror is convex. By equating any two of these formulæ for the magnification we get at once the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$. When numerical values of u, v, f are substituted they must be put in with their proper sign ; if the magnification comes out a negative quantity it signifies that the image is real and inverted, for in that case v and u in (1) have the same sign, *i.e.* image and object are on the same side of the mirror.

Optical Bench.—The formulæ given in this chapter may be most accurately verified on an optical bench, of which a simple form is shown in Fig. 86. A number of stands to hold various pieces of

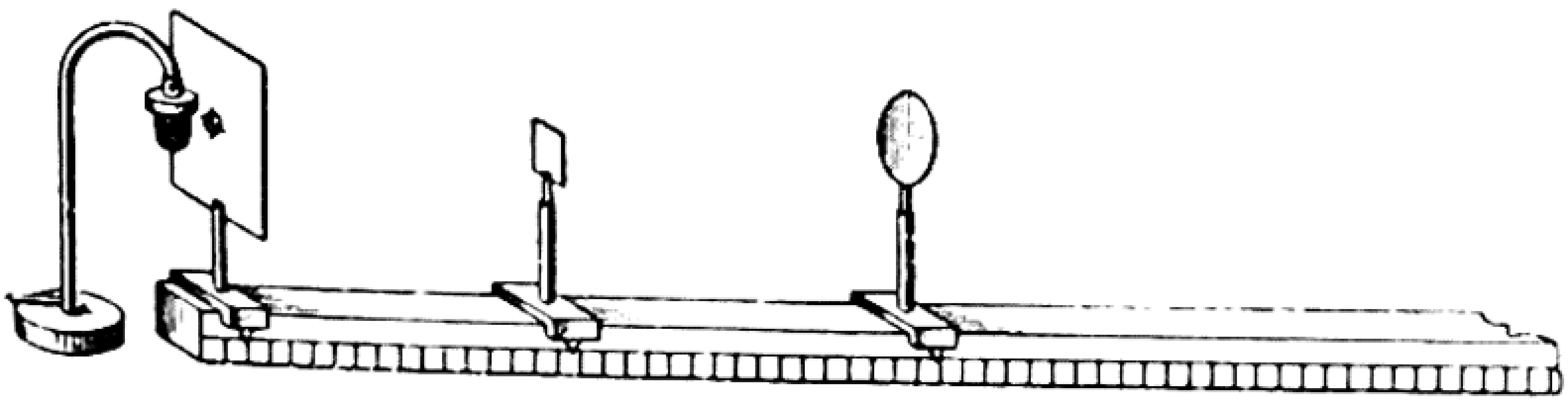


FIG. 86.—Simple Optical Bench.

apparatus slide to and fro along a straight, graduated, wooden bar about two metres long. In one stand is fixed a white cardboard screen containing a circular hole across which two wires are stretched at right angles, these are well illuminated by a lamp placed behind and form the object whose image is to be found. Other stands carry the mirror and a screen to receive the real image ; their positions are given by pointers moving over the scale.

Methods of finding the Focal Lengths of Mirrors.[1]

EXPERIMENT.—Allow a beam of light from a distant object to fall on a concave mirror, the rays coming from any point are parallel and the distance of the image from the mirror is the focal length. This image can be received on a screen and the distance measured. If the experiment is to be done in a small room a distant object may not be available, another mirror (or lens, p. 188) of known focal length may then be used to produce a parallel beam. Fix a candle at the focus of this auxiliary mirror, the reflected rays are parallel, and if they fall on the mirror whose focal length is required an image of the flame will be formed at its principal focus.

EXPERIMENT.—Place a concave mirror on the optical bench facing the cross-wires, move it to and fro until an image is formed on the screen near the

[1] See also Barton and Black, "Practical Physics," pp. 81 and 92.

circular hole; the wires are then at the centre of curvature, for $v = u$ in the ordinary formula, hence $u = r$.

EXPERIMENT.—Use the same apparatus but move the mirror further from the cross-wires; a real image is formed as in Fig. 83. This may be received on a small screen placed between the mirror and source. The distance of the cross-wires and image from the mirror are u and v of the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, whence f can be found; different positions of the mirror should be used. In performing the calculations much time is saved by a set of tables giving the reciprocals of numbers. Thus in an experiment $u = 85{\cdot}6$ cms., $v = 42{\cdot}4$ cms.; from tables,

$$\begin{aligned} \text{reciprocal of } 85{\cdot}6 &= 0{\cdot}01168 \\ \text{,, \quad ,, } 42{\cdot}4 &= \underline{0{\cdot}02358} \\ \text{sum} &= 0{\cdot}03526 = \frac{1}{f} \end{aligned}$$

$$\therefore f = \text{reciprocal of } 0{\cdot}03526 = 28{\cdot}5 \text{ cms. from the tables.}$$

If the formulæ are correct we should get the same value of the focal length by all methods, within the limits of experimental error.

To verify the formulæ for magnification the cross-wires are replaced by a rectangular slit about 2 cms. wide and the sizes of the image and of the slit are measured. If in addition to measuring the magnification the distance of image or object from the mirror is found we can calculate f from the formulæ

$$\text{magnification} = \frac{v - f}{f} = \frac{f}{u - f}$$

With convex mirrors there is the difficulty that the image is virtual and therefore cannot be received on a screen. The following methods can be used in this case (see also p. 199).

EXPERIMENT.—In Fig. 87 P represents the cross-wires on the bench, A the convex mirror, B a small piece of plane mirror with its reflecting face towards A. Rays from P pass by B and form an image at Q. The rays apparently coming from Q are reflected in the plane mirror, and, to an observer on the left of B, appear to come from Q′, where BQ = BQ′. By suitably moving B the image Q′ shifts until the distance BQ′ becomes equal to BP; in this position Q′ will not appear to shift relatively to a point on the screen as the observer moves his head sideways. When this position has been found measure BP and BA, then $v = AQ = BQ - BA = BP - BA$, and $u = PA = BP + BA$, whence f can be calculated. The following variation can also be used:— Replace the cross-wires by a knitting needle and turn the plane mirror with its reflecting face to the right, an image is formed at Q as before. Place a second needle between B and P and arrange by the parallax method that the image of this seen in the plane mirror coincides with the image already at Q. Then BQ

is equal to the distance of the second needle from B and this can be measured directly, also $AQ = BQ - BA = v$ and $PA = u$.

Reflexion of Waves other than Light Waves.—For experiments on these waves a thermopile must be used as detector, since they do not cause the sensation of sight. The position of the image formed by a concave mirror has been deduced from the laws of reflexion, if therefore it is found that other radiations (so-called heat waves, p. 116) are brought to a focus at the same point, these radiations must obey the same laws. Concave metal mirrors may be used in place of glass.

EXPERIMENT.—Find the focal length of such a mirror for rays of light, then use as source of radiation a Bunsen burner with a rose top placed some distance

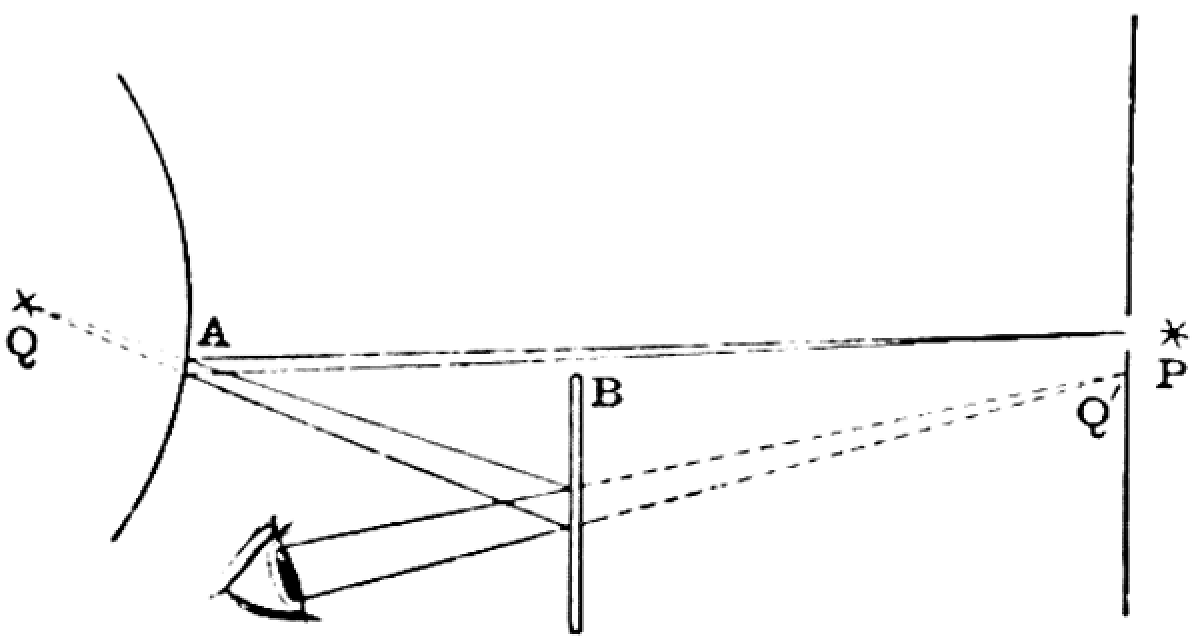


FIG. 87.—Method of finding the Focal Length of a Convex Mirror.

in front of the mirror. Let the reflected rays fall on a thermopile which is connected to a sensitive galvanometer, move this up to the mirror until the maximum deflexion of the needle is produced. The image of the burner is now on the blackened junctions of the thermopile; if the distances of image and object from the mirror are measured the focal length can be calculated from the usual formula. It will be found to be practically equal to that found by optical methods. As the conical reflector on the thermopile collects rays which would not otherwise fall on the junctions it is best removed for this experiment.

EXPERIMENT.—Arrange two concave mirrors to face each other about 10 ft. apart; fix at the focus of one a thermopile, at the focus of the other a lighted candle or a Bunsen burner. The rays from the source are made parallel by reflexion at one mirror and are focussed by the other on the thermopile; a considerable deflexion is produced, but this is greatly reduced if either source or thermopile is moved to one side.

EXPERIMENT.—Arrange two long brass tubes 2–3 inches in diameter as shown in Fig. 88, place a rose burner at the end of one and a thermopile at the corresponding end of the other. Protect the instrument by a wooden screen from direct radiation from the flame. Very little deflexion is produced in these circumstances. Place a reflecting surface at the distant ends of the tubes; when they are equally inclined to it a considerable deflexion of the needle follows.

These experiments show that the long heat waves are reflected according to the same laws as light.

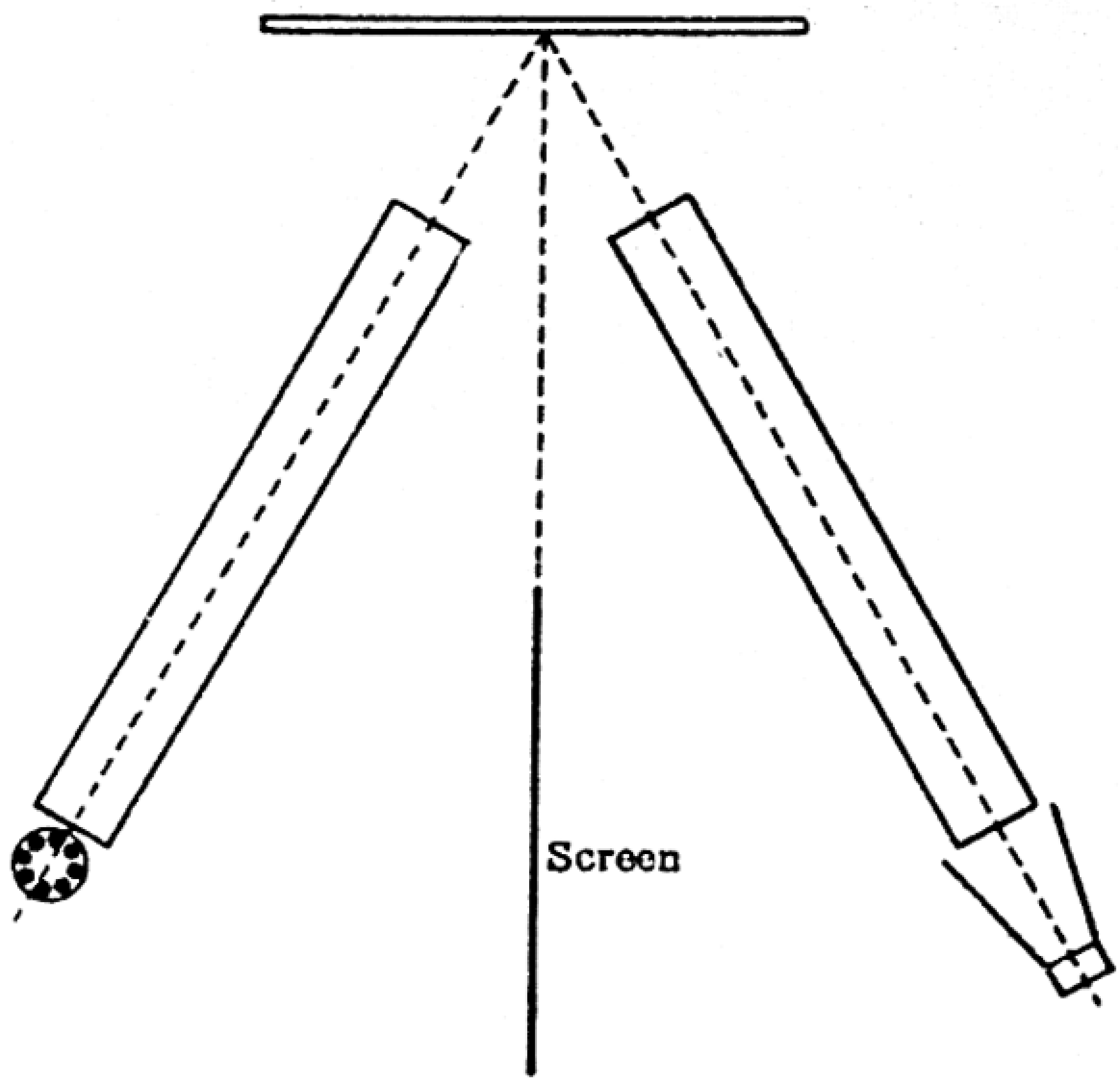


Fig. 88.—Showing Reflexion of Heat Rays.

Measurement of an Angular Deflexion.—Instead of the telescope method of measuring an angular deflexion a concave mirror may be

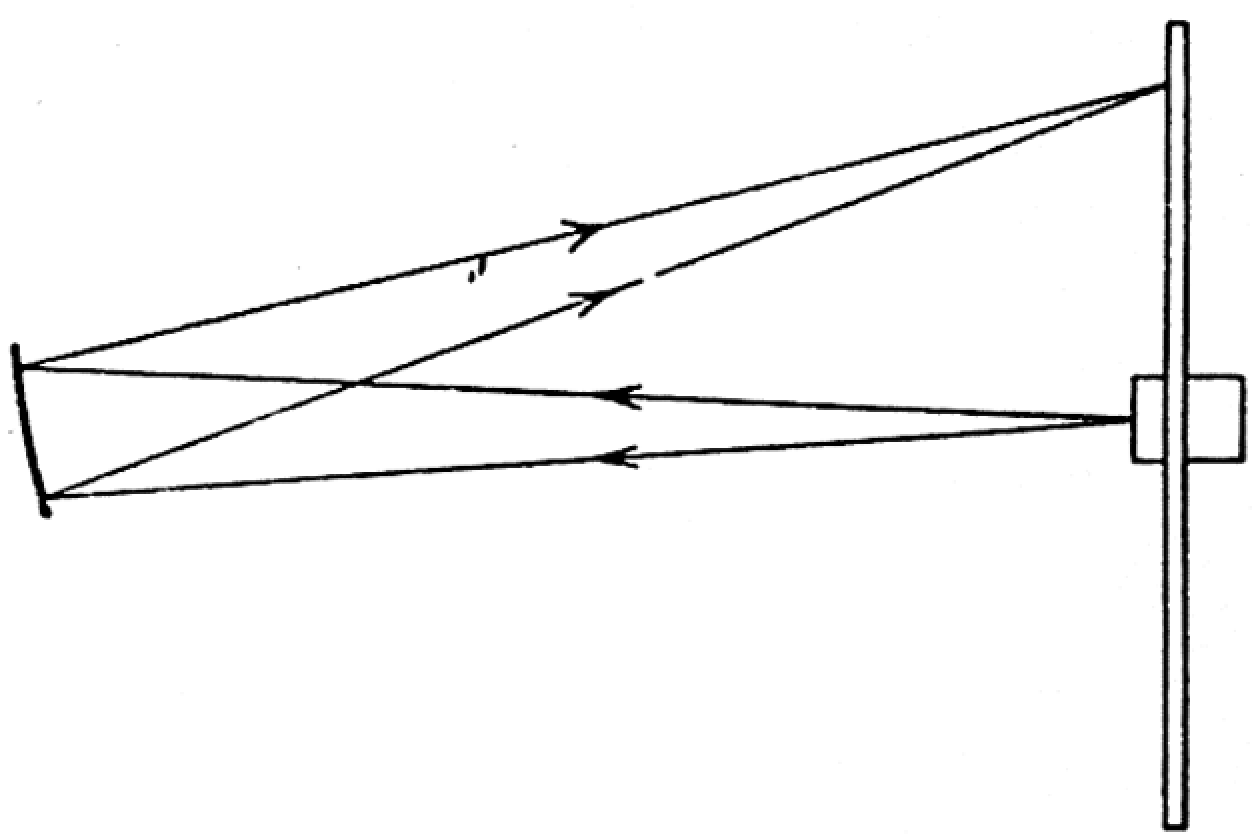


Fig. 89.—Lamp and Scale Method of measuring an Angular Deflexion.

used. The mirror (Fig. 89) is attached to the body whose deflexion is required, and the telescope is replaced by a narrow vertical slit

illuminated from behind by a lamp. The slit is fixed at a distance from the mirror equal to the radius of curvature, and immediately above or below it a divided scale is placed. In these circumstances an image of the slit is thrown on the scale, and the movements of the spot of light are used as in the previous method to measure the deflexion of the mirror. It must be remembered that the angle through which the reflected ray is turned is twice the angular displacement of the mirror.

EXAMPLES ON CHAPTER XV

1. A candle flame is placed 25 cms. away from a concave mirror whose radius of curvature is 80 cms.; find the position and nature of the image.

Here $u = 25, f = 40$,

$$\therefore \frac{1}{v} + \frac{1}{25} = \frac{1}{40}$$

whence, from a table of reciprocals,

$$\frac{1}{v} = 0{\cdot}025 - 0{\cdot}04 = -0{\cdot}015$$

Hence $$v = -66{\cdot}7 \text{ cms.}$$

The magnification $$-\frac{v}{u} = \frac{66{\cdot}7}{25}$$

The image is therefore 66·7 cms. behind the mirror and is virtual, enlarged, and erect (magnification positive).

2. Where must the candle be placed in order that a real image, five times as large as the object, may be formed?

As the image is real v and u have the same sign and $v/u = 5$, or $v = 5u$.

Hence $$\frac{1}{5u} + \frac{1}{u} = \frac{1}{40}$$

and $$u = 48 \text{ cms.}$$

3. In what positions must the candle be placed to give rise to an image four times as large as the object?

4. An object 2 cms. in length is placed 35 cms. in front of a mirror and the real image is found to be 4 mms. high; find the focal length of the mirror. What is the focal length if the image is virtual?

5. When a gas flame is placed 32 cms. in front of a mirror it is found that the image is 12 cms. behind the mirror. Find the radius of curvature.

6. Show that the image formed by a convex mirror is always smaller than the object. (L. '80.)

7. An object is placed 28 cms. from a concave mirror whose focal length is 10 cms.; find where the image is. Is it real or virtual, erect or inverted? What is its size if the object be 4·2 mms. broad by 14 mms. long? (L. '91.)

8. A ray parallel to the principal axis meets a concave mirror at an angle of incidence θ. Prove that the reflected ray cuts the axis at a distance $R/2 \cos \theta$ from the centre, where R is the radius of curvature.

9. A narrow strip of plane mirror is placed between a vertical knitting needle and a convex mirror. The distance between the mirrors is 8 cms. When the needle is 14 cms. from the strip it is found that the two virtual images appear to coincide. What is the focal length of the mirror? If the needle is moved 5 cms. nearer to the convex mirror where must the strip be placed for coincidence of the images to occur?

CHAPTER XVI

LAWS OF REFRACTION. MEASUREMENT OF REFRACTIVE INDICES

Refraction.—When a beam of light falls on the surface of separation of two media we have seen that part is reflected while the remainder continues its course in the second medium; it is with this transmitted portion that we have now to deal.

Experiment.—Fix a semicircular plate of glass on the Hartl disc with its diameter along that of the graduated circle and arrange that a ray of light PN (Fig. 90) travels from a slit towards the centre. The ray NQ which enters the glass is bent towards the normal OO′ as in the figure; this bending of the ray is due to refraction. At points where the ray strikes the surface normally no bending takes place.

In the figure PN is the incident and NQ the refracted ray; if ONO′ is the normal at N, ∠ONP is the angle of incidence and ∠O′NQ the angle of refraction. It is found generally when a ray goes from a rare into a denser medium that the refracted ray is bent towards the normal, making the angle of incidence greater than the angle of refraction. If the slit is moved round to Q the ray travels in the direction QNP, *i.e.* the path of the light is reversible.

Experiment.—Place a coin in the bottom of a basin and arrange a slit at E (Fig. 91) so that on looking through it only the extreme edge of the coin can be seen. When water is poured into the vessel the whole coin is visible; the emergent ray is bent away from the normal at the surface and appears to come from Q. The eye cannot tell, of course, that the ray has been bent.

Laws of Refraction.—The laws of refraction are concerned with the position of the refracted ray. Calling the plane containing the refracted ray and the normal the plane of refraction, the two laws of refraction are the following:—

(1) The plane of incidence coincides with the plane of refraction,

i.e. the incident ray, the refracted ray, and the normal all lie in one plane.

(2) The ratio $\dfrac{\text{sine of the angle of incidence}}{\text{sine of the angle of refraction}}$ is a constant for two media while the same coloured light is used.

The truth of the first law is evident since in the first experiment both rays and the normal lie in the plane of the graduated circle. Denoting the angles of incidence and refraction by i and r respectively the second law states that $\dfrac{\sin i}{\sin r} = \mu$, where μ is a constant called the **refractive index** of the second medium relatively to the first. If

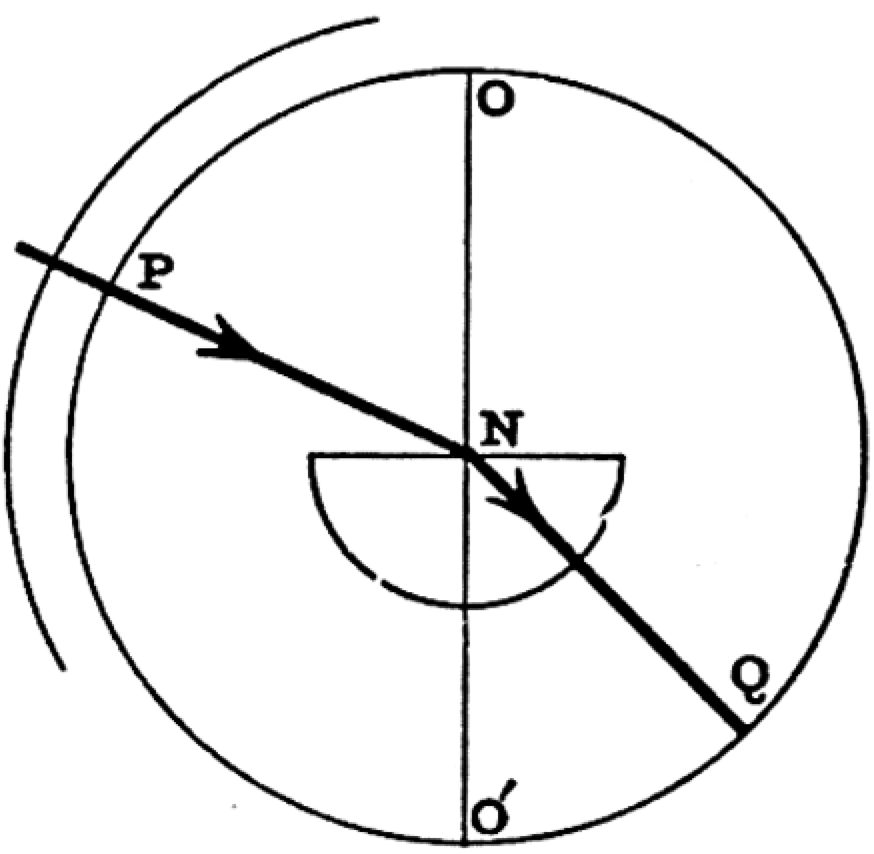


FIG. 90.—Showing Refraction.

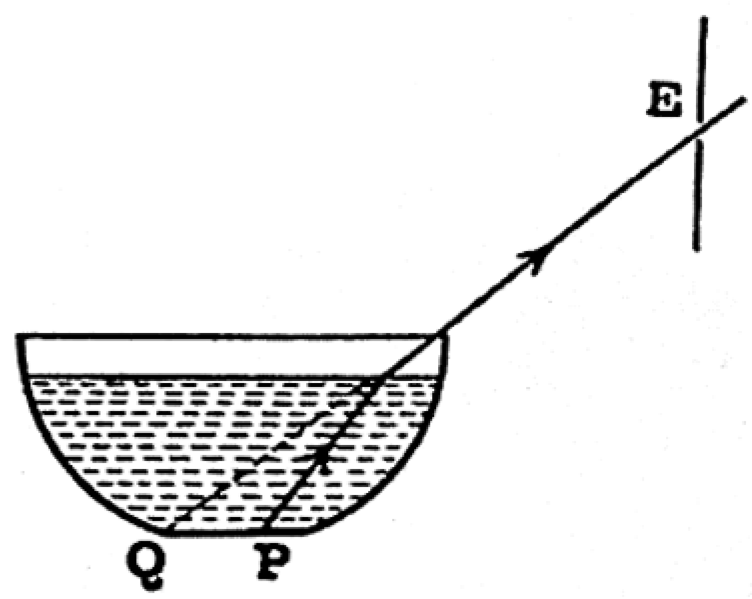


FIG. 91.—Coin in a Basin of Water.

the first medium is a vacuum this constant is called the absolute refractive index, or simply the refractive index, of the second medium; as the angle of refraction in this case is usually less than the angle of incidence the refractive index of most media is greater than unity. For most purposes in giving values of μ it is sufficiently accurate to take air as the first medium, the numbers differ very little from their true values. As regards the colour of the light and its effect on the refractive index more will be said in a subsequent chapter. The numbers in the second table below refer to the yellow light obtained by putting a bead of common salt in a Bunsen flame.

The most accurate proof of the second law lies in the fact that the refractive index of a medium measured by widely different methods does actually come out constant; an approximate proof

may be obtained with the apparatus used in Fig. 90, the angles i and r being read off directly from the graduated circle. The following table contains some values of i, r, and $\mu = \sin i/\sin r$ obtained in this manner.

i.	r.	μ.
31°	20°	1·50
39°	25°	1·49
50°	30°	1·50
60°	35°	1·52
76·5°	40°	1·51
90°	41·5°	1·51

In taking these numbers the path of the light was in the direction QNP (Fig. 90), since the path of the rays is reversible this does not matter. The numbers in the third column show that μ is a constant no matter what the angle of incidence.

Table of Refractive Indices.

Substance.	Refractive index.	Substance.	Refractive Index.
Methyl alcohol . . .	1·33	Glycerine	1·47
Ethyl alcohol . . .	1·36	Turpentine	1·47
Aniline	1·59	Water	1·33
Canada balsam . . .	1·53	Crown glass . . .	1·51

The refractive index depends on the temperature, at higher temperatures the refractive index of a liquid is decreased. If d is the density of a substance it is found that as μ and d alter the fraction $(\mu - 1)/d$ remains constant; this result is known as **Gladstone and Dale's law.**

EXPERIMENT.—*To trace rays through a block of glass and to find its refractive index.* For this experiment a large block of glass with parallel sides is required, the cutting shapes used in trimming photographic prints, ¼-plate size, answer admirably. Lay the glass flat on a drawing board and fix pins at P, Q (Fig. 92), this line represents an incident ray. Look through the glass from S and arrange two other pins R, S, to be apparently in the same straight line as PQ.

RS is the ray after passing through the block. Rule in the outline of the glass and produce PQ, SR to meet it at N and N′; the line NN′ represents the ray in the block. Draw the normal at N; with centre N and a large radius describe a circle, from the points where the incident and refracted rays cut it drop perpendiculars PO, ML, on ON. Then

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin ONP}{\sin MNL} = \frac{OP}{PN} \Big/ \frac{ML}{MN} = \frac{OP}{ML}$$

Measure these lines and prove that μ is independent of the angle of incidence.

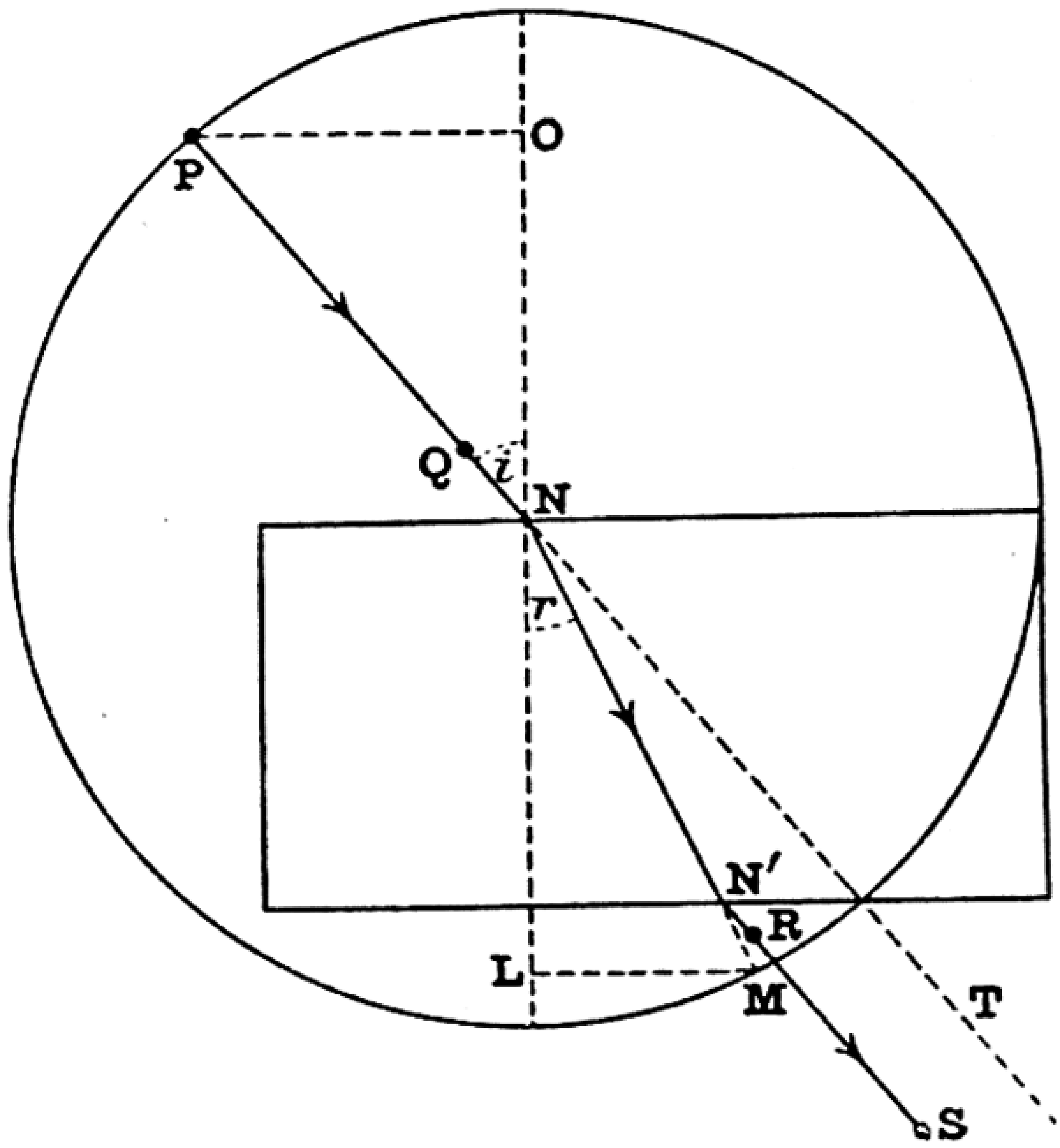


FIG. 92.—Showing the Path of a Ray through a Block of Glass.

Ray passing in succession through Several Media.—A second important conclusion can be drawn from the last experiment, it will be found that the emergent ray RS is parallel to the incident ray PNT. This follows also from the reversibility of the light path, for the angle of incidence in the glass at N′ is r and hence that of emergence is i. If μ_{ag} represents the refractive index going from air to glass and μ_{ga} from glass to air, then $\mu_{ag} = \sin i/\sin r$ and $\mu_{ga} = \sin r/\sin i = \dfrac{1}{\mu_{ag}}$. Hence when rays traverse a slab of a medium whose sides are parallel the rays which emerge are parallel

to their original direction, but they are displaced sideways through a distance ST (Fig. 92). For example, if a straight pole is viewed through such a block of glass, held so that the rays strike it obliquely, the part seen through the block is displaced relatively to the remainder. The emergent and incident rays are still parallel, even when the light traverses several slabs of different media with parallel sides, provided the first and last media are the same.

EXPERIMENT.—Prove this using a thick-sided glass vessel containing water.

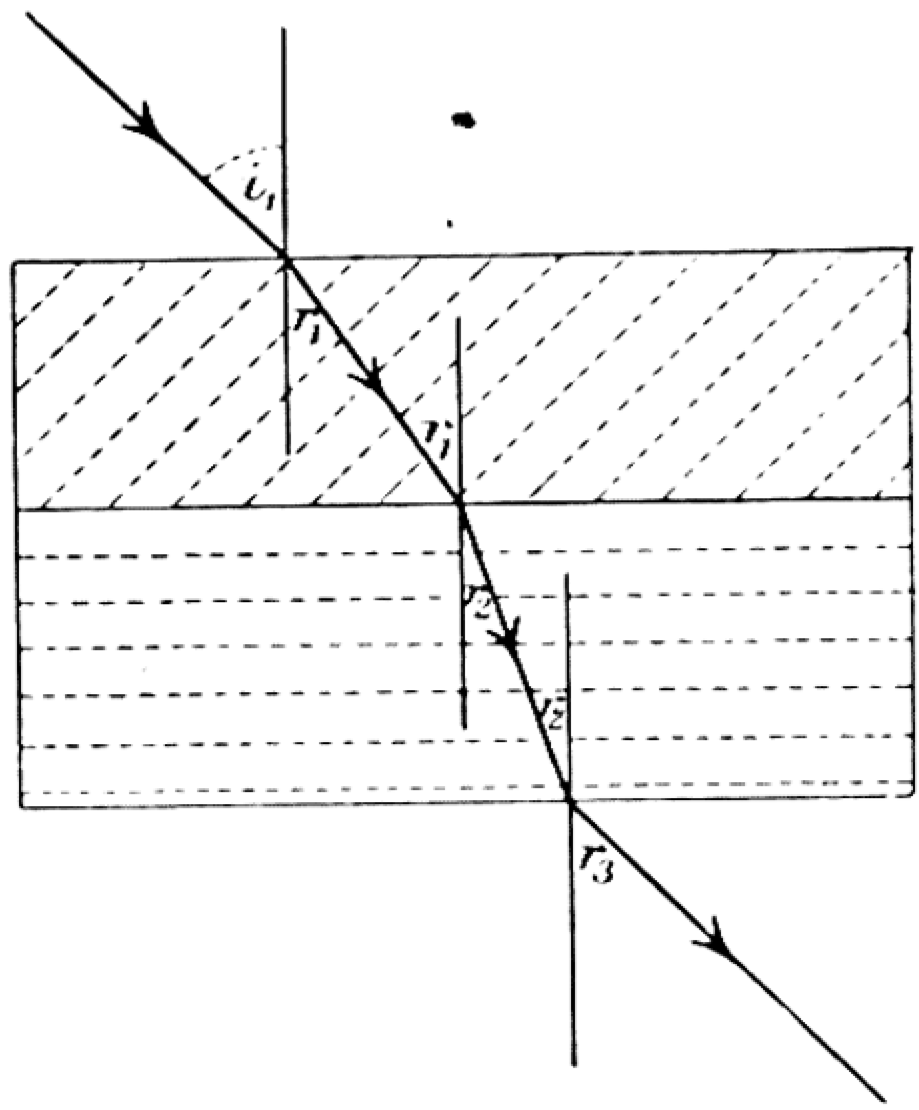


FIG. 93.—Light passing through several Media.

Suppose the first and last media are air and let μ_{12}, μ_{23}, μ_{31} represent the refractive indices in the direction in which the light is travelling, *i.e.* μ_{23} is the refractive index when the light passes from the second to the third medium and so on. Then (Fig. 93) since the incident and emergent rays are parallel $i_1 = r_3$ and

$$\mu_{12} = \frac{\sin i_1}{\sin r_2}, \ \mu_{23} = \frac{\sin r_1}{\sin r_2}, \ \mu_{31} = \frac{\sin r_2}{\sin r_3} = \frac{\sin r_2}{\sin i_1}$$

$$\therefore \mu_{12} \cdot \mu_{23} \cdot \mu_{31} = \frac{\sin i_1}{\sin r_1} \cdot \frac{\sin r_1}{\sin r_2} \cdot \frac{\sin r_2}{\sin i_1} = 1$$

A similar relation holds no matter how many media there are. If there are only two, as in Fig. 92,

$$\mu_{12} \cdot \mu_{21} = 1$$

or

$$\mu_{12} = \frac{1}{\mu_{21}} \text{ as we have already found.}$$

For three media

$$\mu_{23} = \frac{1}{\mu_{12} \cdot \mu_{31}} = \frac{\mu_{13}}{\mu_{12}} \text{ (from the last equation).}$$

Stated in words, this equation tells us that the refractive index of the third medium relatively to the second is obtained by dividing the absolute refractive index of the third medium by that of the second.

EXAMPLES.—The refractive indices of water and glass relatively to air as the first medium are $\frac{4}{3}$ and $\frac{3}{2}$ respectively; hence when light passes from water to glass the refractive index $\mu_{wg} = \frac{3}{2}/\frac{4}{3} = \frac{9}{8}$, and the index going from glass to air is

$$\mu_{ga} = \frac{1}{\mu_{ag}} = \frac{1}{3/2} = \frac{2}{3}$$

Geometrical optics furnishes no reason for the laws of refraction, for this we must refer to books on Physical Optics, where it is shown that

$$\mu_{12} = \frac{\text{velocity of light in the 1st medium}}{\text{velocity of light in the 2nd medium}}$$

As the refractive index varies with the colour of the light it follows that different coloured lights generally travel with different velocities.

Image formed by Refraction at a Plane Surface.—Let O (Fig. 94) represent a small object in any medium, ON the normal to AB the surface of separation of two media; we require to know where O will appear to be to an eye placed above N. On account of the size of the eye-pupil the rays to be considered must emerge near N. Suppose, for example, that the media above and below AB are air and glass respectively; a ray OQ starting from O is bent away from the normal at Q and travels in the direction QM. Produce MQ backwards to meet ON at O′. The normals to AB at Q and N being parallel

$$\mu_{ga} = \frac{\sin i}{\sin r} \text{ in the figure}$$

$$= \frac{\sin \text{QON}}{\sin \text{QO'N}}$$

$$\therefore \mu_{ga} = \frac{\text{QN}}{\text{QO}} \Big/ \frac{\text{QN}}{\text{QO'}} = \frac{\text{QO'}}{\text{QO}} = \frac{\text{O'N}}{\text{ON}}$$

since Q is near N, and therefore OQ = ON, and O′Q = O′N approximately. This gives the position of O′ the point at which the emergent ray intersects the normal; if any other ray is taken coming from O and incident near N it will be found that it cuts ON at the same

point, hence all the emerging rays appear to diverge from O′ and this point is the image of O. Calling u the distance of the object and v the distance of the image from the surface AB, we have $v = \mu u$, where μ is the refractive index *in the direction in which the light is travelling.*

The object appears to be displaced from its true position by an amount $OO' = ON - O'N = ON(1 - \mu)$. Owing to this effect of refraction a swimming bath appears to be less deep than it really is. If the position of O′ can be found experimentally the above result can be used to find the refractive index μ_{ga}.

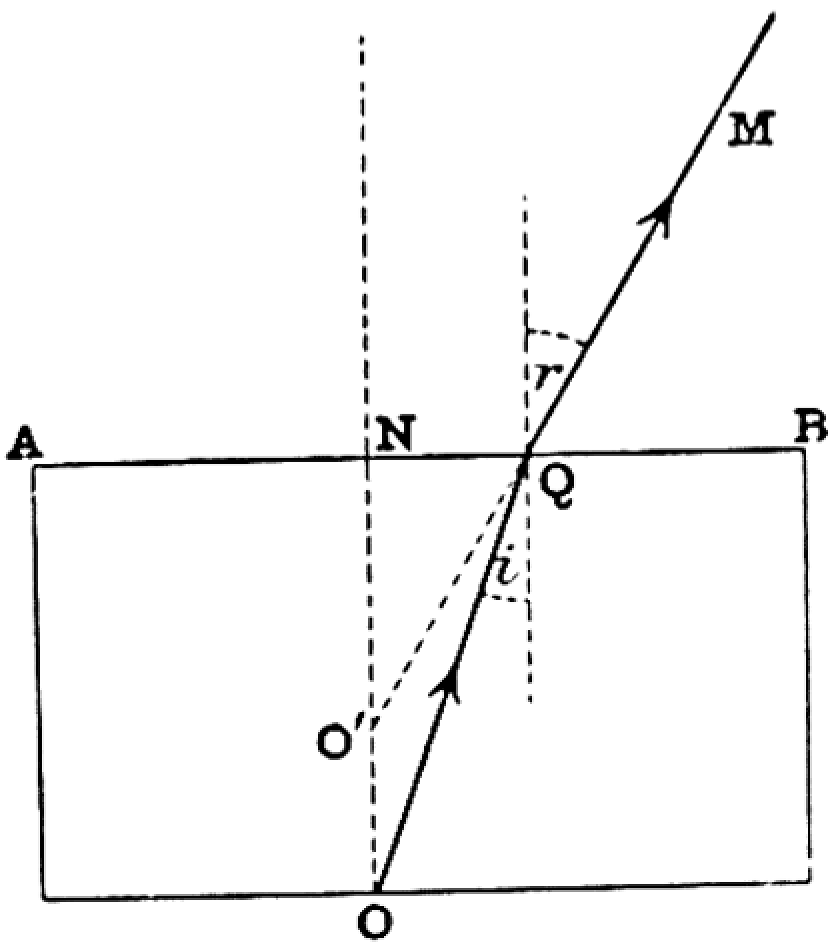


FIG. 94.—Image formed by Refraction at a Plane Surface.

EXPERIMENT.—*Pin method.* Place the photographic cutting shape on a drawing board and stick a pin immediately behind and in contact with it to serve as the object O. View the pin through the glass and fix two other pins at Q, M (Fig. 94) to appear in the same straight line with this image. QM fixes the direction of one emergent ray, others can be found in a similar way; the point where they meet is O′. Measure the distances of O and O′ from AB along the normal ON, then

$$\frac{1}{\mu_{ga}} = \mu_{ag} = \frac{ON}{O'N} = \frac{\text{actual thickness of the block}}{\text{apparent thickness}}$$

EXPERIMENT.—*Microscope method.*[1] A microscope whose objective has a focal length of from one to three inches is required; it should be capable of movement along a vertical graduated scale attached to the stand and its position relatively to this scale should be given by a vernier. Make a pencil mark on a piece of paper and stick it to the bench; focus the microscope with its axis vertical on this mark and read the vernier. Put a thick block of glass on the paper, *e.g.* a cubical paper weight; as the mark is now apparently raised through a distance OO′ (Fig. 94) it is no longer in focus. Move the microscope along the scale until the mark is clearly seen and again read the vernier. Finally scatter a few grains of chalk on the upper surface of the block, focus the microscope on these and read the vernier. The instrument has now been focussed in succession on points corresponding to O, O′, N (Fig. 94), hence the difference between the first and last vernier readings gives the distance ON, and that

[1] Barton and Black, "Practical Physics," p. 86.

between the second and third readings the length O'N, μ_{ag} can therefore be found. The same method can be applied to find the refractive index of a liquid. A piece of marked paper is stuck to the inner side of the bottom of a beaker and the microscope focussed on it as before. Liquid is poured in and the microscope focussed in succession on the mark and on the upper liquid surface. The calculation is made as in the last case.

It is only when we limit ourselves to the rays emerging near N that a definite image is formed. If we find the position of a number

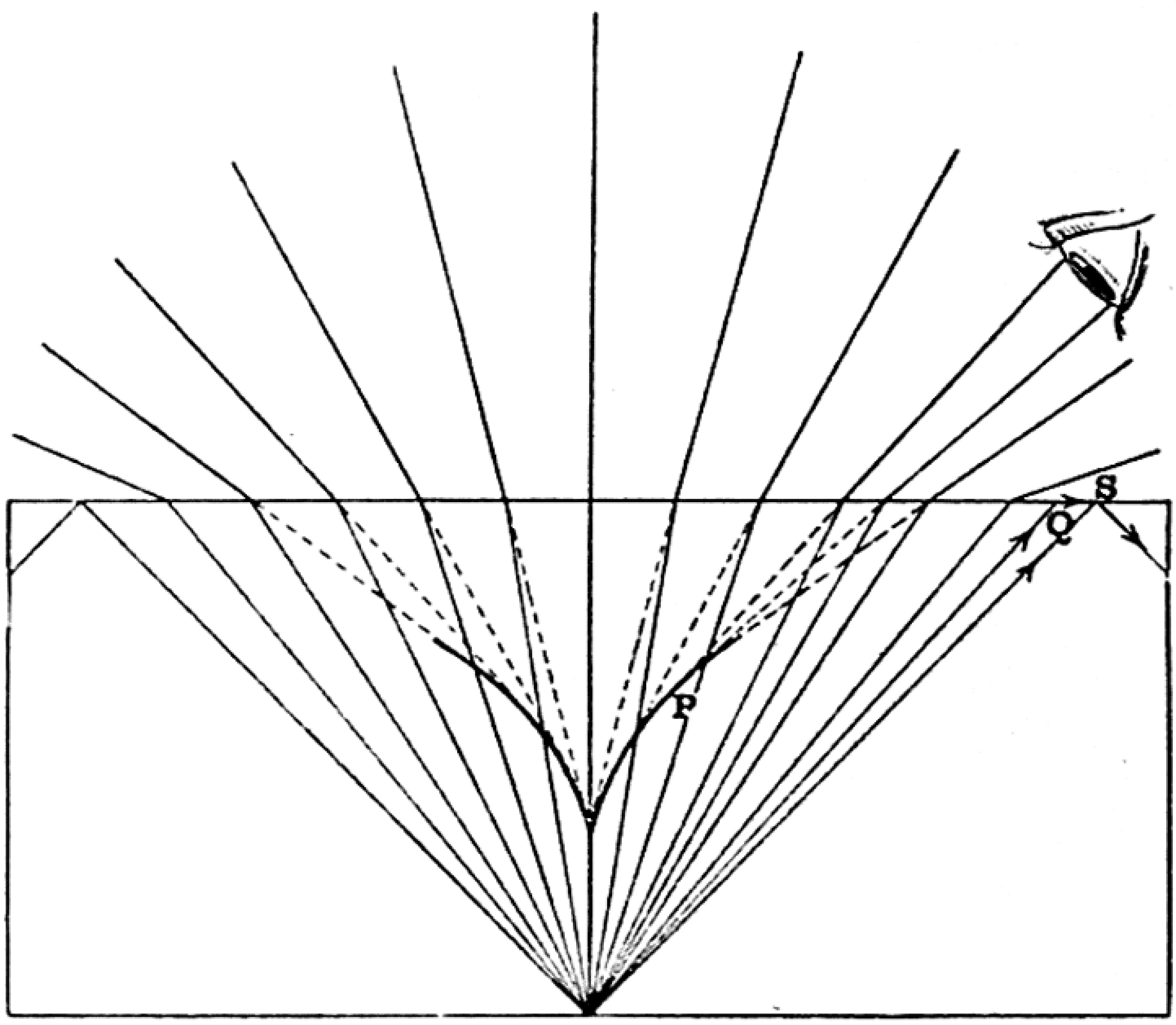


FIG. 95.—Caustic by Refraction at a Plane Surface.

of refracted rays, by the pin method given above, results are obtained similar to those shown in Fig. 95. The extreme rays do not diverge even approximately from a point, and the apparent thickness of the block varies with the position of the observer. Thus if the eye is placed as shown in the figure the pin appears to be at P. The curve joining the points of intersection of adjacent refracted rays is called the caustic by refraction; it is shown by the thickened line in the figure.

Total Reflexion.—When light passes from a rare into a dense medium the refracted ray is bent towards the normal, so that even

when the angle of incidence is nearly 90° a refracted ray is possible. It is otherwise when the first medium is denser than the second. Thus in Fig. 90, if the light is travelling in the direction QNP and the angle of incidence O′NQ is gradually increased, a stage is arrived at where the angle of refraction is 90°, the refracted ray then travels parallel with the surface of the glass. If the angle of incidence is increased still more there is no refracted ray, all the light is reflected back again into the first medium; this is called **total internal reflexion.** The angle of incidence at which total reflexion begins is called the **critical angle.** The table on p. 165 shows that the critical angle for the glass used is 41·5°. If μ_{ga} is the refractive index from glass to air and θ is the critical angle

$$\mu_{ga} = \frac{\sin \theta}{\sin \frac{\pi}{2}} = \sin \theta$$

hence the refractive index of the glass

$$\mu_{ag} = \frac{1}{\mu_{ga}} = 1/\sin \theta = \text{cosec } \theta$$

In Fig. 95 the ray OQ is incident at the critical angle, the ray OS is totally reflected. It follows from this that if we stand on the side of a swimming bath we shall not be able to see the more distant points at the bottom; the rays coming from such points in the direction of the eye are totally reflected at the surface of the water. A crack in a window-pane looks brightly reflecting for a similar cause; rays travelling in the glass strike the air film at an angle greater than the critical angle and are totally reflected.

Experiment.—Hold an empty test-tube in an inclined position in a beaker of water and view it from above. The sides of the tube look like a brightly polished mirror owing to the light which comes through the sides of the beaker being totally reflected.

Whenever light is reflected from a glass mirror a certain amount of it is lost since part is refracted; this loss can be avoided by making use of total reflexion. Fig. 96 shows a total reflexion prism in a form frequently used to turn the path of a beam of light through 90°. It is a right-angled isosceles prism of glass having a refractive index about 1·51; for such a glass, as the table on p. 165 shows, the critical angle is 41·5°. Rays fall normally on one of the short faces

and travel on without refraction; they meet the hypotenuse at an angle of 45°, which is greater than the critical angle, and are therefore totally reflected without loss.

Application of the Critical Angle to the measurement of Refractive Index.—The last paragraph shows that the refractive index of the denser medium referred to air is given by $\mu = \text{cosec}\,\theta$, where θ is the critical angle for light travelling from the medium into air. This equation provides a simple and at the same time most accurate method of measuring refractive indices. It is the method which is most commonly used for liquids. Fig. 97 shows a section of a simple form of apparatus which may be used. D is a rectangular glass vessel containing the experimental liquid; in this is immersed a thin film of air AB enclosed between two glass plates. This part of the apparatus is made by separating two glass plates at their edges by a narrow strip of tin-foil and smearing their whole perimeter with

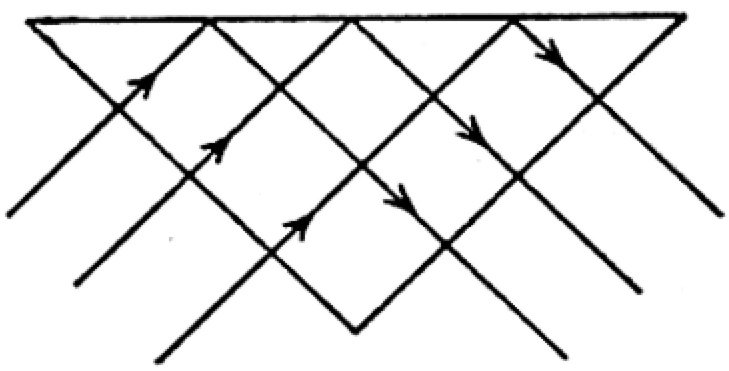


Fig. 96.—Total Reflexion Prism.

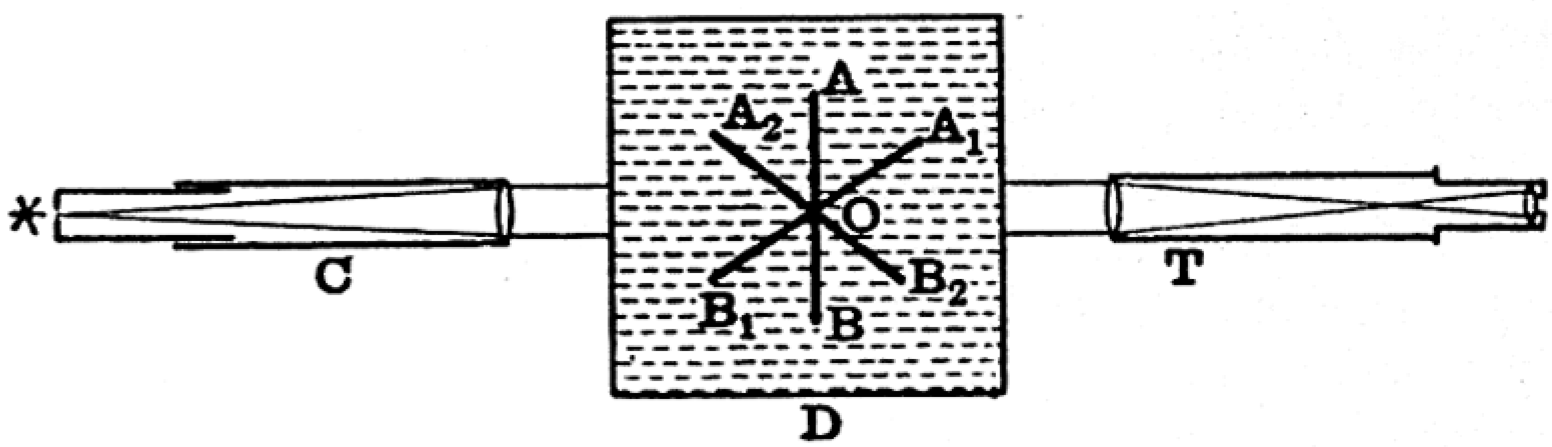


Fig. 97.—Measurement of Refractive Index by Total Reflexion.

shellac varnish so as to make an air-tight seal. The plates are supported by a vertical rod which carries a horizontal pointer moving over a graduated circle. Light coming from a salted Bunsen flame passes into the liquid and through the film into a telescope T. Suppose the air-film to be initially in the position AB perpendicular to the path of the light; the normal to AB is along the rays and the angle of incidence on the film is zero. If the plates are turned to A_1B_1 the angle of incidence is equal to $\angle AOA_1$; when this is the critical angle for water-air no light enters the telescope. The plates are next turned through their first position into that shown at A_2B_2 where total reflexion again occurs. Evidently the $\angle A_1OA_2$ is twice

the critical angle θ, hence this can be found from the graduated circle by noting the positions of the pointer when the light is suddenly cut off, then $\mu = \operatorname{cosec}\theta$. In order that the light shall disappear suddenly it is necessary that all the rays shall strike the film at the same angle; to ensure this they are made parallel by means of a collimator C. This consists of a tube carrying a vertical slit at one end and a convex lens at the other; it will be seen later that when the slit is at the principal focus of the lens the transmitted rays are parallel. It should be noticed that the glass plates have no influence on the

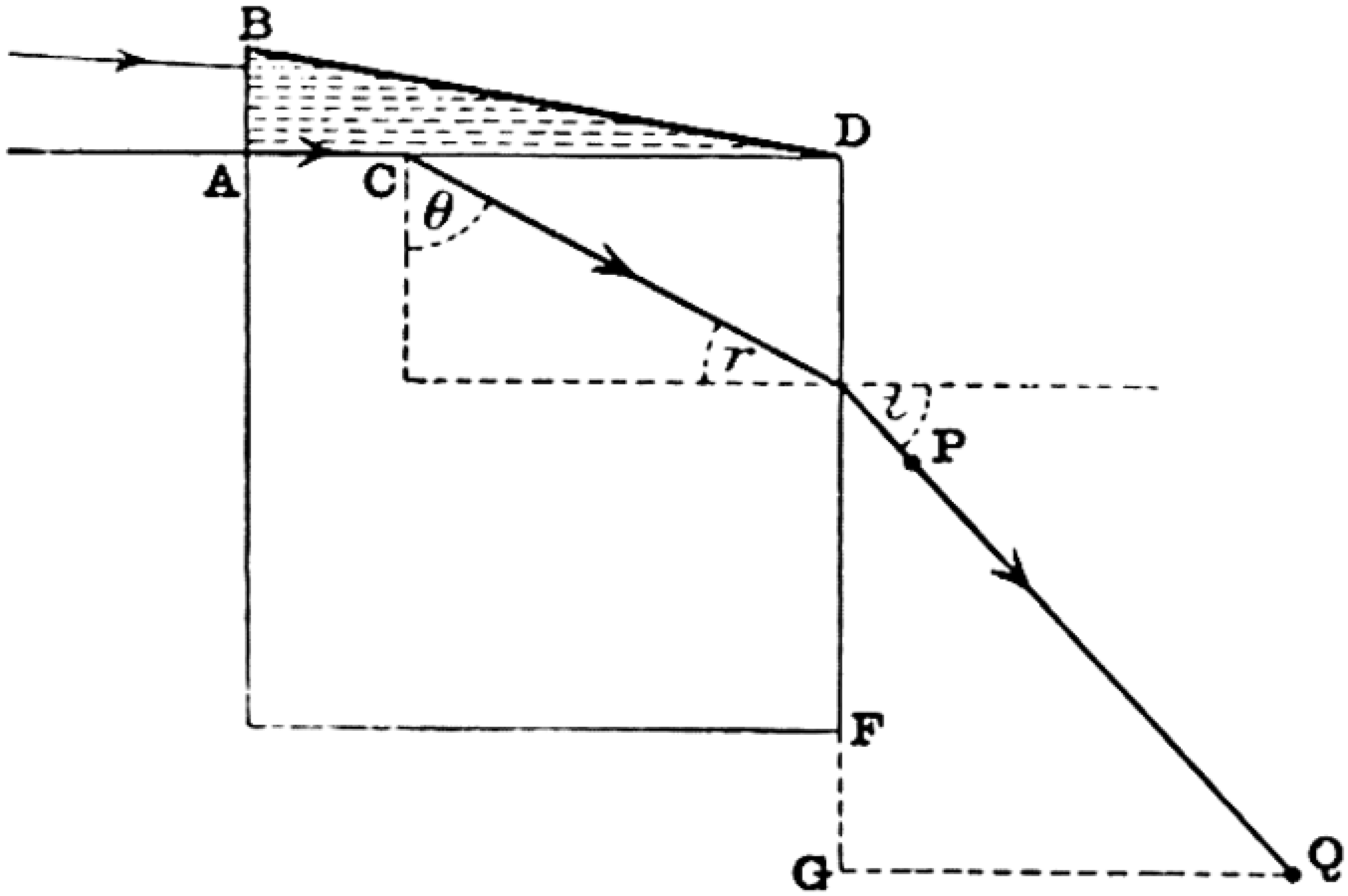


FIG. 98.—Principle of the Pulfrich Refractometer.

result since (p. 166) they do not alter the direction of the rays but merely displace them sideways.

EXPERIMENT.—Use the apparatus to measure the refractive index of water for red and yellow light. Suitable red light is obtained by placing a bead of lithium chloride in a Bunsen flame. It will be found that the refractive index is less for the red than for the yellow light.[1]

Another method is shown in Fig. 98. A cubical block of glass (*e.g.* a paper weight) has cemented to one face a metal plate BD and a piece of glass AB to form a small chamber; the shape of this is immaterial provided it does not project beyond the edge at D. Water is poured into this chamber, the block is fixed on a drawing-board, and a well-illuminated white card is placed a short distance to the left of AB. Those rays which are incident on the water-glass

[1] For a simpler apparatus, see Barton and Black, "Practical Physics," p. 97.

surface at an angle just less than 90° are refracted into the glass at the critical angle θ for these substances and travel in the direction CPQ. All other rays enter the glass at an angle of refraction smaller than θ. Just as the eye of an observer focusses into a point all the parallel rays coming from a star, so will an eye placed at Q focus into a straight line all the rays which emerge parallel to PQ. The face AD of the cube will therefore be divided into bright and black halves, and the rays coming from the line of division enter the glass at an angle θ. Let N and μ be the refractive indices of glass and water respectively, μ_{wg} be the refractive index going from water to glass.

Then $$\mu_{wg} = \frac{N}{\mu} \quad \text{(p. 167)}$$

Also $$\mu_{wg} = \frac{\sin\frac{\pi}{2}}{\sin\theta} = \frac{1}{\sin\theta} \quad \text{(from the figure)}$$

$$\therefore \sin\theta = \mu/N$$

But $$N = \sin i/\sin r \text{ (see figure)}$$

and as the angle at D is a right angle

$$\sin r = \cos\theta$$

$$\therefore N = \frac{\sin i}{\cos\theta}$$

or $$\cos\theta = \frac{\sin i}{N}$$

Squaring and adding the expressions for $\sin\theta$ and $\cos\theta$ we get

$$\frac{\mu^2}{N^2} + \frac{\sin^2 i}{N^2} = \sin^2\theta + \cos^2\theta = 1$$

or $$\mu^2 + \sin^2 i = N^2$$

Hence if the angle i is measured we can calculate either μ or N provided the other is known. Usually we shall know N, and the apparatus can then be used for different liquids. To measure i pins are placed at P and Q, about 30 cms. apart, so that they appear in line with the dark edge of the field, a ruler is placed along DF and this straight line produced. From Q the line QG is drawn perpendicular to DF, then cos QPG $= \sin i$ can be found by measuring PG and PQ.

EXAMPLE.—In a certain experiment water was placed in the chamber, and the following measurements (in inches) were made. PG = 10·9, PQ = 15·5, hence $\sin i = 0{\cdot}703$. Taking $\mu = 1{\cdot}333$ we find from a book of squares N = 1·507. This value can then be used to find μ for other liquids.

This simple apparatus illustrates the principle of the Pulfrich refractometer, one of the best means for the determination of refractive indices. In this instrument the angle i is measured by means of a telescope moving over a graduated circle as in the spectrometer (p. 238).

Illustrative Experiments.—The following two experiments are to be regarded as exercises on the laws of refraction rather than means of measuring refractive indices.

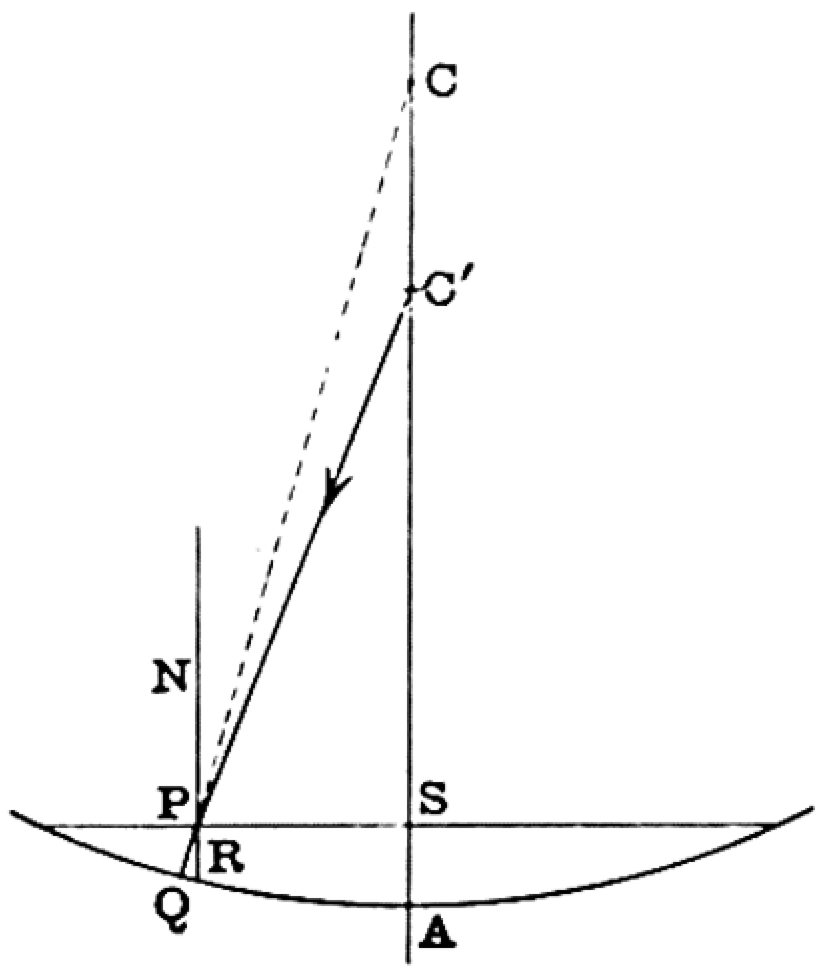


FIG. 99.—Measurement of Refractive Index with a Concave Mirror.

EXPERIMENT.—*To measure the refractive index of water with a concave mirror.* Place a concave mirror of 40–60 cms. radius of curvature on the floor and arrange a bit of white cardboard to be at the centre C (Fig. 99) by the parallax method (p. 139). Pour water on to the mirror and again arrange the pointer to be at the same distance from the mirror as its image, let C′ in the figure be its new position. Then the refractive index of water μ = CA/C′A, measure these distances and find μ. Let NPR be a normal to the water surface; if the axis of the mirror is vertical this line is parallel to CA. A ray C′P is refracted along PQ, and since it retraces its path to C′ after reflexion at Q it must strike the mirror normally, hence QP if produced backwards passes through C.

$$\text{Then} \qquad \mu = \frac{\sin \mathrm{C'PN}}{\sin \mathrm{QPR}} = \frac{\sin \mathrm{C'}}{\sin \mathrm{C}} \text{ (since NR and CA are parallel)}$$

$$\therefore \mu = \frac{\mathrm{PS}}{\mathrm{PC'}}\Big/\frac{\mathrm{PS}}{\mathrm{PC}} = \frac{\mathrm{PC}}{\mathrm{PC'}}$$

As the depth of the water is small and Q is near A we may put CP = CQ = CA, and similarly C′P = C′A, hence μ = CA/C′A.

EXPERIMENT.—Fig. 100 shows a wooden box in which two equal, upright, pieces of brass PN′, QN, are fixed; between these, at the bottom of the box, is a mm. scale with its zero at Q. Pour water into the box and level by the

screw A until the liquid surface is just above N and N′, the scale is then horizontal. Look through a cardboard slit B over the top of QN and note the division O that can just be seen, ONB is the path of the ray. Run the water out through a hole in the bottom of the box and again look through B, note the division M which is now just in view, BNM is a straight line. Supposing

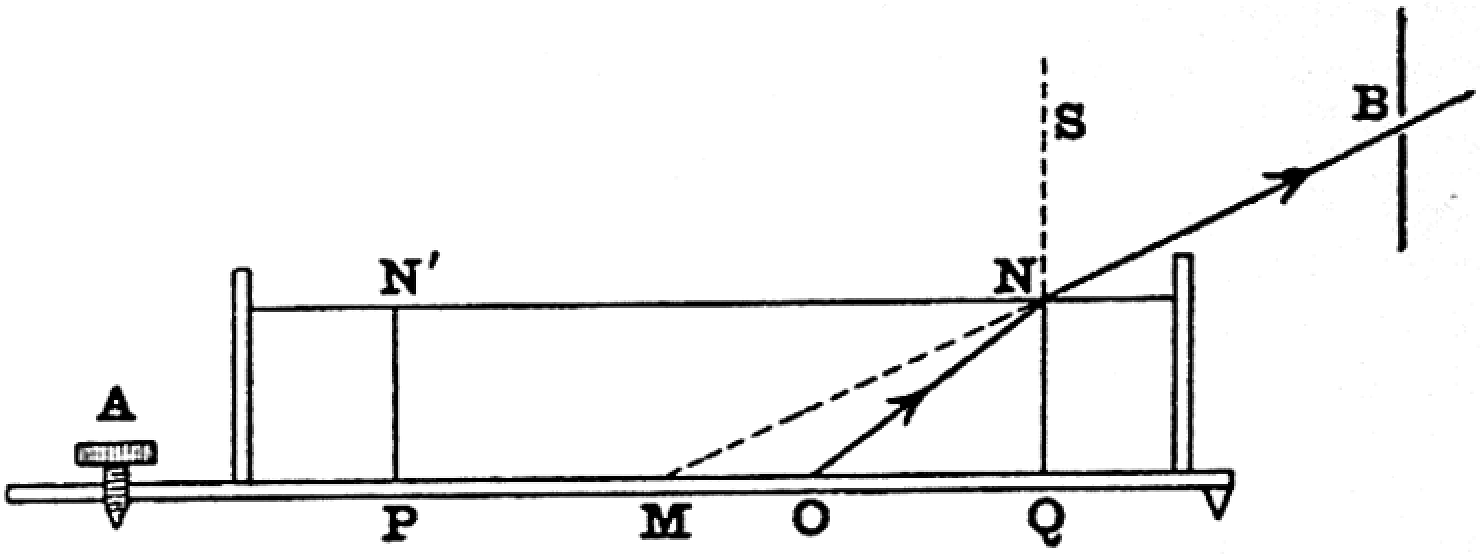


FIG. 100.—Measurement of Refractive Index by a Displacement Method.

the path of the light to be reversed we see that a ray travelling along BNM is refracted by the water along NO, hence

$$\mu = \frac{\sin BNS}{\sin QNO} = \frac{\sin QNM}{\sin QNO}$$

Measure QN and make a large drawing to scale of the part QOMN. Draw QL′, QL perpendicular to MN, ON respectively, then

$$\mu = \frac{QL'}{QN} \Big/ \frac{QL}{QN} = \frac{QL'}{QL}$$

hence μ can be found.

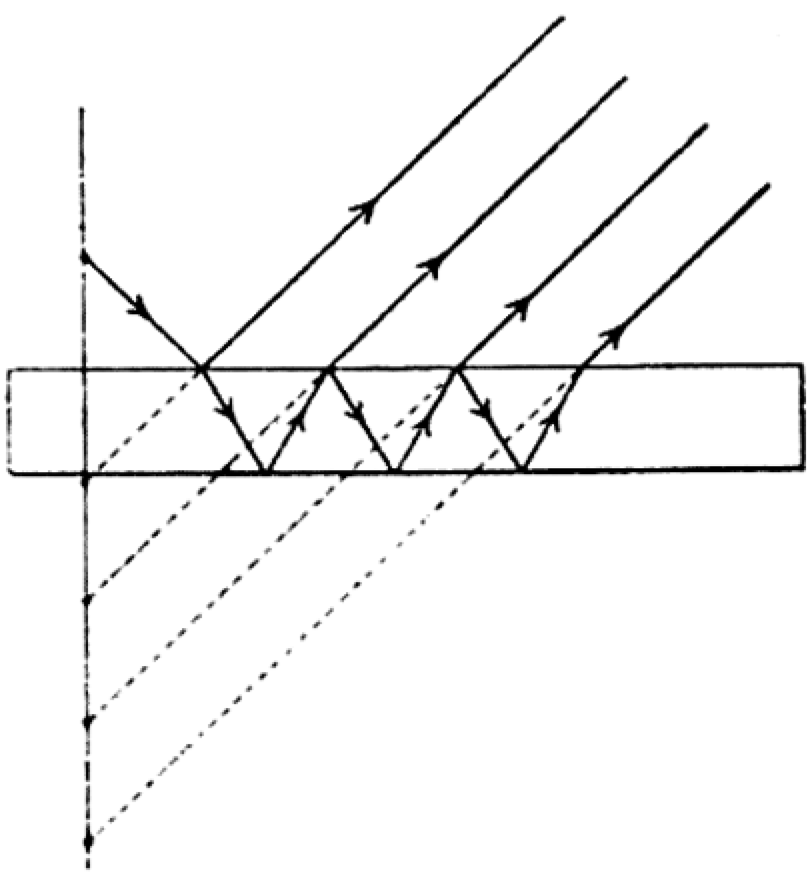


FIG. 101.—Reflexion by a Thick Mirror.

Images in a Thick Mirror.—When a lighted candle is held in front of a thick mirror a number of images placed one behind the other can be seen, especially if the light strikes the reflecting surface very obliquely. Fig. 101 shows how these images arise. A ray is partially reflected at the first face but the rest of the light enters the glass and is reflected to and fro between its faces; each time one of these multiply-reflected rays strikes the front surface of the mirror a portion of the light emerges into the air. It is to these emergent rays that the images are due. When the candle is very distant,

and the front and back surfaces of the mirror are parallel, the images are not seen, for in that case the incident and therefore the emergent rays are parallel, hence they appear to come from a single distant image. If in any instance a distant candle is used and the series of images is still visible it shows that the faces of the mirror are not parallel; this, in fact, provides a simple means of testing the parallelism of the front and back faces.

EXAMPLES ON CHAPTER XVI

1. An object is viewed through a thick plate of glass so that the rays meet the plate at nearly normal incidence. Prove that its apparent displacement towards the observer is independent of its small distance from the glass.

2. A ray of light passes obliquely through a plate of glass with parallel sides. Show that the distance between the emergent ray and the incident ray produced is $\frac{e \sin (i - r)}{\cos r}$, where e is the thickness of the plate, and i and r are the angles of incidence and refraction respectively.

3. Explain why a thick plate of glass produces no appreciable displacement in the apparent position of a distant object viewed through the plate. The rays are supposed to meet the plate normally. (L. '88.)

4. A substance has a refractive index $\sqrt{3}$. Draw as nearly as you can to scale the path of a ray incident on a parallel plate of the substance 1 in. thick, the angle of incidence being 60°. What is the distance between the incident ray produced and the emergent ray? (L. '95.)

5. A pencil of light from a point is incident on a plate of a refracting substance. Show that, if the pencil is nearly normal, then within the plate it proceeds as if it came from an image μ times as far from the surface as the luminous point. Draw a figure for the case in which $\mu = 2$. (L. '96.)

6. Draw to scale a diagram showing the directions in water in which a ray of light, incident at 45° on the surface of the water, will travel, assuming that the refractive index of water is $\frac{4}{3}$. (L. '97.)

7. Show that if a horizontal concave mirror is filled with liquid its apparent radius of curvature is diminished in the ratio of the refractive index of the liquid. (L. '07.)

8. Prove that to an eye under the surface of water all objects that can be seen above the surface appear in a cone whose semi-vertical angle is the critical angle.

9. A cubical block of glass is placed on a black glass plate with a film of water between them and the whole is placed before a window with one face of the cube vertical. On looking through the opposite vertical face from a certain position the base is seen to be divided into bright and dark halves. Explain

this. If the ray coming from the dividing line of the two halves emerges into the air making an angle i with the horizontal, show that $\mu^2 = N^2 - \sin^2 i$, where μ and N are the refractive indices of water and glass respectively.

10. The refractive indices of water and turpentine are 1·33 and 1·47 respectively; find the critical angle for a ray passing from the latter to the former liquid.

11. A vertical microscope is focussed on a mark on the bench, a plate of glass 2 in. thick is then interposed. Find how much the microscope must be raised for the mark still to be in focus. (μ for glass $= 3/2$.)

12. A beaker containing liquid is placed on the table underneath a microscope which can be moved along a vertical scale. The microscope is focussed through the liquid on to a mark on the table when the reading on the scale is a. It is next focussed on the upper surface of the liquid and the reading is b. More liquid is added and the observations are repeated, the corresponding readings being c and d. Show that the refractive index of the liquid is

$$\frac{d - b}{a + d - b - c}$$

CHAPTER XVII

APPLICATIONS OF THE LAWS OF REFRACTION

Passage of Light through a Prism.—A portion of a medium between two plane faces inclined to each other at an angle is called, for optical purposes, a prism. The line of intersection of the faces is called the refracting edge, and a section perpendicular to this line is a principal section. The angle between the faces is called the angle of the prism. In what follows we shall deal only with rays in a principal section and we shall further suppose the light is such as is obtained from a salted Bunsen flame. The general features attending the passage of light through a prism are best studied on the Hartl disc.

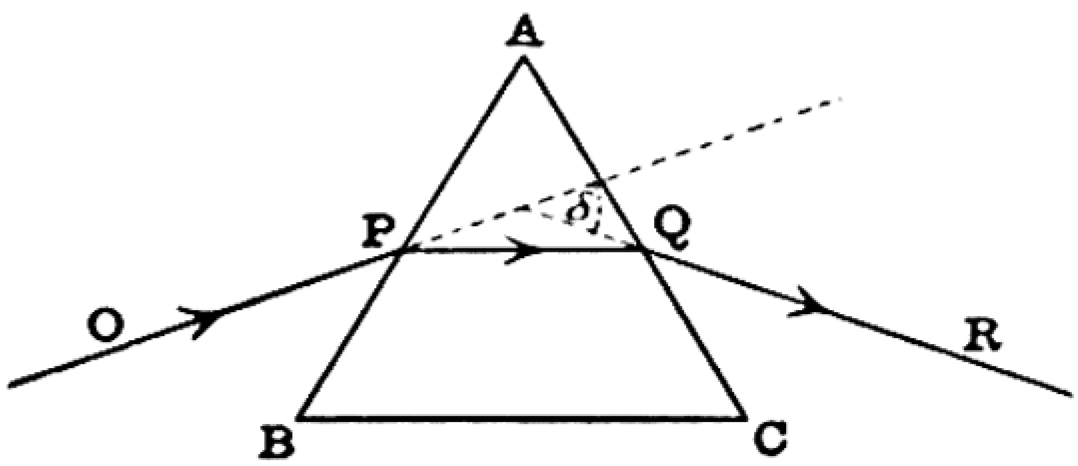


FIG. 102.—Path of Light through a Prism.

EXPERIMENT.—Fix a prism ABC (Fig. 102) on the disc and arrange the slit so that a ray OP falls on one face; the path through the prism is shown by OPQR, and the ray is bent away from the refracting edge. The angle between the initial and final directions of the ray, δ in the figure, is called the deviation produced by the prism. If the prism is rotated round the point A the deviation changes; turn it continually in that direction which causes the deviation to diminish, it is found that the emergent ray QR gradually approaches a direction parallel to OP, but before reaching this position it stops and finally moves back again. Hence for a particular angle of incidence the deviation is a minimum, when this is reached it can easily be shown by measurement that the $\angle$ OPB $= \angle$ RQC, *i.e.* in the minimum deviation position the light passes symmetrically through the prism.

Experiment.—*To plot a curve showing how the deviation varies with the angle of incidence on the first face.*[1] Pin a sheet of paper on a drawing-board and rule lines OX, YY′ at right angles to each other (Fig. 103). Draw also lines OP_1, OP_2, etc., making angles of 10°, 20°, etc., with OX. Place the prism with one face along OX and its refracting edge vertical at A. Let YO represent a ray incident normally on the face AB, this ray travels straight along to N and, unless total reflexion occurs, emerges in the direction NM. The angle between NM and OY′ is the deviation produced. Stick two pins some distance apart on OY, and, looking through the prism in the direction MN, fix two other pins at M and N to be in the same straight line as the images of the first two. The line joining the second pair of pins is the emergent ray. Remove

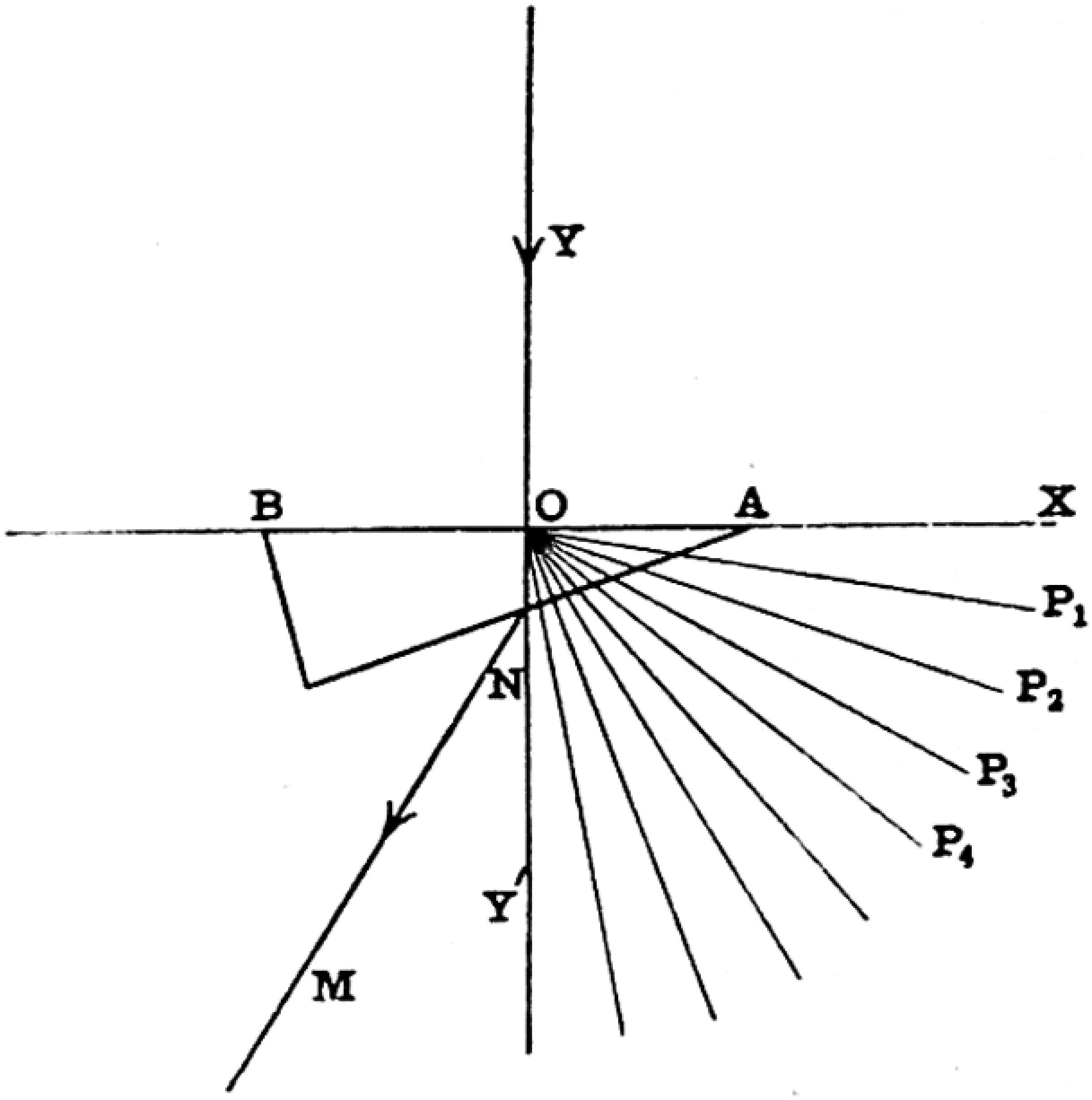


Fig. 103.—Change in the Deviation when the Angle of Incidence is varied.

the prism and measure the deviation with a protractor. Next fix the prism with its face AB along OP_1, the angle of incidence of the ray along YO is 10° and the emergent ray and deviation can be found as before. Repeat with the face AB along OP_2, OP_3, etc.; plot a curve showing the deviation for different angles of incidence. Read from your curve what is the angle of incidence at minimum deviation, place the prism in the corresponding position and find the emergent ray. By joining the points at which the incident and emergent rays meet the prism get the path of the ray in the glass; show that it is equally inclined to the prism faces. Rule in the outline of the prism and measure its angle for future use.

[1] I am indebted to Mr. F. J. Harlow for this method of carrying out the experiment.

EXPERIMENT.—Trace a ray through a prism as in the last experiment, draw the normal to one face at the point where the ray intersects it and measure the angles of incidence and refraction; hence calculate the refractive index of the glass.

Image produced by a Prism.—Let P (Fig. 104) represent a source of light and let the ray PQ pass through a prism with minimum deviation. Two near rays PR, PS, are incident at slightly different angles, but an inspection of the curve obtained above shows that near the minimum the deviation varies very slowly with the angle of incidence, hence the deviation of these rays is practically equal to that of PQ. It follows that the inclination of the rays to each other is unaltered by their passage through the prism, hence if the

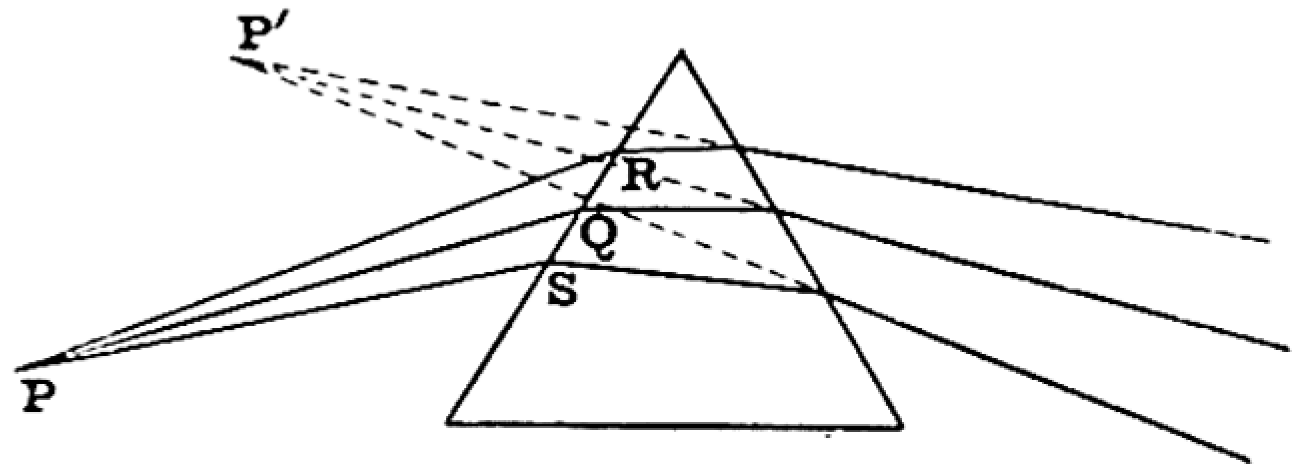


FIG. 104.—Formation of an Image by a Prism.

emergent rays are produced backwards they will meet at a point P′ whose distance from the prism is equal to that of P. (This is not quite true unless the thickness of the glass is neglected, P should be a considerable distance away.) P′ is the virtual image of P. When the prism is in any other position the corresponding rays are unequally deviated, they no longer diverge from a point after refraction and no true image is formed. Whenever it is necessary to produce a well-defined image the prism must be placed in the minimum deviation position.

EXPERIMENT.—Use a vertical pin as object and trace rays through a prism. Show that the emerging rays do not diverge from a point except in the minimum deviation position. Fix another pin by the parallax method to coincide with the image in the latter case; for this purpose the second pin must be long enough to be seen over the top of the prism. Show that image and object are equidistant from the first face.[1]

Measurement of Refractive Index by means of a Prism.—Let PQRS (Fig. 105) be the path of a ray through a prism and δ the deviation produced. Draw the normals QN, RN to the prism faces and let

[1] Barton and Black, "Practical Physics," p. 87.

the angles of incidence and refraction be as shown in the figure. Then from $\triangle$ OQR

$$\begin{aligned}\delta &= \angle OQR + \angle ORQ \\ &= (\angle OQN - \angle RQN) + (\angle ORN - \angle QRN) \\ &= (i_1 - r_1) + (i_2 - r_2) \\ &= i_1 + i_2 - (r_1 + r_2)\end{aligned}$$

But the interior angles of the quadrilateral AQNR together equal four right angles, and as the angles at Q and R are right angles

$$\angle A + \angle N = 2 \text{ rt. } \angle s$$

Also
$$r_1 + r_2 + N = 2 \text{ rt. } \angle s$$

$$\therefore A = r_1 + r_2$$

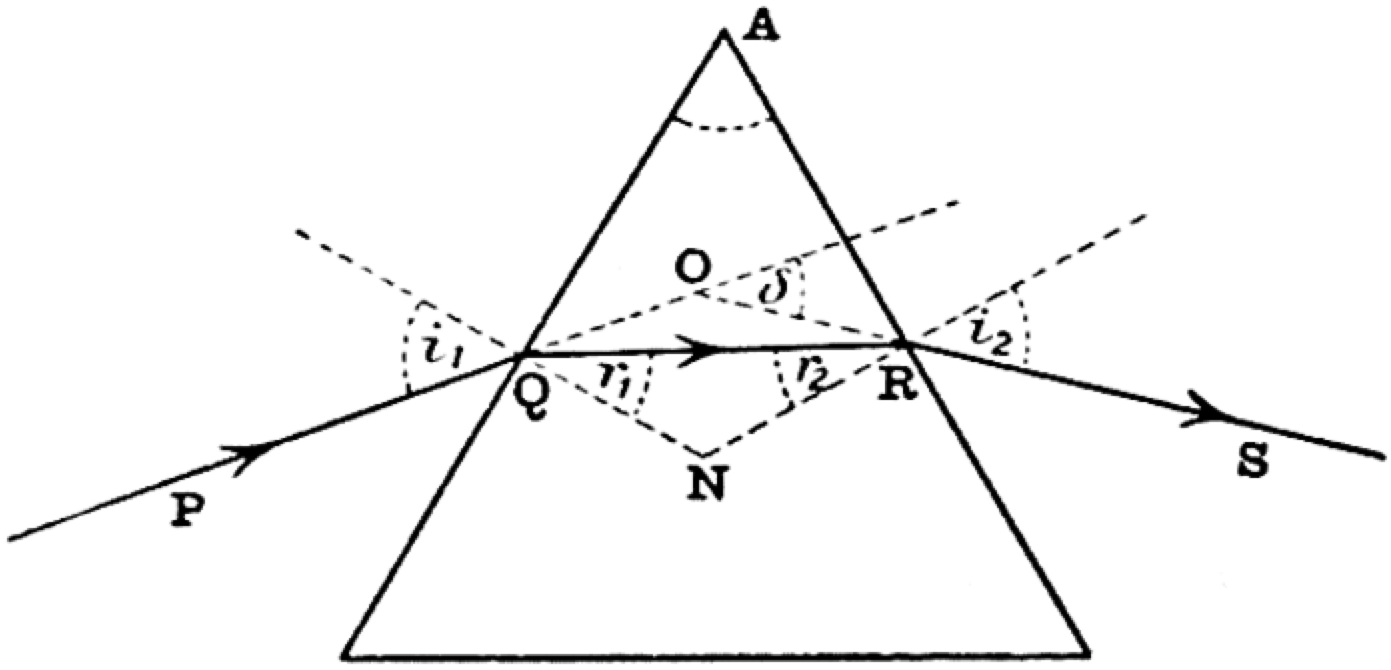


FIG. 105.

In the minimum deviation position $i_1 = i_2$ and $r_1 = r_2$

$$\therefore A = 2r_1$$

and
$$\begin{aligned}\delta &= 2i_1 - 2r_1 \\ &= 2i_1 - A\end{aligned}$$

$$\therefore i_1 = \frac{\delta + A}{2}$$

and
$$r_1 = \frac{A}{2}$$

Now the refractive index of the prism material

$$\mu = \sin i_1 / \sin r_1$$

$$\therefore \mu = \frac{\sin \dfrac{\delta + A}{2}}{\sin \dfrac{A}{2}}$$

This equation shows that μ can be calculated when the angle of the prism and the minimum deviation have been measured. An accurate method of making these measurements will be given later.

EXAMPLE.—The minimum deviation and angle of a prism have been measured in a previous experiment (p. 180), calculate the refractive index from these results.

When the prism angle is very small $(\delta + A)$ is also small, hence in the above equation the sines may be replaced by the angles themselves and

$$\mu = \frac{\delta + A}{A}$$

$$\therefore \delta = (\mu - 1)A$$

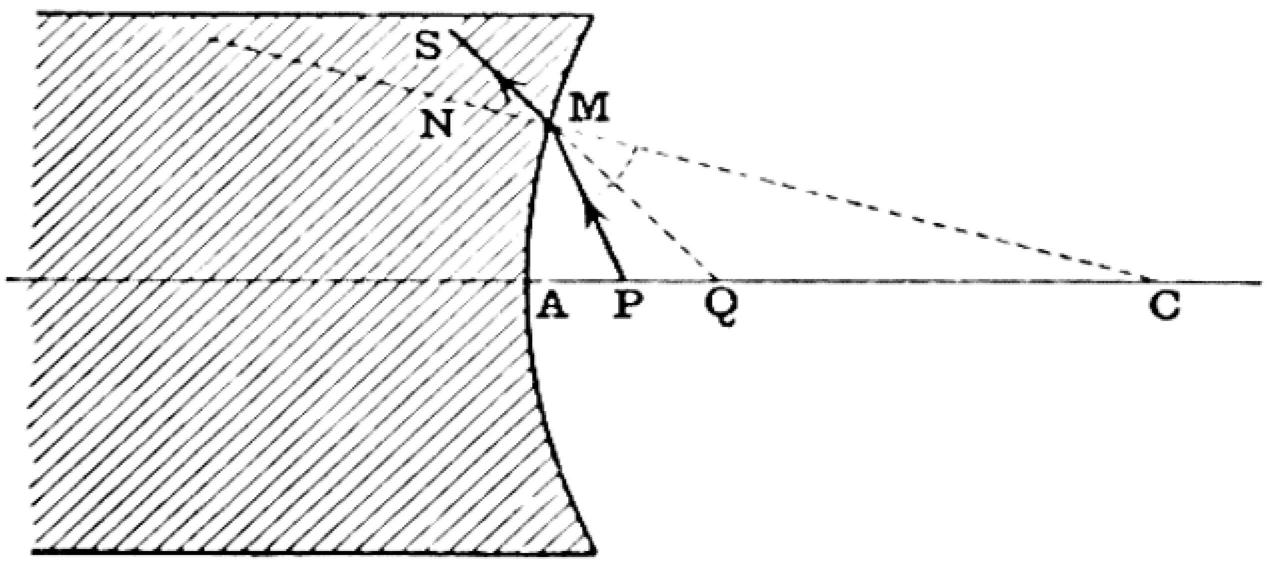


FIG. 106.—Refraction at a Concave Spherical Surface.

EXPERIMENT.—Make a hollow prism out of glass plates with an angle not greater than 10°. Fill it in succession with water and aniline and measure the deviation by the pin method. Taking μ for water as 1·33 find the refractive index of aniline using the last equation.

REFRACTION AT SPHERICAL SURFACES

Image formed by Refraction at a Spherical Surface.—Let AM (Fig. 106) represent a concave spherical surface whose centre of curvature is C and pole A, and let the medium on the left have a refractive index μ relatively to the medium on the right; *e.g.* let the medium on the right be air, that on the left glass. Let P be a small object on the axis, PM any ray meeting the surface; we require to find where the refracted ray MS cuts the axis. Produce SM to meet

the axis at Q, then Q is the point whose position is to be calculated. CMN is the normal at M, hence $\angle$PMC is the angle of incidence i and $\angle$SMN $= \angle$QMC the angle of refraction r.

Then $$\mu = \frac{\sin \text{PMC}}{\sin \text{QMC}}$$

From $\triangle$ PMC $$\frac{\text{PM}}{\text{PC}} = \frac{\sin \text{C}}{\sin i}$$

from $\triangle$ QMC $$\frac{\text{QM}}{\text{QC}} = \frac{\sin \text{C}}{\sin r}$$

dividing the 2nd by the 1st $$\frac{\text{QM}}{\text{QC}} \cdot \frac{\text{PC}}{\text{PM}} = \frac{\sin i}{\sin r} = \mu$$

Limiting ourselves to the case where M is near A, as in the corresponding case for reflexion, we may put QM = QA, PM = PA,

and $$\mu = \frac{\text{QA}}{\text{QC}} \cdot \frac{\text{PC}}{\text{PA}}$$

Put PA $= u$, QA $= v$, CA $= r$ and measure all distances from A,

then $$\mu = \frac{\text{QA}}{\text{CA} - \text{QA}} \cdot \frac{\text{CA} - \text{PA}}{\text{PA}} = \frac{v}{r - v} \cdot \frac{r - u}{u}$$

$$\therefore \mu ur - \mu uv = vr - vu$$

Divide throughout by uvr and rearrange the terms, then

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$$

This equation enables us to calculate the distance QA when the other quantities are given. Exactly the same result holds for all the rays starting from P, provided they are incident near A, hence after refraction all rays diverge from Q and this point is the image of P. The same equation is true for a convex surface if the usual sign convention is used. Thus in Fig. 107, where the lettering is the same as before,

$$\mu = \frac{\sin \text{PMN}}{\sin \text{QMN}} = \frac{\sin i}{\sin r}$$

From $\triangle$ PMC $\qquad \dfrac{PM}{PC} = \dfrac{\sin C}{\sin PMC} = \dfrac{\sin C}{\sin (\pi - i)} = \dfrac{\sin C}{\sin i}$

Similarly from $\triangle$ QMC $\quad \dfrac{QM}{QC} = \dfrac{\sin C}{\sin r}$

and $\qquad \dfrac{QM}{QC} \cdot \dfrac{PC}{PM} = \dfrac{\sin i}{\sin r} = \mu$

$$\therefore \mu = \frac{QA}{QC} \cdot \frac{PC}{PA} = \frac{v}{v + (-r)} \cdot \frac{u + (-r)}{u} = \frac{v}{v - r} \cdot \frac{u - r}{u}$$

as in the first case. It should be remembered that u and v are the distances of the object and image respectively from A, and that all

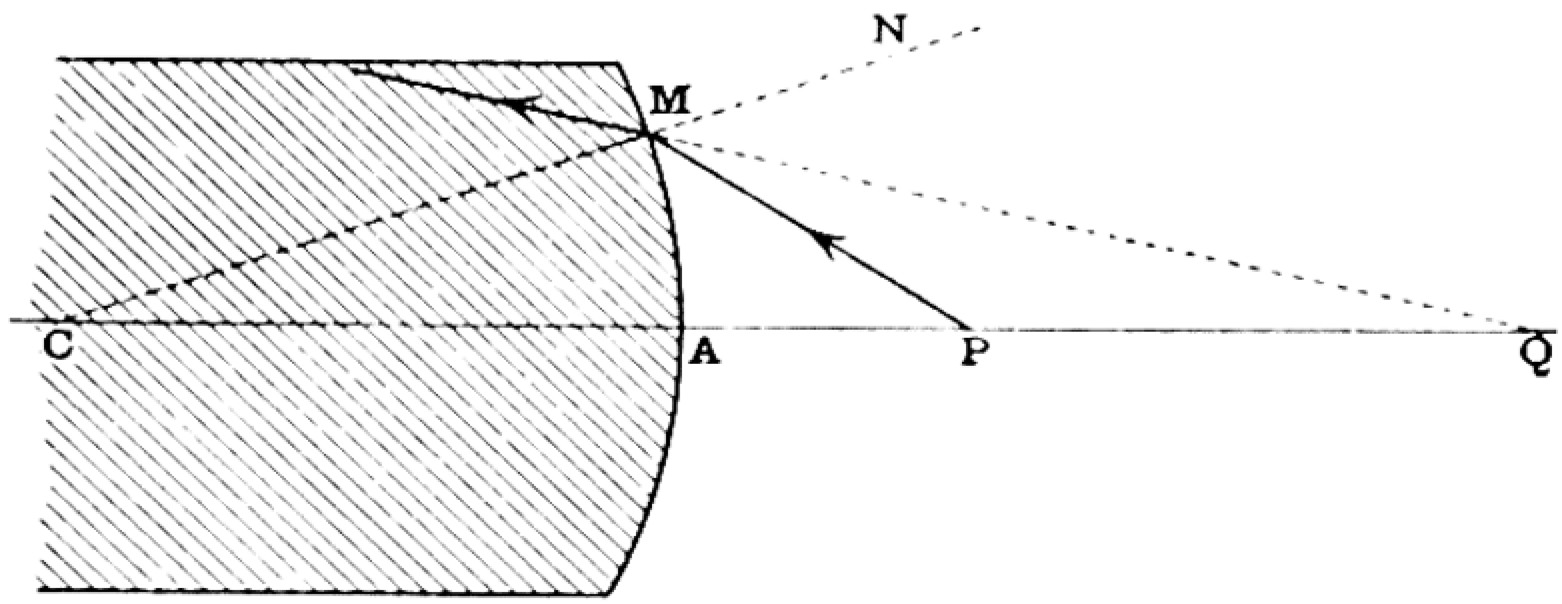


FIG. 107.—Refraction at a Convex Spherical Surface.

distances are to be measured from this point with the usual sign convention. If an eye could be placed in the medium on the left the point P would appear to be at Q. The apparatus used in the following experimental verification of this formula is due to Dr. R. S. Clay.

EXPERIMENT.—Pour water into a circular glass crystallising dish 15 cms. or more in diameter and place it on a drawing board. Fasten a pin into a flat block of lead and put it in the water at P (Fig. 108). Look into the water from S and find by the usual pin method the direction of several rays emerging near A. Measure PA $= u$, and the diameter of the dish; taking $\mu = 1/1{\cdot}33$ calculate the distance of the image from A. Rule in the outline of the dish and produce the refracted rays backwards until they meet; measure the distance QA and compare it with the calculated value.

EXAMPLE.—The experiment was used to find the refraction index of turpentine; the following numbers were obtained, $v = 18$ cms., $u = 12{\cdot}1$ cms., $r = 7$ cms., whence $\mu = 1{\cdot}45$, in the direction air to turpentine.

Graphical Construction of Images.—Suppose P in Figs. 106 and 107 is very distant, then the incident rays are parallel with the principal axis, and all the refracted rays meet at a point called the second principal focus; the distance of this point from A is the second focal length. This length f_2 can be found if we put $u=\infty$, and therefore $1/u=0$, in the above equation; we get $v=f_2=\dfrac{\mu r}{\mu-1}$. Similarly there is a point such that all the rays coming from it are parallel after refraction; this is called the first focal point, and its distance from A is the first focal length f_1. In this case the image is infinitely distant and therefore $\mu/v=\mu/\infty=0$; hence from the equation we obtain $u=f_1=-\dfrac{r}{\mu-1}$. We can make use of these focal points to construct graphically the image of any small object in a manner similar to that employed on p. 153. This is left as an exercise for the student. Refraction at spherical surfaces only becomes of practical importance when the light emerges into the air again, for this two refractions are necessary and we have to deal with a lens.

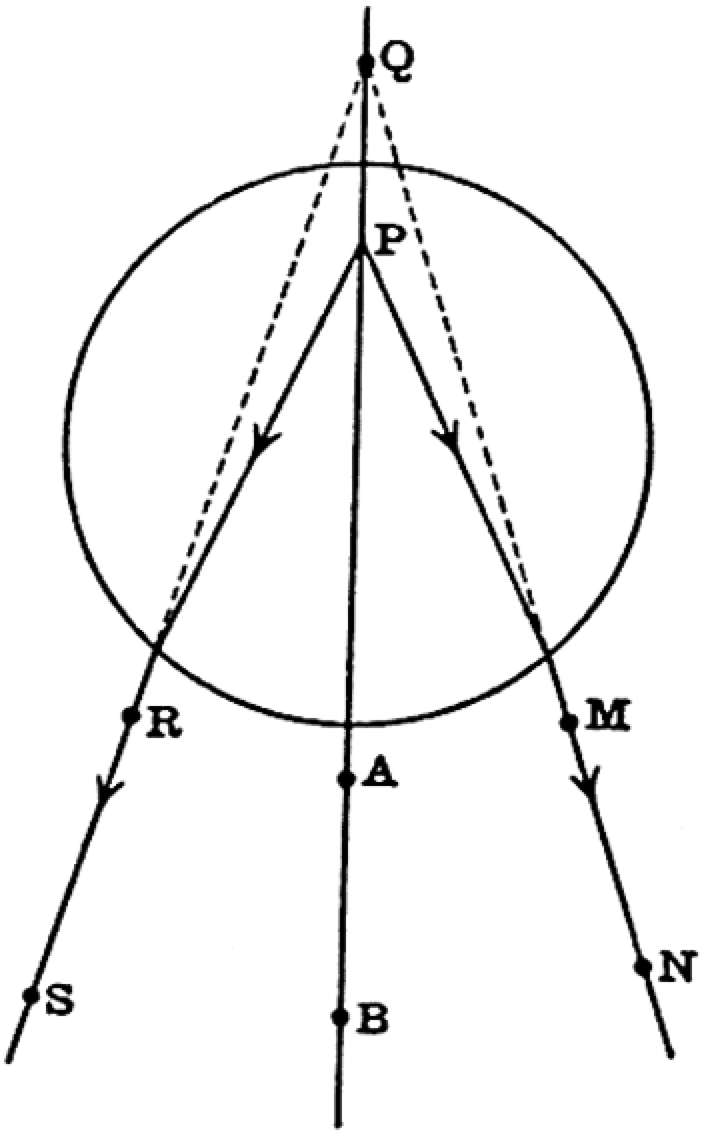


FIG. 108.—Apparatus to verify the Formula for Refraction at a Spherical Surface.

LENSES

Passage of Light through a Lens.—A lens may be defined as a portion of a transparent, refracting, medium bounded by two surfaces which are most frequently parts of spheres or cylinders. The lenses we shall consider are those bounded by spherical surfaces having a common normal, the plane being regarded as a sphere of infinite radius. Lenses are divided into two classes, those which are thicker at the middle than the edges are called convex, those which are

thinnest at the middle are concave. Fig. 109 shows three of each type. The **principal axis** of a lens is the line joining the centres of curvature of the faces. If one surface is plane the principal axis is perpendicular to this face and passes through the centre of curvature of the other.

EXPERIMENT.—Place a glass convex lens on the Hartl disc and allow a number of rays to fall on it parallel with the principal axis ; the beam is rendered

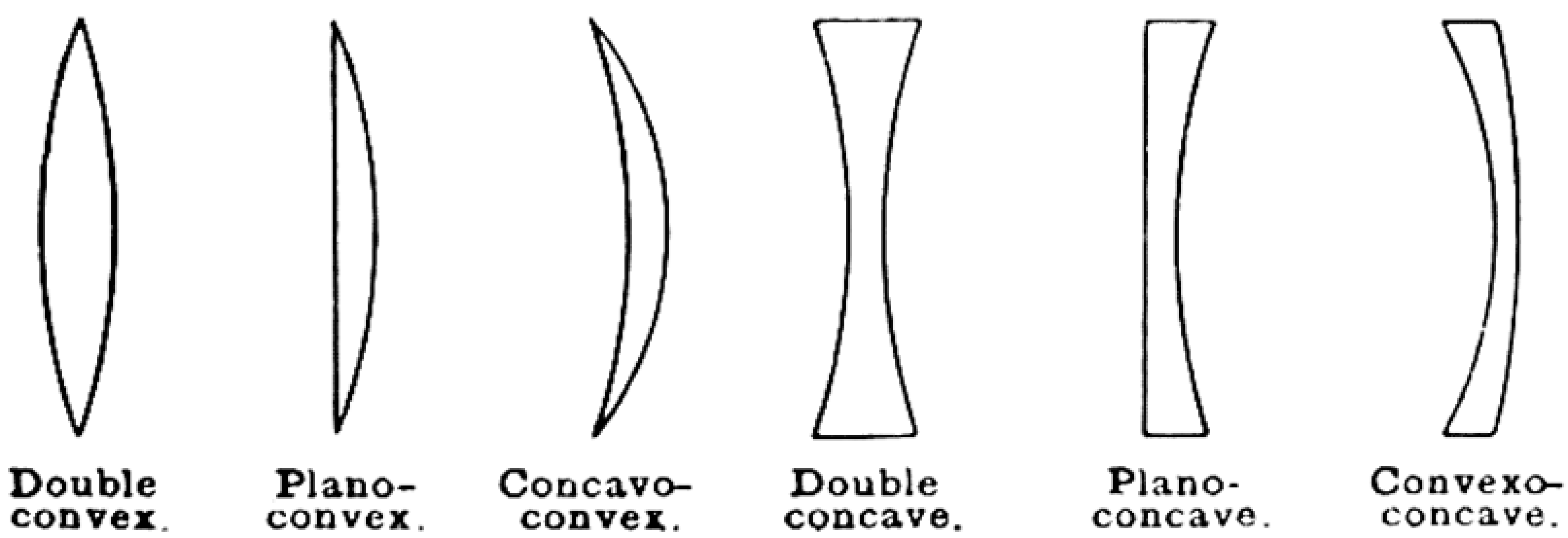


FIG. 109.—Types of Lens.

convergent and provided we deal only with the part near the axis all the rays meet at a point F behind the lens (Fig. 110, A). A concave lens causes the rays

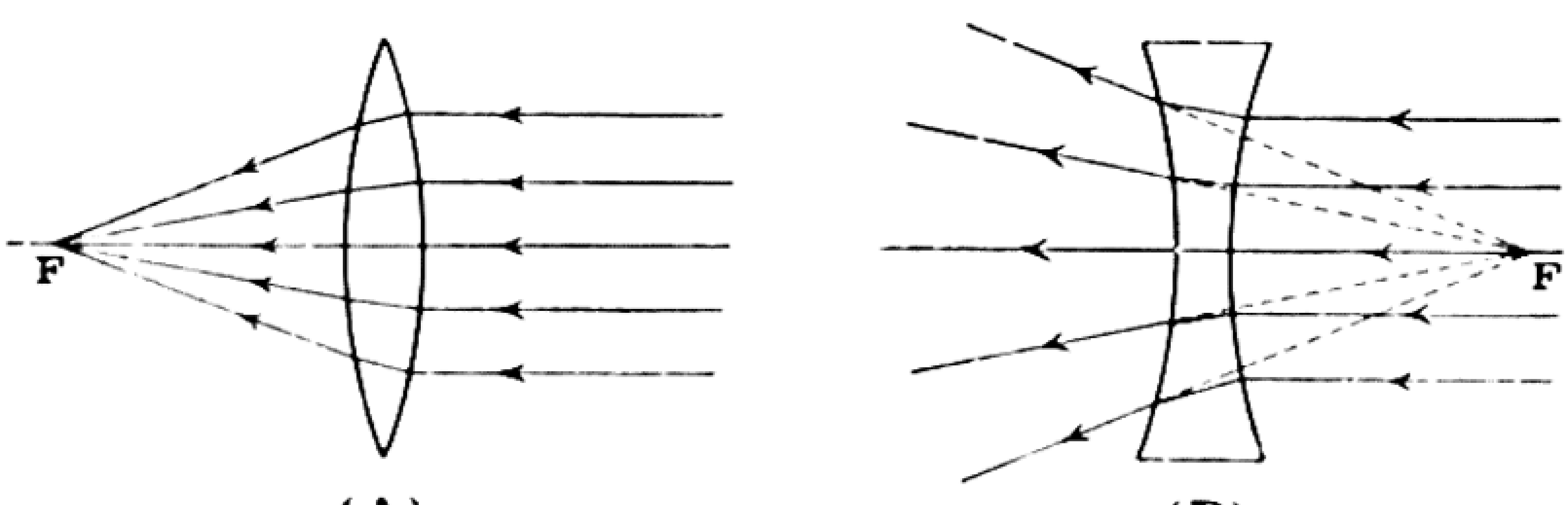


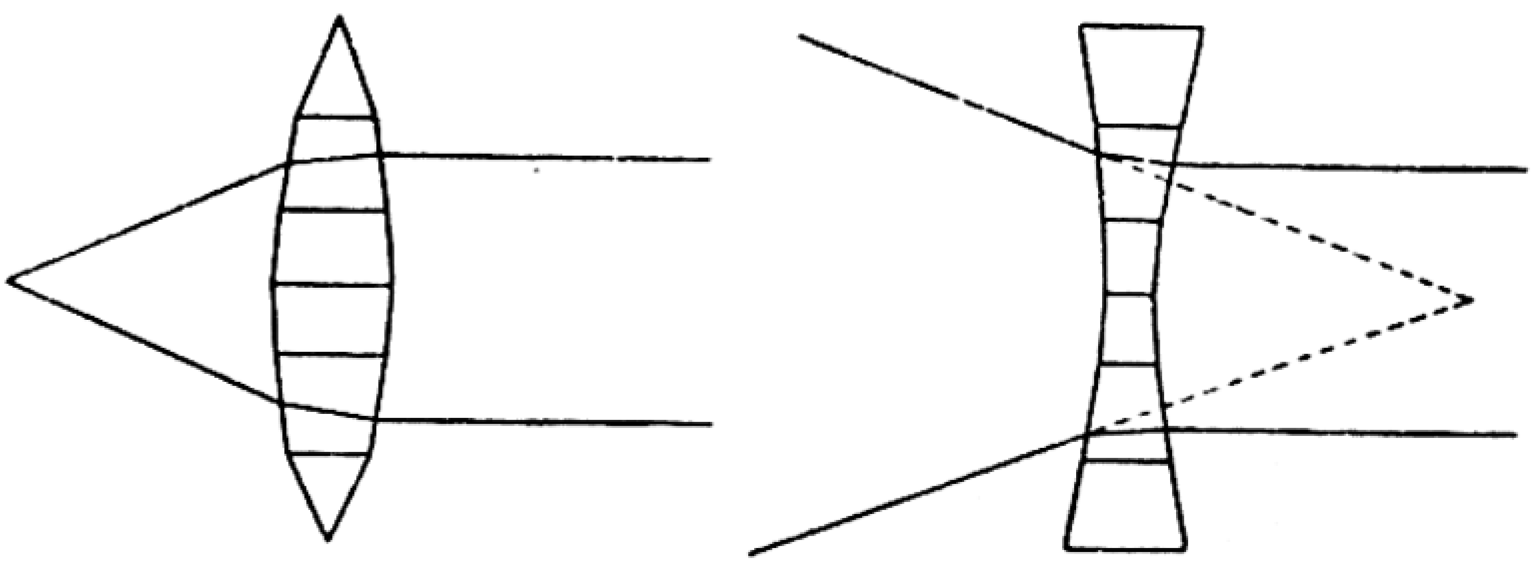
FIG. 110 (A and B).—Passage of Light through a Lens.

to diverge, but if the emergent rays are produced backwards they meet at a point F in front of the lens (Fig. 110, B).

When a number of rays fall on a lens parallel with the principal axis they are made to converge to or diverge from a point ; this point is the **principal focus** and its distance from the lens is the **focal length** of the lens. With the usual sign convention the focal length of a convex lens is negative, that of a concave lens is positive, all distances being measured from the lens. The focal length is the same

no matter which surface is presented to the incident light. A point on the opposite side of the lens to the principal focus and the same distance away is called the first focal point, the principal focus just defined is then called the second focal point.

This converging or diverging effect of a lens is explained by a reference to Fig. 111, A and B. In the first we have two sets of truncated prisms of different angles arranged symmetrically about an axis with the base of each prism turned towards this line. The prisms furthest away from the axis have the largest angle and they produce the largest deviation of a ray. Since a prism bends rays towards its base such an arrangement will bend all rays towards the axis, or will make a beam more convergent. If the number of



(A) (B)

Fig. 111 (A and B).—Illustrating the Action of a Lens.

prisms is largely increased while their height is correspondingly diminished the figure approximates to the section of a double convex lens. Similarly the second figure shows that the section of a concave lens may be regarded as built up from a large number of truncated prisms with their refracting angles turned towards the axis. The refraction produced by such an arrangement will increase with the refractive index of the material and the angles of the prisms. If a lens is immersed in a liquid whose refractive index is greater than that of the glass the rays are deviated in a direction which is opposite to that obtained in the above experiment; a convex lens then causes a parallel beam to become divergent while a concave lens makes it converge.

Suppose the path of the light to be reversed in Fig. 110, A and B, then F becomes the first focal point and it is seen that if the incident rays are directed towards or from this point they leave the lens parallel with the principal axis.

Optical Centre of a Lens.—Let C, C′ (Fig. 112) be the centres of curvature of the faces of a double convex lens, CC′ the principal axis. Draw from C any radius CP of the right-hand face and from C′ draw a radius C′P′ of the other face parallel to CP. Let Q′P′ be a ray which enters the lens at P′ and emerges at P in the direction PQ. Since the faces at P, P′ are parallel the ray passes through as if the lens were a sheet of glass with parallel sides, the emerging ray is displaced sideways but is parallel to its original direction. Let us

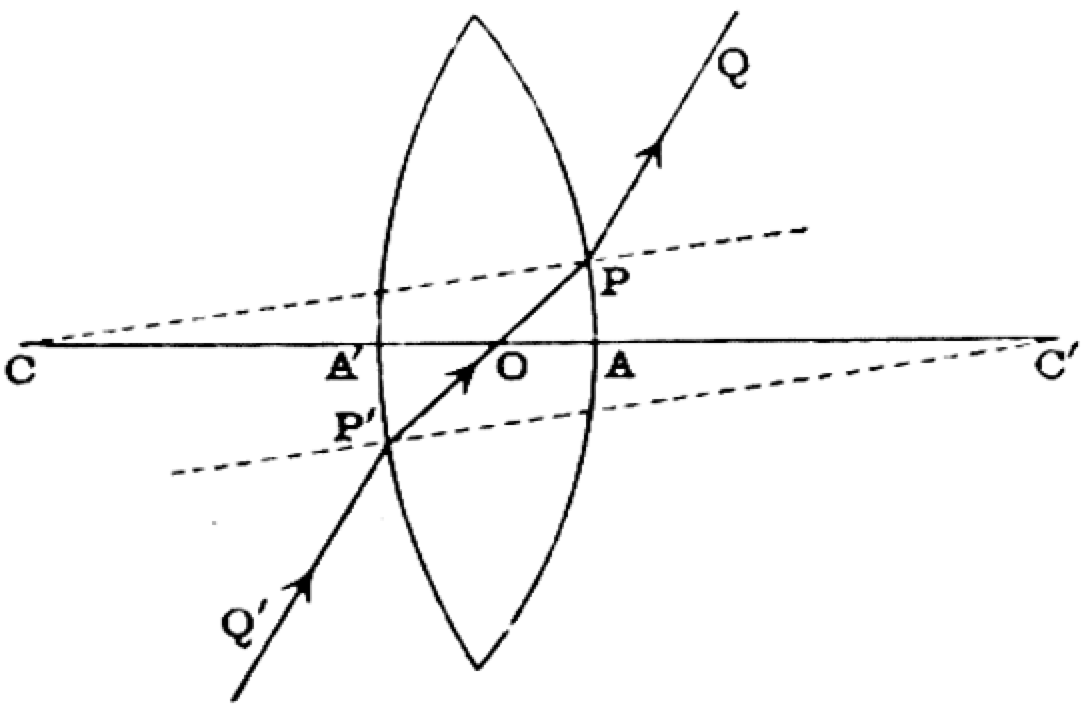


FIG. 112.—Optical Centre of a Lens.

find the position of the point O where the ray in the lens cuts the axis. The △'s OP′C′, OPC are similar,

$$\therefore \frac{OC}{OC'} = \frac{CP}{C'P'} = \frac{CA}{C'A'} = \frac{CA - OC}{C'A' - OC'}$$

the last result following from a well-known theorem in algebra.[1]

Hence $$\frac{CA}{C'A'} = \frac{OA}{OA'}$$

which shows that the point O divides AA′ in the ratio of the radii of the faces. It is therefore a fixed point no matter what pair of parallel radii such as CP, C′P′ are drawn. This point is called the

[1] If $\frac{a}{b} = \frac{c}{d} = k$ say, then $a = kb$ and $c = kd$, hence $\frac{a-c}{b-d} = \frac{kb - kd}{b - d} = k$. Thus each of the original fractions is equal to $\frac{a-c}{b-d}$.

optical centre of the lens; it is characterised by the fact that all rays which pass through it leave the lens parallel to their original direction. Conversely if the initial and final directions of the ray are parallel it must pass within the lens through the optical centre. When the radii of curvature of the faces have the same sign, as with convexo-concave or concavo-convex lenses, the point at which PP′ cuts the axis is *virtual*, *i.e.* it lies on PP′ produced. In such cases the optical centre lies outside the lens and it divides AA′ externally in the ratio of the radii; hence, as before, $OA/OA' = CA/C'A'$. If, as we shall suppose, the lens is thin the lateral displacement of the rays is small, and it may be said without appreciable error that rays passing through the optical centre continue their course in a straight line.

Graphical Construction of Images.—We are now in a position to find by graphical construction the position of the image formed by a lens. The principles used are similar to those employed for mirrors (p. 153). In the figures here given F is the principal focus as defined on p. 187, for convex lenses it is on the side remote from the source, for concave lenses on the same side as the source, O is the optical centre, and F′ is a point on the opposite side of the lens to F such that $OF = OF'$, *i.e.* F′ is the first focal point. The directions of three refracted rays are known for—

(1) Any ray incident parallel with the principal axis passes really or virtually through F after leaving the lens.

(2) Any ray passing through O is undeviated.

(3) Any incident ray which passes through F′ is parallel with the axis after refraction by the lens.

To find the image of an object two of these rays are drawn from any point on it and we find where they meet after refraction; this gives one point of the image, others may be found in a similar manner.

To keep the figures clearer the object PQ is supposed to be entirely on one side of the axis. In Fig. 113 *a* it is at a distance from the lens greater than the focal length and the image P′Q′, for which all three rays are drawn, is real and inverted. In Fig. 113 *b* the object is nearer the lens than F′ and rays (1) and (2) are employed. It is seen that the image is virtual, erect, and magnified. Fig. 113 *c* represents the formation of an image by a concave lens; three rays are drawn showing that the image is virtual, erect, and smaller than the object.

Conjugate Points.—The distance of the image from the lens can be deduced directly from the formula for refraction at a spherical surface. Let u be the distance of the object from the lens, r_1 and r_2 the radii of curvature of the first and second faces respectively, μ

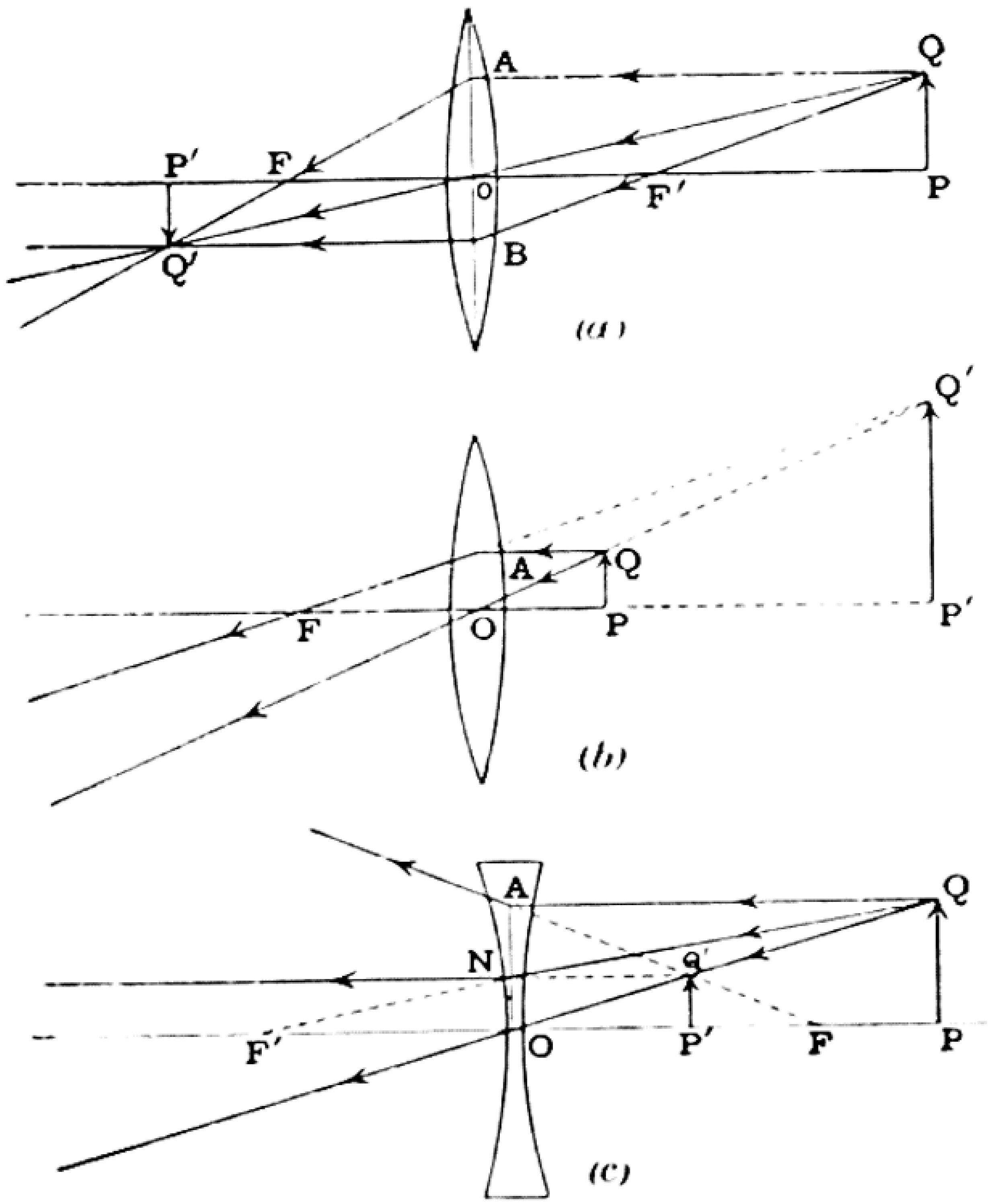


FIG. 113 (a, b, c).—Graphica Construction of Images.

the refractive index of the lens material. For the image formed by refraction at the first face we have

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{r_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where v' is its distance from this face. In the lens the rays appear to diverge from this image, we may therefore regard it as the object when applying the formula to the second face. Let v be the distance of the final image from the lens; neglecting the thickness of

the glass, and replacing μ by $1/\mu$, since the light is passing from the lens into air, we have at the second face

$$\frac{\frac{1}{\mu}}{v}-\frac{1}{v'}=\frac{\frac{1}{\mu}-1}{r_2}$$

or, multiplying by μ,

$$\frac{1}{v}-\frac{\mu}{v'}=-\frac{\mu-1}{r_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Adding equations (1) and (2) together we get

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

The right side is a constant for the lens. When the object is very distant the emergent rays pass through the principal focus, in this case $v=f$, the focal length, $u=\infty$ and $\frac{1}{u}=0$. Substituting these values we have

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Combining this with the last equation we get finally

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

a formula very similar to that obtained for the image formed by reflexion from a spherical mirror. Equations (3) and (4) are important and should be remembered. When numerical values are substituted the proper signs must be used; thus for convex lenses f is negative, and for a double convex lens r_1 is negative and r_2 positive. Equation (4) can be used to investigate how the image moves when the object changes its position, although generally it is most useful to draw a figure as in the preceding paragraph. We will take the case of a convex lens; from the equation we have $v=\frac{uf}{u+f}$, dividing above and below by u and putting the negative sign before f this becomes $v=\frac{-f}{1-f/u}$. When u is infinitely great

$f/u = 0$ and $v = -f$, *i.e.* the image of an infinitely distant object is at the principal focus. For smaller values of u, but such that $u > f$, f/u is < 1, the denominator is positive and < 1, hence $v > f$ and is negative, showing that as the object moves towards the lens from the right the image moves further away to the left. When $u = f$, *i.e.* when the object is at F′ (Fig. 113), $v = -\infty$ or the rays leave the lens parallel with the principal axis. If the object is nearer to the lens than F′ u is less than f and $f/u > 1$, hence the denominator of the fraction is negative and v is positive, meaning that the image is on the same side of the lens as the object and is therefore virtual. For values of u only slightly smaller than f, v is very great and is positive, but as the object approaches the lens the image moves in the same direction and the two coincide at the lens itself. It will be noticed that when the object passes F′ the image moves round from $-\infty$ to $+\infty$.

The reciprocal of the focal length is called the **power of a lens**; if f is given in metres the power is given in diopters. Thus a lens whose focal length is $\frac{1}{2}$ metre has a power 2 diopters.

Linear Magnification.—Expressions for the linear magnification, as defined on p. 156, can readily be deduced from Fig. 113 *a*, *b*, *c*. To keep the signs consistent it must be remembered that if PQ (figure, *a*) is taken as positive then P′Q′ must be considered negative since it is drawn in the opposite direction. In order to avoid confusion we shall also find it convenient to put $OF' = f'$ with proper sign and substitute $-f$ for this in the final results, since f and f' are measured in opposite directions. In Fig. 113 *c* we have, from △'s OPQ, OP′Q′

$$\frac{\text{Image}}{\text{Object}} = \frac{P'Q'}{PQ} = \frac{OP'}{OP} = \frac{v}{u} \quad . \quad . \quad . \quad . \quad (1)$$

In △'s FP′Q′, FOA,

$$\frac{\text{Image}}{\text{Object}} = \frac{P'Q'}{OA} = \frac{FP'}{FO} = \frac{f - v}{f} \quad . \quad . \quad . \quad (2)$$

From △'s F′QP, F′NO,

$$\frac{\text{Image}}{\text{Object}} = \frac{ON}{PQ} = \frac{OF'}{PF'} = \frac{-f'}{u + (-f')} = \frac{f}{u + f} \quad . \quad . \quad (3)$$

Exactly the same formulæ hold for a convex lens; thus in Fig. 113 *a*, from △'s OP′Q′, OPQ

$$\frac{-\text{Image}}{\text{Object}}=\frac{\text{OP}'}{\text{OP}}=\frac{-v}{u}$$

In △'s FP′Q′, FOA,

$$\frac{-\text{Image}}{\text{Object}}=\frac{\text{FP}'}{\text{OF}}=\frac{\text{OP}'-\text{OF}}{\text{OF}}=\frac{-v-(-f)}{-f}=\frac{-(f-v)}{f}$$

And from △'s F′PQ, F′OB,

$$\frac{-\text{Image}}{\text{Object}}=\frac{\text{OF}'}{\text{PF}'}=\frac{\text{OF}'}{\text{OP}-\text{OF}'}=\frac{f'}{u-f'}=\frac{-f}{u+f}$$

Changing signs on both sides the last three are the same as before.

When numerical values are substituted in these expressions the proper signs must be used, if the result comes out negative it means that the image is inverted. Conversely if the magnification is put in in (2) or (3) with its proper sign (negative for an inverted image) the focal length can be calculated if either v or u is known.

If any two of these expressions are equated the ordinary lens formula is at once obtained. Thus from (1) and (3)

$$\frac{v}{u}=\frac{f}{u+f}$$

$$\therefore\ uv+vf=uf$$

dividing by uvf and rearranging the required result follows. This is perhaps the simplest method of getting the equation as it is not necessary to follow the details of the refraction at each face, all that is required is an experimental knowledge of the properties of the focal points and the optical centre.

Two Lenses in Contact.—For some purposes it is necessary to pass light through two lenses placed in contact; let us calculate the focal length F of the combination, when f_1 and f_2 are the focal lengths of the separate lenses. Let u be the distance of the object from the system, then by refraction at the first lens an image is formed at a distance v', where

$$\frac{1}{v'}-\frac{1}{u}=\frac{1}{f_1}$$

Regarding this image as the object for the second lens, the distance v of the final image from the system is given by

$$\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2}$$

Adding the two equations, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}$$

but
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{\mathrm{F}}$$

hence
$$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}$$

The focal lengths must be used with their proper signs. The equation tells us that the power of the combination is the sum of the powers of the components.

Methods of measuring the Focal Lengths of Convex Lenses.—*1st method.* The lens is mounted on the optical bench and a real image of the cross-wires is focussed on a screen which can be moved to and fro for this purpose. The distances from the lens of image and object are measured, f is then calculated from the equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ with the help of a table of reciprocals. If the same value of f is obtained with different values of u the correctness of the equation is verified.

2nd method. The image of a very distant object is focussed on a screen; the distance between lens and screen is the focal length.

3rd method. If in the first method the wires are placed at the first focal point the rays are parallel after leaving the lens. Suppose they then strike a plane mirror at nearly normal incidence; they will retrace their path and form an image near the cross-wires. Hence mount the lens facing the cross-wires and place behind it on another stand a piece of plane mirror of good quality. Move the lens about until a clear image is obtained beside the cross-wires, the distance of this from the lens is the focal length. An image may be formed by light reflected from the back face of the lens, but this may easily be distinguished from the one sought for since it does not move when the mirror is tilted.

4th method. This is a modification of the 3rd method in which a different means is adopted to test the parallelism of the emergent rays. Look through a telescope at a distant object and pull out the eye-piece until a clear image is obtained; if the instrument is now directed to any other object a badly defined image will be seen unless the light received is practically parallel. Fix the lens in front of the object glass and look through the combination at a page of printed matter. Move the latter about until it is clearly seen; the rays entering the telescope are then parallel and the print is at the first focal point of the lens. The advantage of this and the last method over the first is that they require shorter distances between lens and object, they are therefore preferable for long focal lengths.

5th method. Let P be the cross-wires and S the screen on the optical bench (Fig. 114). When the lens is placed at A an enlarged

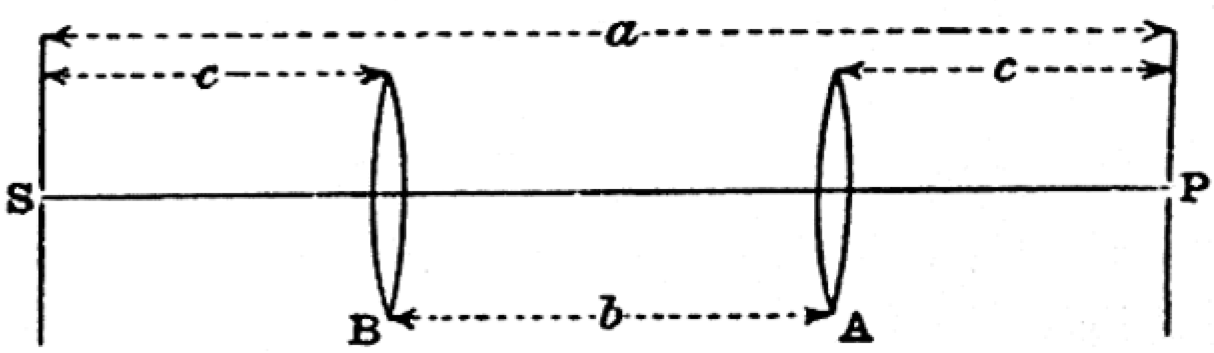


FIG. 114.

image is formed at S; since the path of the light is reversible if an object were placed at S a diminished image would be produced at P, hence if the lens is shifted to B, where PA = BS, a diminished image of the wires is formed on the screen. Let $PA = BS = c$, $PS = a$, $AB = b$, then in either position of the lens we have from the equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{b+c} - \frac{1}{c} = -\frac{1}{f}$$

Also
$$a = b + 2c$$

or
$$c = \frac{a-b}{2}$$

Substituting this value in the equation we find

$$f = \frac{a^2 - b^2}{4a}$$

Hence fix the screen at a distance a from the cross-wires so that two

images can be found and note how much the lens has to be shifted to change from one to the other; f can then be calculated from the above formula. If the screen is brought nearer to the wires the distance b is diminished until at a certain position only one image, the same size as the object, can be obtained. In this case $b = 0$ and $f = a/4$. It can be shown that the image is now as near to the object as it is possible to get it, hence if this position be found experimentally the focal length is one-quarter of the distance between wires and screen.

6th method. If the magnification and either v or u be measured the focal length can be found from the second or third formulæ on p. 193. As the image is inverted the magnification must be put negative. A slit exactly 1 cm. wide is used as the object, and the image is focussed on a mm. paper scale from which the magnification is read off directly. The following variation gives correct results even for thick lenses. Arrange that the magnification is unity, then, keeping the lens fixed, move the slit and scale until an image is obtained twice, three times, etc., as large as the object. Note the distance through which the scale has been moved from one image to the next; this is the focal length. For, with proper sign,

$$-1 = \frac{f - v_1}{f}$$

or

$$-f = f - v_1$$

Similarly when the magnification is two

$$-2f = f - v_2$$

subtracting one equation from the other

$$f = v_2 - v_1$$

7th method. The following convenient method of determining the power of a lens is due to Prof. S. P. Thompson. The same apparatus is used as in the last method, but the scale is fixed one metre from the lens and the slit is moved until a clear image is obtained. Let this image be m cms. long, then the power of the lens is $(m + 1)$ diopters. For, in the second expression (p. 193) for the magnification, putting $v = -1$ we get, since the image is inverted,

$$-m = \frac{f + 1}{f}$$

whence

$$-\frac{1}{f} = \text{power in diopters} = (m + 1)$$

8th method. All the above methods are inconvenient when the focal length is several metres; in such cases the incident light should be rendered convergent by means of an auxiliary lens of shorter focus. The two lenses may be placed in contact and the formula $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$ employed, or the arrangement shown in Fig. 115 can be used, where C represents the cross-wires, A the auxiliary lens, and B the lens whose focal length is required. With the lens B removed an image is first obtained on a screen at P and AP is measured. B is next placed in position and, the light now being more convergent, the screen has to be moved to Q to receive the image. AB and BQ are measured whence $BP = AP - AB$ can be

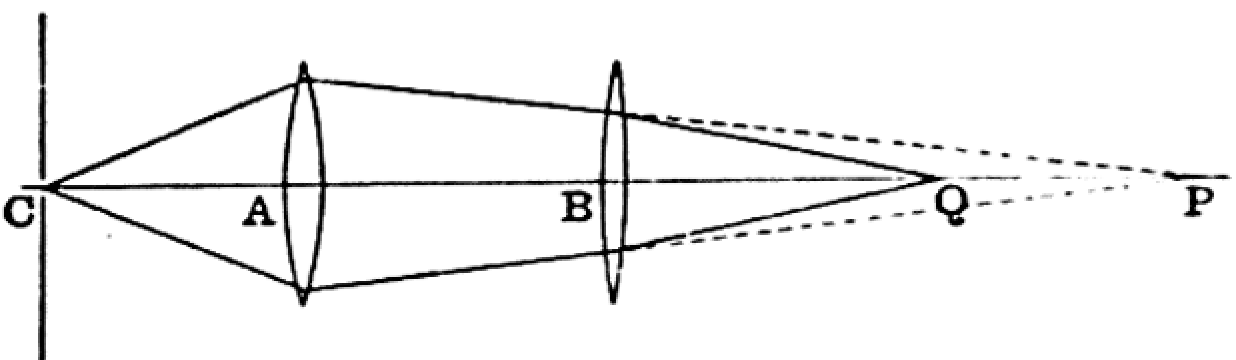


FIG. 115.

found. The image at P is to be regarded as a virtual object and Q is its image formed by the second lens, hence f can be found from the equation

$$-\frac{1}{BQ}+\frac{1}{BP}=\frac{1}{f}$$

Methods of measuring the Focal Lengths of Concave Lenses.—The difficulty in this case is that the image, being virtual, cannot be received on a screen.

1st method. This will be best understood by a reference to Fig. 116. P is a knitting needle which is used as object and Q is its image, M is a small piece of plane mirror with its reflecting surface turned towards a second needle O. An observer on the right sees two images, one of P formed by the lens, the top of this is seen over the edge of M, the other the image of O in the plane mirror; by adjusting the distance OM these can be brought into coincidence at Q, as tested by the parallax method. Then $MQ = OM$, and $LQ = OM - ML$; by measuring OM, ML the distance $LQ = v$ can be found, also $PL = u$ can be measured and f obtained from the equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

2nd method. The lens is placed in contact with a convex lens of shorter focus, the combination forms a convex lens whose focal length F can be determined, and the required focal length f_2 can be calculated from the equation $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$.

3rd method. The concave lens may replace B of Fig. 115; in this case the beam is rendered more divergent and the final image is to the right of P. The calculation is the same as before. If the concave lens is shifted until BP is equal to its focal length the emergent beam is parallel. This parallelism can be tested by either of the methods (3) or (4) of the last paragraph and $BP=f$. Owing, however, to optical defects it is difficult to get a well-defined image.

4th method. If a number of convex lenses of known focal lengths

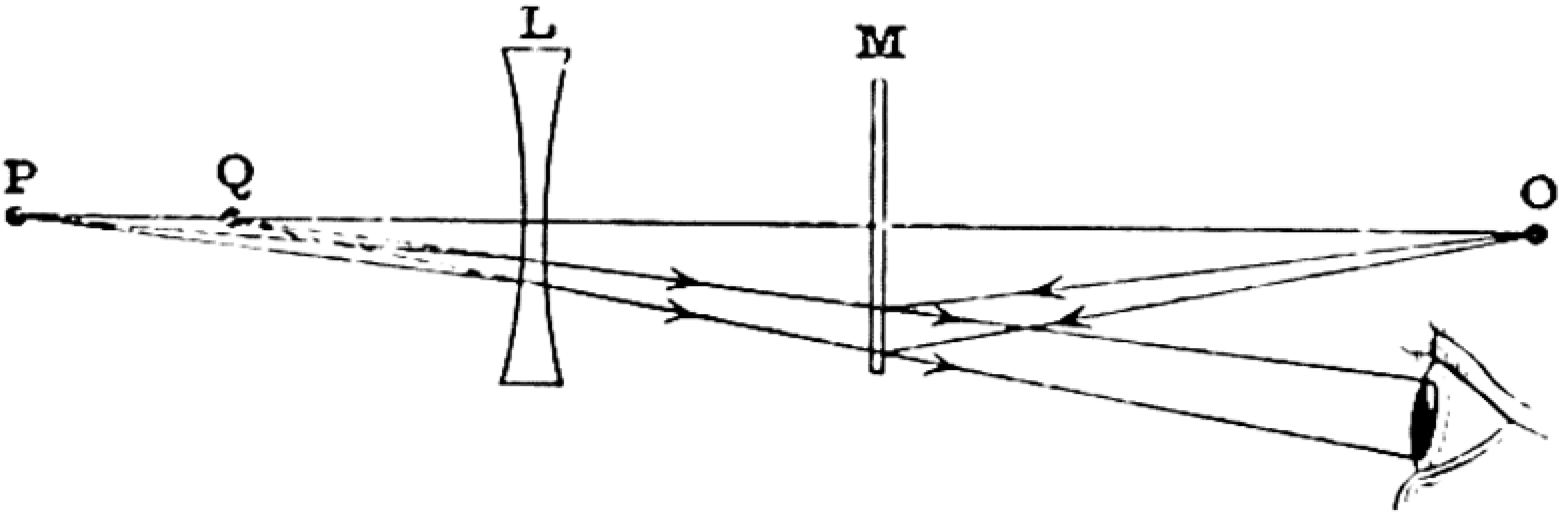


FIG. 116.—Method of finding the Focal Length of a Concave Lens.

are available it may be possible to choose one which just neutralises the concave lens.

EXPERIMENT.—Hold a concave lens *close* to the eye and look through it at a window frame; if the lens is moved up or down the frame appears to move in the same direction. Repeat with a convex lens, the motions of lens and frame are opposed. Make use of this to find two lenses which just neutralise each other, their focal lengths are equal and opposite in sign. The focal length of the convex lens can be found by the methods given above.

Radii of Curvature of the Faces of a Lens.—Part of the light which falls on a lens is reflected; this may be used to measure the radii of curvature of the faces. When the face is concave the methods of p. 157 are applicable, if it is convex either of the following can be employed, the first of which is useful for convex mirrors generally.

1st method. Light from the cross-wires of an optical bench passes

through a short focus convex lens A (Fig. 117) and falls on the convex surface B. By shifting A or B it can be arranged that the rays meet the surface at nearly normal incidence; when this happens the reflected portion retraces its path and forms an image near the wires. If the rays are produced to the right they evidently meet at C, the centre of the surface. The distance AB is measured, B is removed and a screen is placed at C to receive a well-defined image. The distance AC is measured, whence the radius $BC = AC - AB$ can be found.

2nd method. In this it is arranged that the light which enters the lens meets the second face normally, the rays reflected from this face therefore return along their path and form an image near the source while the transmitted rays emerge into the air without further

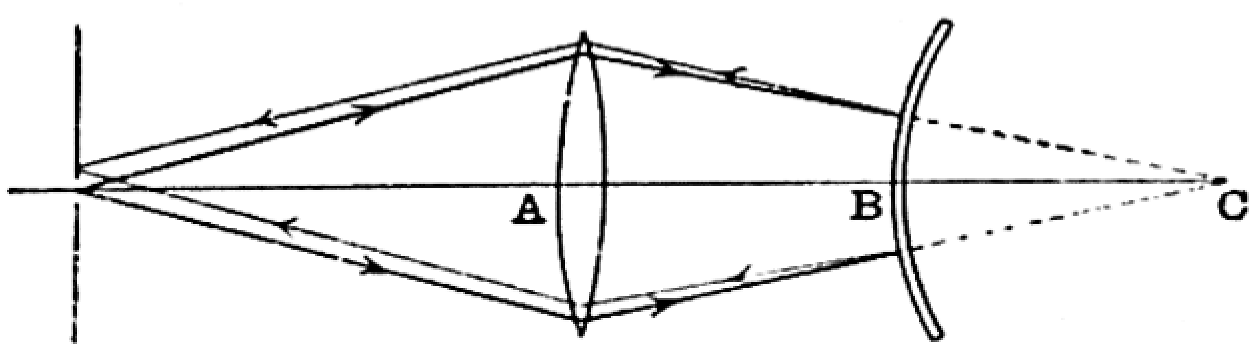


Fig. 117.—Method of finding the Radius of a Convex Spherical Surface.

deviation. In Fig. 118 P represents the source and C′ the point from which the rays diverge after refraction into the lens; since they meet the second face normally C′ is the centre of curvature of this face, it is also the virtual image of P formed by the light which passes through the lens. Its position is therefore given by the usual lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, and as $v = R_2$, the radius of the back face,

$$\frac{1}{R_2} - \frac{1}{u} = \frac{1}{f}$$

or

$$R_2 = \frac{uf}{u+f}$$

where f is to be used with its proper sign. To carry out the experiment the lens is placed on the optical bench with the convex face whose radius is required turned away from the wires; it is moved about until a clear image is obtained by reflexion from this face. The distance between source and lens is then u of the formula. The focal length must be measured by a separate experiment before R_2

can be calculated. The lens may also be floated on a small quantity of mercury to increase the amount of reflected light, a small cardboard pointer is placed above it and this is moved about until it coincides with its own image as tested by parallax. This gives u above.

Refractive Index of a Lens.[1]—When the radii of curvature and the focal length have been measured, the refractive index of the material can be calculated from the equation $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.

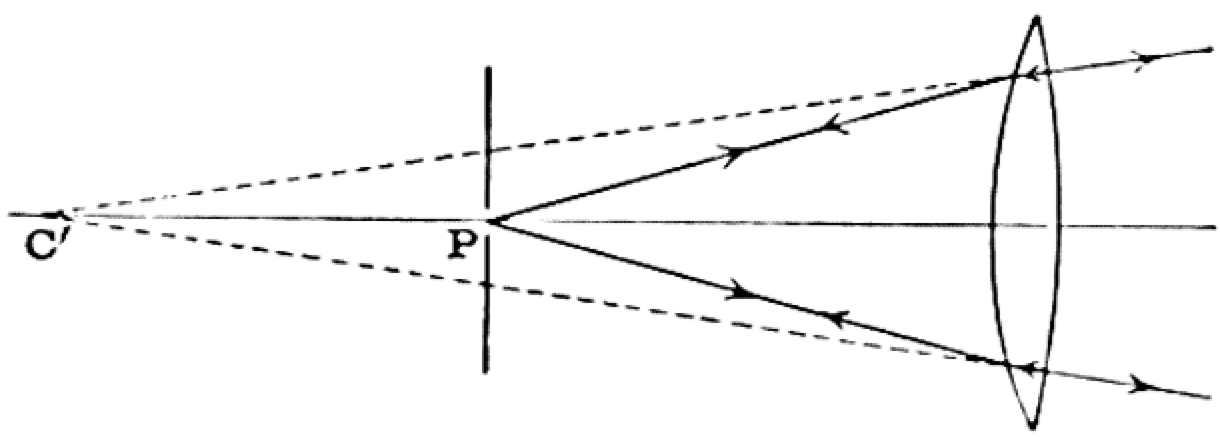


FIG. 118.—Path of Rays reflected from the Back, Convex Face of a Lens.

As exercises on the preceding methods the following experiments are instructive.

EXPERIMENT.—Lay a piece of plane mirror on the floor and place on it a convex lens. Support a cardboard pointer in a stand above the two and move it up and down until it coincides with its real image; the distance from lens to pointer is the focal length f_1. (This is simply a modification of the 3rd method, p. 195). Run a film of water between the lens and mirror, this forms a plano-concave liquid lens whose upper face has a radius of curvature R_2, the same as the lower face of the glass lens. Measure the focal length F of the combination by the same method, then the focal length f_2 of the liquid lens is given by $\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1}$, hence f_2 is known.

Also
$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R_2}$$

where μ is the refractive index of the liquid.

If R_2 is measured by any of the preceding methods μ can be found.

EXPERIMENT.—Fill a watch-glass with liquid, cover it with a plate of glass to ensure a flat surface and place it on a mirror as in the last experiment. Find,

[1] Barton and Black, "Practical Physics," p. 94.

as before, the focal length of this plano-convex liquid lens. Replace the first liquid by water and again find the focal length; then from Equation 3, p. 192,

$$\frac{\mu_1 - 1}{\mu_2 - 1} = \frac{f_2}{f_1}$$

Taking $\mu_2 = 1{\cdot}33$ for water find the refractive index μ of the first liquid.

EXAMPLES ON CHAPTER XVII

1. A small air bubble in a sphere of glass 4 in. in diameter appears, when looked at so that the bubble and the centre of the sphere are in a line with the eye, to be 1 in. from the surface. What is its true distance? ($\mu = 1{\cdot}5$.) (L. '87.)

2. A small object is enclosed in a sphere of solid glass 7 cms. in radius. It is situated 1 cm. from the centre and is viewed from the side to which it is nearest. Where will it appear to be if the refractive index of the glass be 1·4? (L. '91.)

3. A block of transparent jelly of refractive index 1·33 is bounded on one side by part of a convex surface of a sphere of radius 8 mms. Find the position of the principal focus within the mass of material. (L. '98.)

4. Construct the path of a ray passing through a spherical boundary of a dense medium of given refractive index. Calculate the position of the place to which parallel rays passing nearly perpendicularly through the surface would converge if the refractive index be 1·7 and the radius of curvature 6 ft. (L. '92.)

5. A ray of light passing through a prism meets the second face at perpendicular incidence. If i is the angle of incidence on the first face and A the angle of the prism, show that $\mu = \sin i/\sin A$.

6. What is the greatest allowable angle of a prism in order that a ray, incident on the first face at an angle of 60°, may emerge from the second face? ($\mu = 1{\cdot}64$.)

7. The refracting angle and the minimum deviation for a given prism are each 60°. Find, by means of a diagram showing the course of the minimum ray, the refractive index of the glass. (L. '10.)

8. A lens forms an image one-third the size of an object and 2 ft. distant from itself. What is the focal length of the lens? Where is the object? Consider the case of virtual as well as of a real image. Draw diagrams to illustrate your answer. (L. '90.)

9. A bright point is situated on one wall of a room 9 ft. wide. A convex lens, 1 ft. focal length and 2 in. in diameter, is placed 3 ft. from the wall in the normal from the point. What is the width of the circle of light thrown by the lens on the opposite wall? (L. '96.)

10. Show how the focal length of a convex lens depends on the curvature of its faces. (L. '05.)

11. A flat object, whose surface is a sq. mm., is placed facing an ordinary magnifying glass at a distance u from it; an image of the object is formed at a distance mu from the lens. Prove that the size of this image will be such that its area is m^2a. Will it make any difference whether the image be real or virtual? (L. '97.)

12. Find the size and position of the image formed, by a convex lens of 12 in. focal length, of the following object, viz. an arrow 2 ft. long, lying along the axis of the lens with its middle point 30 in. from the lens. What would be the size and position of the image if the arrow were turned through 90° about its middle point? (L. '99.)

13. Give a sketch of the arrangement of a lamp, slit, lens, and scale by means of which the image of the slit formed by the lens and reflected by a plane mirror may be thrown on to the scale. Trace in your sketch the course of a pencil of rays. (L. '01.) [This is a modification of the lamp and scale method of measuring deflexions, see Fig. 89.]

14. A convex lens 2 in. focal length is held 1 in. from the eye by a person with distance of distinct vision of 9 in. so as to look at a small object. Where must the small object be placed? Illustrate your answer by a figure. (L. '02.)

15. Draw a curve showing in the case of a convex lens the connexion between the distance of the object from one principal focus and the distance of the image from the other. (L. '03.) [See next example.]

16. If the distance of an object from the first focal point of a convex lens is x and the distance of its image from the second focal point is y, show, without regard to sign, that $xy = f^2$.

17. A convex lens of 10 cms. focal length is held in a horizontal position just above the surface of a liquid filling a tank 20 cms. deep. The image of a point 30 cms. above the centre of this lens is brought to a focus on the bottom of the tank. Draw a diagram of the path of the rays and calculate the index of refraction for the liquid. (L. '08.)

18. The focal length of a convex lens is 10 in. It is placed in a small tank with parallel sides. Where is the image of a distant object formed if the tank is filled with (1) water, (2) a liquid of refractive index 1·63? Take μ for lens and water as 1·53 and 1·33 respectively.

19. In method five of finding the focal length of a convex lens l is the length of the object and l_1, l_2 the lengths of the two images. Prove that $l = \sqrt{l_1 l_2}$.

20. Use the result of Example 16 to show that the least distance between real image and object is $4f$ for a convex lens.

21. When a luminous point is placed on the principal axis of a convex lens (A) and at a distance a from it an image is formed 10 in. from the lens on the other side. If a second lens (B) is placed close to A the image is 15 in. off. Find the focal length of lens B and state whether it is convex or concave. (L. '85.)

22. An object is 20 ft. from a screen. Given two convex lenses respectively of 9 in. and 18 in. focal length explain how you will obtain (1) an erect and magnified, (2) an inverted and magnified, image of the object on the screen. (L. '86.)

23. Two thin lenses have each a focal length of 1 in. Draw to scale the path of a beam of light from a distant object (1) when the lenses are in contact, (2) when they are separated by $1\frac{1}{2}$ in., (3) when they are separated by 3 in. (L. '94.)

24. The plane side of a plano-convex lens is silvered and the lens then acts like a concave mirror of 30 cms. focal length. The refractive index of the lens is 1·5. Calculate the radius of curvature of the convex face. (L. '09.)

25. A plano-convex lens is silvered on its plane side and then acts like a concave mirror of 20 cms. focal length. When the convex side is silvered it acts like a concave mirror of 7 cms. focal length. Calculate the refractive index of the lens. (L. '09. Hons.)

26. Two convex lenses each of focal length f are placed at a distance $3f$ apart. For what position of the object will a real image be formed by this combination of lenses? (L. '10.)

CHAPTER XVIII

DISPERSION, PHOSPHORESCENCE AND FLUORESCENCE

Passage of White Light through a Prism.—In the experiments on refraction that have been studied in the previous pages it has been assumed that the light is of a definite colour, *e.g.* that given by a sodium flame. We will now see what differences are introduced when sunlight, or the light from an arc lamp or gas flame, is used. The fundamental observations are due to the illustrious Newton, who experimented with sunlight which passed through a small slit in the side of a darkened room. We shall find it more convenient to use an arc lamp as the source of light.

EXPERIMENT.—Fix a glass prism with its refracting edge A vertical (Fig. 119), and place about 1 m. away from it a cardboard screen in which a narrow vertical slit S has been cut. Illuminate this slit from behind by means of a sodium flame and place on the other side of the prism a vertical white screen. With suitable adjustments light passes through the prism and produces a yellow patch on the screen at Y. Substitute an arc or incandescent gas lamp for the sodium flame; in place of the yellow slit at Y there now appears an extended patch of light of different colours. The order in which the colours come is as follows:—red, orange, yellow, green, blue, indigo, violet. The red end is deviated less and the violet more than the yellow light which appears in its original position. (S′ is the patch produced by the undeviated light when the prism is removed.)

This patch of coloured light is called a **spectrum** and the white light is said to have undergone **dispersion.** If an eye is placed to receive the light which comes from the prism the spectrum appears on the backward prolongation of the rays at R′—V′. Since the rays do not actually come from R′—V′, but only appear to do so, R′—V′ is a virtual spectrum. It will be noticed that its colours are much purer and more brilliant than those of the real spectrum R—V.

EXPERIMENT.—Turn the prism round its refracting edge until the minimum deviation position is reached; it will be found that the spectrum is shorter,

but its colours are much more brilliant and pure. Hence in the production of spectra it is advantageous to place the prism in the position of minimum deviation.

The question to be answered now is, Does the prism colour the light during its passage through the glass or does it merely separate colours which are already present? If a piece of blue glass is held in the path of the rays between slit and prism only the violet end of the spectrum appears, if a piece of red ruby glass is used instead only the red end is seen. The prism is therefore unable to turn violet light into red, or *vice versâ*, and we conclude that the different

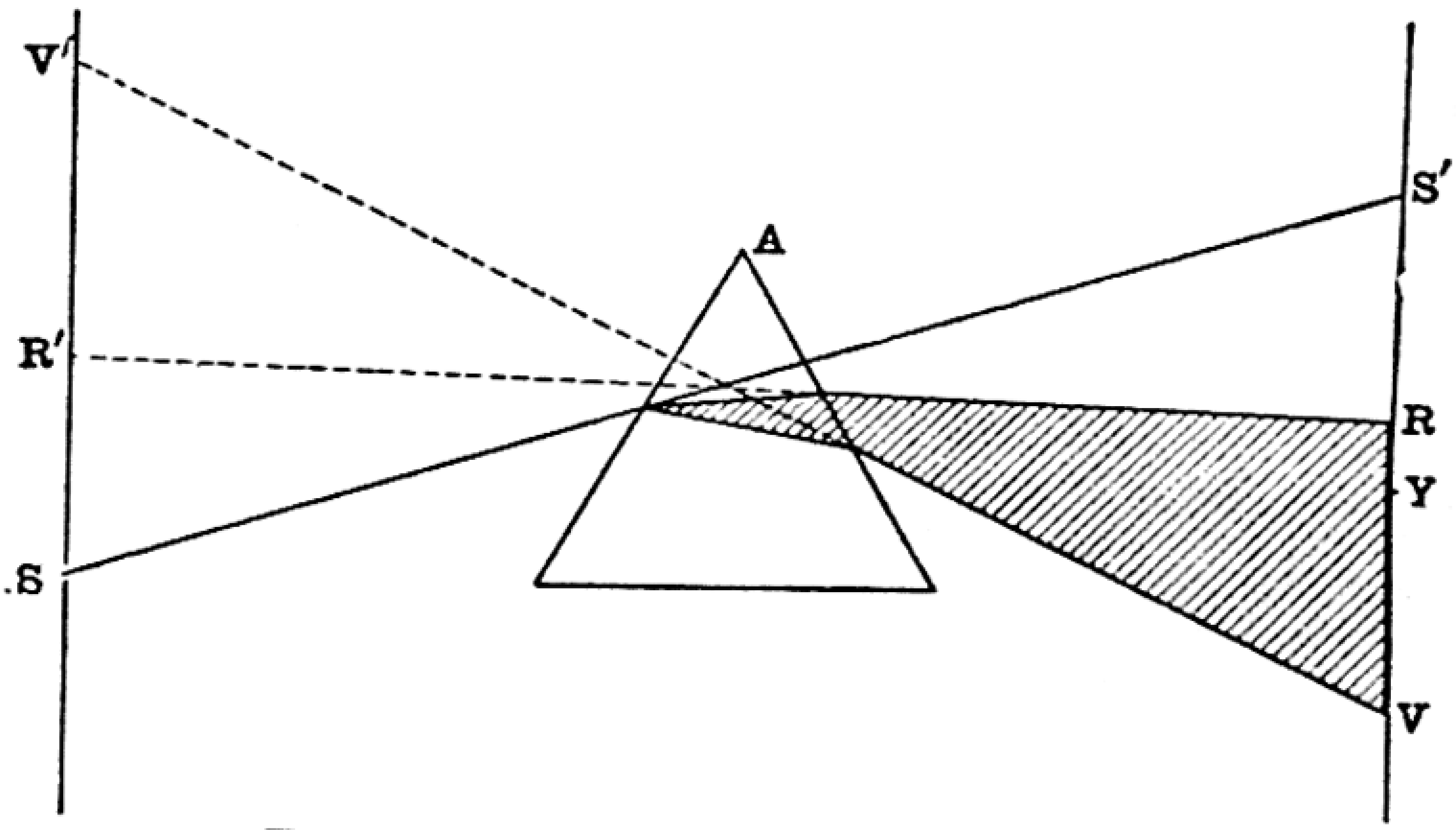


FIG. 119.—Dispersion of Light by a Prism.

colours were originally present in the white light; all the prism does is to make them visible by separating them from each other. Since the different coloured rays are deviated by different amounts it is clear that the refractive index of the prism varies with the colour of the light; the violet rays, which are deviated most, are said to be more refrangible than the red which are deviated least. It is owing to this difference in refrangibility that the rays are separated. That this is the correct explanation of dispersion is shown by Newton's experiment of the crossed prisms.

EXPERIMENT.—In the first experiment above the refracting edge of the prism is *vertical* and the spectrum is horizontal; hold between the prism and the screen a second prism with its refracting edge *horizontal* and its base uppermost. The red rays coming from the first prism fall on the second and are bent upwards, the violet rays fall on a different part and, owing to their greater refrangibility, are bent upwards by a larger amount. The final spectrum is

therefore inclined to the horizontal as in Fig. 120, where A shows the spectrum produced by the first prism and B that produced by the two. The direction of the rays, but not their colour, is altered by the second prism.

We have spoken of seven different spectral colours, in reality there are a much larger number; an artist would see many more than seven, while a person whose colour sense is badly developed would probably see less. It is known from physical optics that the wave-length of the red rays is nearly double that of the violet. Light of a definite wave-length, and therefore of definite colour, is called **monochromatic.**

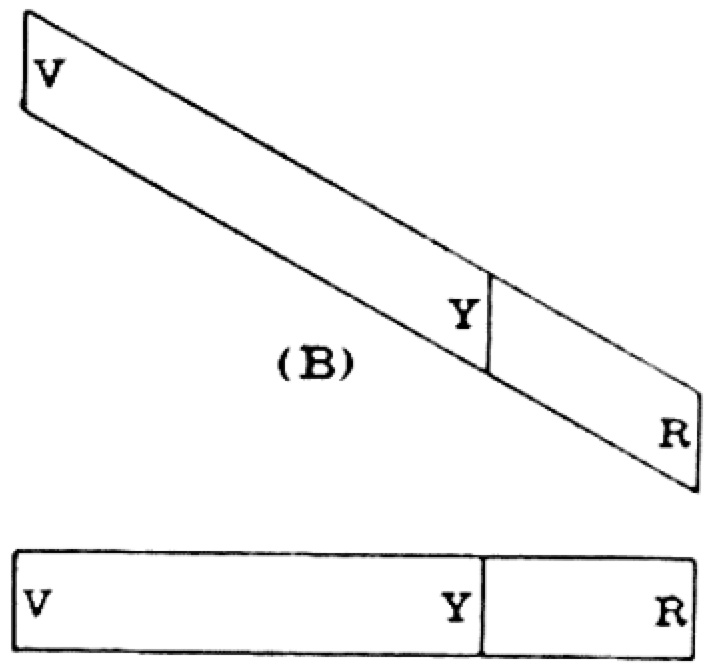


FIG. 120.—Newton's Experiment with Crossed Prisms.

Recomposition of White Light.—Since white light is a mixture of colours it ought to be possible to combine different coloured rays so as to produce white light. This can be done in several ways :—

(1) Fig. 121 represents two prisms, exactly alike, with their refracting edges in opposite directions; the dispersion produced by the first is then just cancelled by the second and the emergent beam is white. The arrangement, in fact, acts like a parallel plate (p. 166) and all the rays emerge parallel to

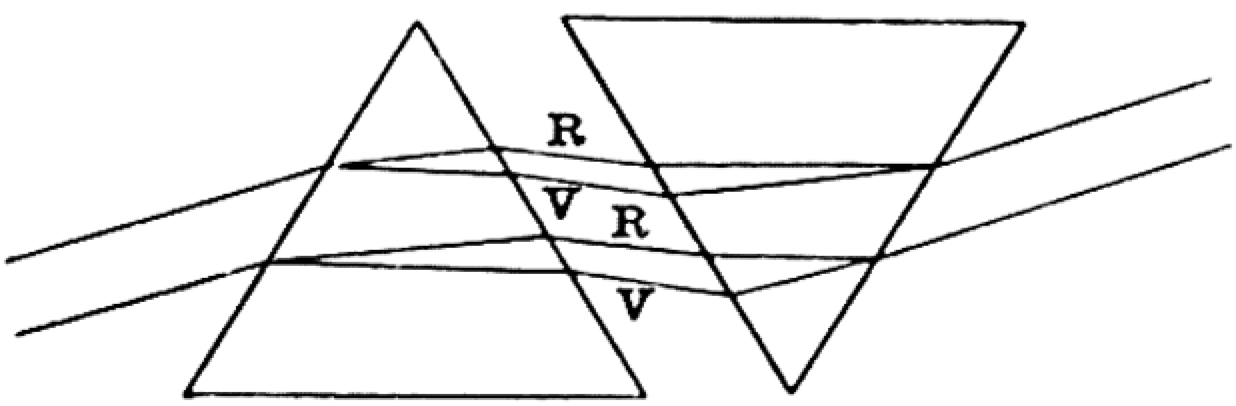


FIG. 121.—Recomposition of White Light by two Prisms.

their original directions. If a piece of cardboard is held between the prisms to obstruct some of the light the patch is again coloured.

(2) The spectrum produced by a prism may be thrown on to a number of strips of plane mirror from which the light is reflected to a screen. A number of coloured patches are seen which may be made

to coincide by properly adjusting each mirror, the mixture produces white light.

(3) *Newton's Colour Disc.*—An impression produced on the retina of the eye persists for a small fraction of a second after the exciting cause has been removed, thus the paper sails of a child's "whirling mill" appear to coalesce into a continuous disc when they are revolving rapidly. This is used to mix colour impressions. A circular cardboard disc, capable of rapid revolution round an axis perpendicular to its plane, is divided into seven unequal sectors each of which is painted with one of the spectral colours. It is well illuminated and rotated rapidly when, owing to the persistence of visual impressions, the various colour sensations coalesce and the disc appears to be greyish-white. It is grey rather than white owing to the lack of light; if it were white cardboard, each part would contribute a portion of, say, red light to the image on the retina, whereas now only one sector does this, hence the effect is that produced by a badly lighted white screen, *i.e.* grey.

A Pure Spectrum and how to produce it.—It has been seen that unless a prism is in the minimum deviation position no definite image is produced by the transmitted rays (p. 181). Suppose now white light passes through the slit in the experiment on p. 205, each colour produces its appropriate image, and unless these are well defined they will overlap and the colours will be mixed. Such a spectrum is said to be impure. As the images are more definite when the prism is placed for minimum deviation it is clear why the spectrum is more brilliant under these circumstances. There will also be less risk of overlapping if the object slit and therefore each of the coloured images is very narrow. A still further improvement is obtained if the images are focussed on the screen by a lens, which may be placed in either of the positions shown in Fig. 122 A and B. All the red rays are then brought to a focus at R and the violet ones at V. Even when the prism is placed for minimum deviation no true image is formed unless the incident rays are nearly parallel (p. 181), hence it is advisable to use two lenses as in the spectroscope. Fig. 123 shows the optical parts of this instrument.

The apparatus used to produce a parallel beam consists of a narrow slit, S, placed at the principal focus of a convex lens C; slit and lens are mounted at the opposite ends of a brass tube. This part of the apparatus is called a collimator. Light which starts from the slit and passes through the lens emerges as a beam of

parallel rays. The beam undergoes dispersion in the prism and then falls on a second convex lens A which focusses the rays into a pure spectrum at VR. In the spectroscope A is the objective and B the eye-piece of a telescope ; the function of the latter lens is to produce a magnified, virtual, image of the spectrum at V′R′. (For Tele scope, see p. 235.)

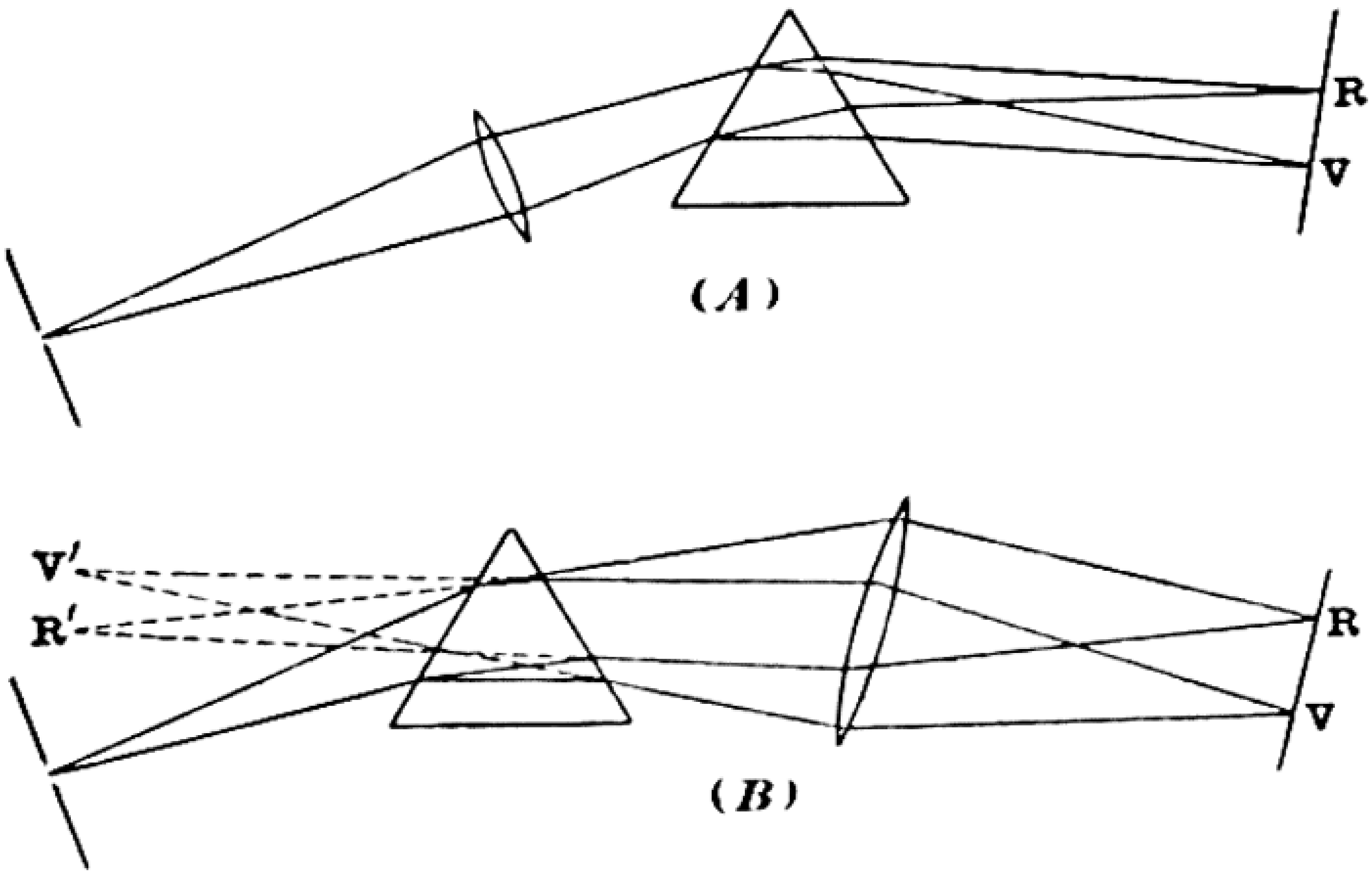


FIG. 122.—Production of a Pure Spectrum.

Fig. 122 B shows how it is that the virtual spectrum in the experiment on p. 205 is purer than the real one, for in that case the

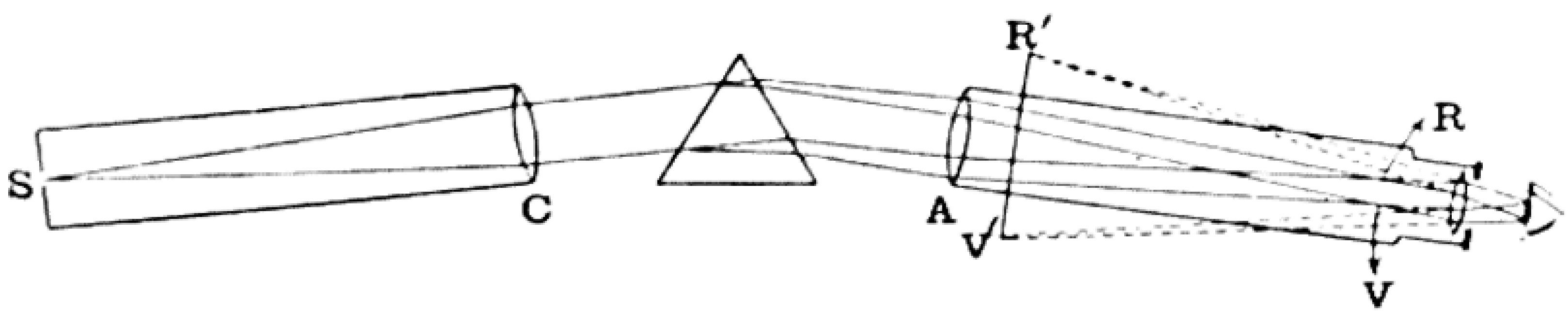


FIG. 123.—Optical Parts of a Spectroscope.

lens and screen of the figure are replaced by the lens and retina of the eye.

In order to produce a pure spectrum it is seen that the following conditions must be fulfilled :—

(1) A narrow slit.
(2) Prism in the position for minimum deviation.
(3) Incident rays nearly parallel.
(4) A lens to bring the emergent rays to a focus.

Spectrum Analysis.—The spectrum formed by the light coming from an incandescent solid is usually continuous like those studied above, but if a substance is heated under suitable conditions its spectrum is found to consist of a number of bright lines on a dark ground. These lines are characteristic of the substance and may be used to identify it for purposes of chemical analysis. Thus if a little strontium chloride is heated to volatilisation in a Bunsen flame its spectrum is found to consist of a number of bright red and green lines. Other methods of heating can also be used.

Angular Dispersion. Dispersive Power.—The angle between two differently coloured rays after they emerge from the prism is called the **angular dispersion** for those rays. It varies with the angle of the prism and the nature of the material. If A is the prism angle, and δ_v the deviation produced in a violet ray for which the refractive index is μ_v, then *when A is small*

$$\delta_v = (\mu_v - 1)A \text{ (p. 183)}$$

Similarly for a red ray the deviation δ_r is

$$\delta_r = (\mu_r - 1)A$$

Hence the angle between the rays, *i.e.* the dispersion, is

$$\theta = \delta_v - \delta_r = (\mu_v - \mu_r)A$$

If μ is the refractive index for the mean ray (say yellow light), the quantity $\omega = \dfrac{\mu_v - \mu_r}{\mu - 1}$ is called the **dispersive power of the material.** Since refractive indices are constants for a given material it is clear that the dispersive power depends only on the nature of the prism and not upon its refracting angle.

Achromatism. Direct Vision Spectroscope.—For many purposes it is necessary that rays of light should be deviated without dispersion or be dispersed without deviation. The possibility of either of these can be seen from the following considerations. Let a number of prisms, *made of different kinds of glass*, be caused in turn to form a spectrum under similar conditions, and let the deviation of the yellow ray and the length of the spectrum be measured in each case. It will be found that these quantities do not increase in the same ratio when we go from one kind of glass to another. For example, if the deviation produced by prism A for the yellow ray is twice that produced by B, the length of spectrum A will not usually

be twice spectrum B. By properly choosing the prism angles it may be arranged that either (1) the lengths of the spectra, *i.e.* the angular dispersions, are equal, or (2) the deviations of the yellow ray are equal. If two prisms of different glass are made to fulfil condition (1) and are then arranged as in Fig. 121, the dispersion of the first is cancelled by the second, but the deviations of the mean ray do not cancel, *i.e.* the beam is deviated without dispersion. Such a combination is called an **achromatic prism.** If instead the prisms fulfil condition (2), then, when placed as in Fig. 121, they will not deviate the yellow ray, but the beam as a whole will be dispersed. This is the principle of the direct vision spectroscope; the two prisms are placed in a tube which carries a slit at one end and a magnifying lens at the other, when the slit is direct towards a source

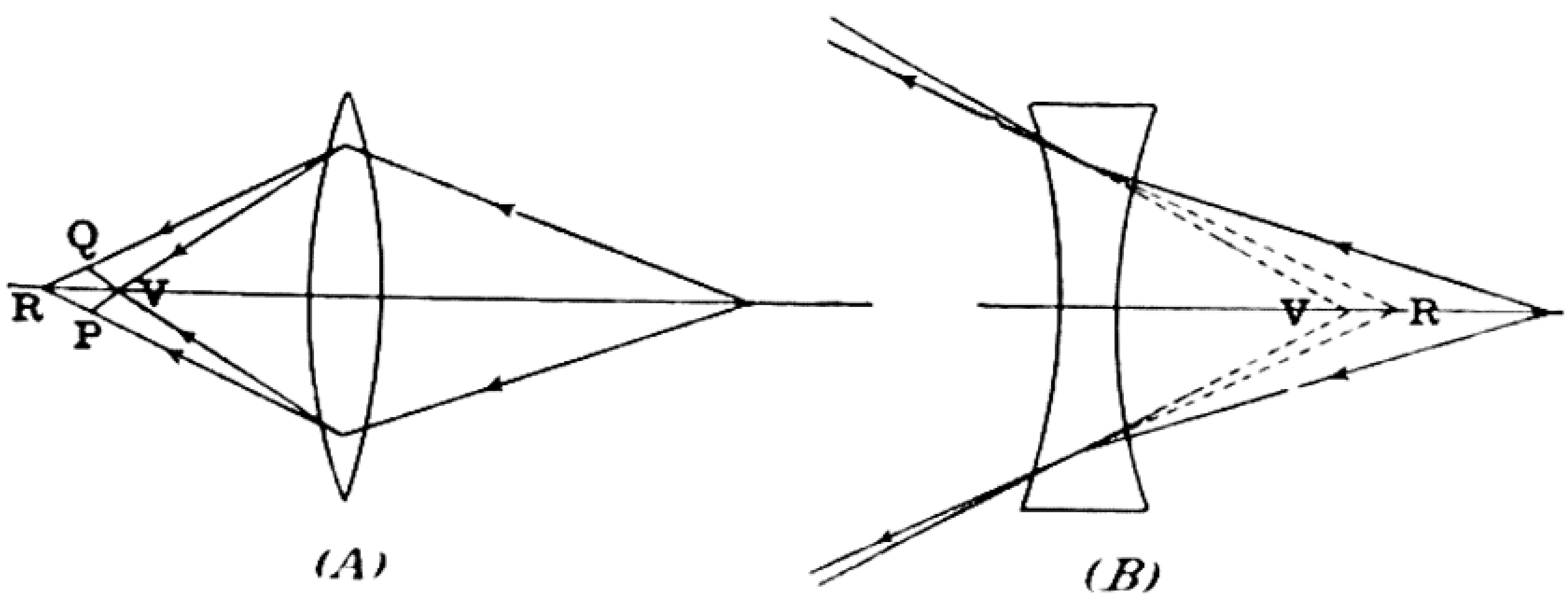


FIG. 124.—Dispersion in a Lens.

of light the spectrum can be seen through the lens. This form of spectroscope is more portable than that shown in Fig. 123 but the dispersion it produces is less.

Dispersion in Lenses.—As the refractive index of a material varies with the colour of the light, when white light passes through a lens dispersion will take place and the differently coloured rays will be brought to different foci. The violet rays being the most refrangible their focus will be nearest to the lens. Fig. 124 shows the path of the rays. PQ (figure *A*) shows the position at which a card must be held to obtain the best-defined image. If a screen is held to the right of this point the outer edge of the image will be coloured red, while further to the left it will be violet. These colour effects are usually seen at the edges of the field when a cheap opera glass or telescope is used. For many instruments it is important that they

should be suppressed. This can be done by combining a convex lens with a concave one of different focal length made from another kind of glass. For in Fig. 124 *A* the violet focus is too far to the right while in *B* it is too far to the left; in the combination these two effects are made to cancel each other and all the rays come to a focus at the same point as in Fig. 125. In practice the two lenses are placed in contact. Such a combination is called an achromatic lens; its section is built up of a number of pairs of prisms forming achromatic combinations.

The Infra-red and Ultra-violet Spectra.—Up to this point our attention has been confined to those radiations which can be detected

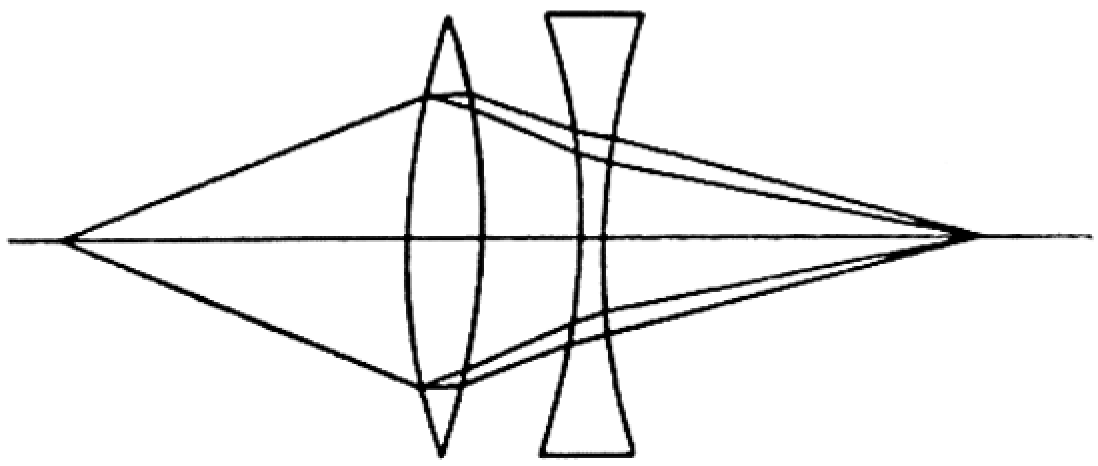


FIG. 125.—Achromatic Lens.

by the eye; these form the visible spectrum, but it is easy to show that there are radiations extending beyond its limits.

EXPERIMENT.—Form an arc-light spectrum and arrange that it falls on a thermopile which is connected to a sensitive galvanometer. The needle is deflected. Move the thermopile towards the red, the deflexion increases and is still quite appreciable when the red end has been passed; in fact, if a rock-salt or quartz prism is used the maximum deflexion occurs beyond the red. If the thermopile is moved towards the violet the deflexion decreases but it is still noticeable beyond the limits of the visible spectrum, especially with a quartz prism and lenses. (Glass readily absorbs these invisible radiations.)

This shows that there are some radiations less refrangible than the red and others more refrangible than the violet rays; these form respectively the infra-red and ultra-violet portions of the spectrum.

EXPERIMENT.—Expose a thermopile to the radiations from a Bunsen flame and note that the galvanometer deflexion is greatly reduced by the interposition of a sheet of glass. Glass readily absorbs infra-red radiations.

EXPERIMENT.—Throw an arc-light spectrum in a darkened room on to a long strip of photographic printing-out paper (P.O.P.). After a short time the

paper becomes darkened, but not uniformly. The rays less refrangible than the blue have practically no photographic effect, while the maximum blackening occurs in the extreme violet or in the ultra-violet.

Owing to their power of promoting chemical changes such as this the rays in the ultra-violet are frequently called the actinic rays. If the light is made to pass through ruby glass before it reaches the P.O.P. there is no darkening as all rays except the red are absorbed. Hence it is possible to handle photographic plates and papers freely provided they are exposed only to red light. (There are special plates which form an exception to this rule.)

If a clean sheet of zinc is insulated and charged with negative electricity it is found that the charge escapes when the plate is illuminated with ultra-violet light. Other effects of the rays are studied in the next paragraphs.

As the infra-red rays suffer refraction in a prism they should be capable of being focussed by a lens. Tyndall showed this by a striking experiment. A solution of iodine in carbon bisulphide is opaque to the rays of the visible spectrum but transmits the infra-red radiations freely. Sunlight was passed through such a solution and the invisible rays were focussed by a large rock-salt lens on to a thin strip of blackened platinum which quickly became red hot under their influence. Similarly the rays from a Bunsen burner can be focussed on a thermopile, if the pile is moved sideways the galvanometer deflexion decreases to zero.

Phosphorescence and Fluorescence.—In Tyndall's experiment the platinum glows owing to its rise in temperature, but there are numerous cases where substances emit light without their temperature rising appreciably when they are exposed to suitable rays. Thus it has been known for hundreds of years that a diamond after exposure to sunlight glows for hours afterwards when examined in a dark room. Calcium sulphide (Balmain's luminous paint) behaves in a similar manner. This phenomenon is termed **phosphorescence.** Other substances, such as solutions of aniline dyes, glow only while they are exposed to the exciting rays; they are said to **fluoresce.** There is probably no sharp line of demarcation between the two phenomena.

Experiment.—Expose some calcium sulphide to sunlight, or to the rays from an electric arc, for a minute. If it is afterwards examined in a darkened room it appears to glow with a bluish light. Heat the substance gently over a

Bunsen flame; the light flashes out strongly but dies away in a few minutes. In order to make it phosphoresce again it must be re-exposed to the light.

Experiment.—Fill a large beaker with water and add a few drops of an alcoholic solution of eosine. Focus the rays from an arc on some point within the liquid; their path is made visible by the green fluorescent light. If an opaque screen is interposed the glow ceases at once.

It is probable that under the action of the light some instable chemical compounds are formed, which break up again and emit light during the process. There are several other methods of exciting phosphorescence and fluorescence, with these we shall not deal beyond giving a well-known application. When a surgeon examines a patient's hand with the assistance of Röntgen rays, he places it on a screen made of some substance which fluoresces under their action. Now flesh is transparent and bone is opaque to the rays, hence when they pass through the hand and fall on the screen fluorescence is produced except in the shadow of the bones. If the screen is viewed from the further side the position of a fracture or of any foreign body can easily be located.

Effect of Different Parts of the Spectrum.—It is easy to show that it is the most refrangible rays which cause phosphorescence.

Experiment.—Heat some Balmain's paint until its phosphorescence is destroyed, then spread it on a strip of paper in a darkened room and throw on to it an arc light spectrum. Mark the position of the yellow, green, etc., portions. After a few minutes' exposure cut off the light. It will be found that the maximum phosphorescence has been produced by the violet or ultra-violet rays, while the part exposed to the red does not glow at all.

A second important conclusion can be drawn from this experiment, viz. that the phosphorescent light is less refrangible, or of greater wave-length, than that which excited it. For the light emitted by the phosphorescing substance is bluish-green while the exciting rays were violet or even ultra-violet. This conclusion was supposed to hold in every case of phosphorescence and fluorescence and was known as **Stokes' law**, but numerous exceptions to it have been discovered in recent years. The change in the colour of the light can be vividly shown with canary glass (glass coloured with uranium oxide).

Experiment.—Concentrate the light from an arc on to a piece of canary glass, but remove the orange-yellow and green rays by interposing a sheet of dense blue cobalt glass. The exciting rays are now blue-violet, but the glass shines with a brilliant green light.

We have already seen that heat destroys phosphorescence ; this property can be used to demonstrate the presence of the infra-red rays.

EXPERIMENT.—Throw an arc-light spectrum on to some Balmain's paint which has previously been made phosphorescent. Where the red and infra-red rays fall the glow is more vivid for a few seconds, then dies away; the rise in temperature they produce causes a more rapid emission of the previously absorbed energy.

If a weak solution of eosine is exposed to light the path of the rays can be traced through the whole vessel, but as more eosine is added the fluorescence is concentrated at the side where the light enters. This is due to the strong absorption of the active rays by the solution, all the violet light is stopped within a short distance of the surface. If the transmitted light is thrown into a spectrum it will be found that the violet end is missing. The energy of the rays is absorbed and transformed into the fluorescent light.

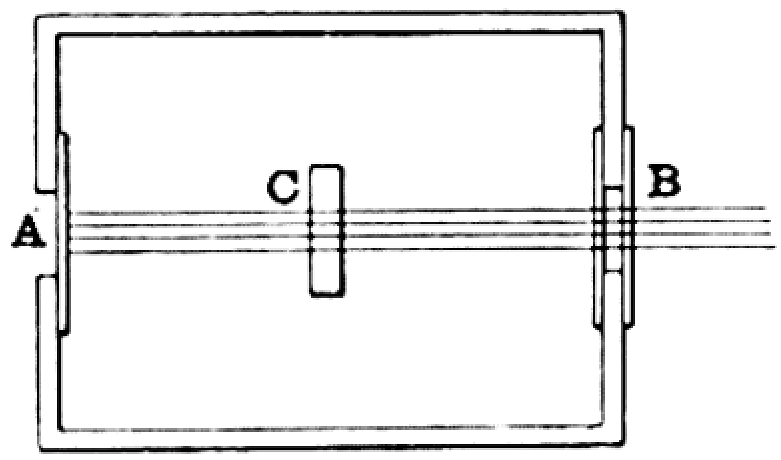


FIG. 126.—Stokes' Method of Detecting Fluorescence.

Stokes' Method of Detecting Fluorescence.—When the fluorescence is weak it may be masked by the dazzling effect of the exciting beam. Stokes overcame this difficulty by making use of the fact that the fluorescent light was more refrangible than that which caused it. The substance to be examined was placed at C (Fig. 126) in a box blackened on the inside and pierced with apertures at A and B. B was covered with two sheets of glass, one dark blue, the other green, these stopped all the red, yellow, and green light coming from a source placed just outside. A was covered with a yellow screen which did not transmit blue or violet light. Suppose the substance at C did not fluoresce, the blue light which entered at B was stopped by the screen at A and an eye placed near the latter received no light, thus the substance was invisible. If, however, there was fluorescence the blue light was transformed into green, and this could pass through the yellow glass, making the object visible.

Becquerel's Phosphoroscope.—If the phosphorescent light emitted by a substance disappears in a small fraction of a second after the exciting cause has been removed, special means will be required to

detect it. Becquerel made an apparatus, called a phosphoroscope, to overcome this difficulty. The substance to be examined is placed in a box at A (Fig. 127). A beam of light is admitted through a window at B, and the observer's eye is placed at C. D and E are circular metal discs which can turn rapidly round an axis XX, they are pierced with a number of holes which come past A at each revolution; the holes are not opposite each other, but a hole in D passes A completely just before the corresponding hole in E comes alongside it. Hence the sequence of events is as follows:—The substance is exposed to the light coming through D, this disc moves forward and cuts off the rays, shortly afterwards the observer can view the specimen through a hole in E. Since it is receiving no light at this instant it can be seen only if it phosphoresces. Evidently if the speed of the discs and the distance between corresponding holes on each are known, it is possible to measure the time of duration of the phosphorescence.

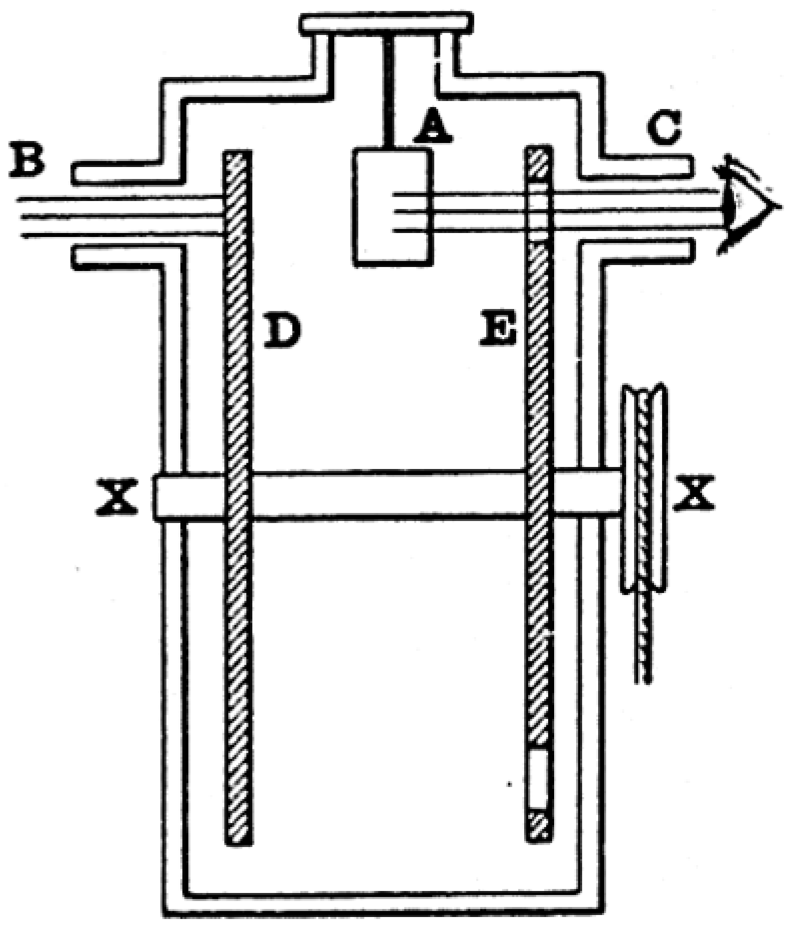


FIG. 127.—Becquerel's Phosphoroscope.

By this instrument Becquerel discovered cases in which the phosphorescence lasted for only a few thousandths of a second. The light from fluorescent liquids disappeared so quickly that he was unable to see them no matter how rapidly the discs were turned.

EXAMPLES ON CHAPTER XVIII

1. A prism, whose refracting angle A is small, is to be combined with a second prism so as to produce no deviation in a given beam of monochromatic light. If μ and μ' are the refractive indices of the prisms, calculate what must be the angle of the second prism.

2. White light falls on a prism of small angle; calculate the angular dispersion of two rays for which the refractive indices are μ and μ'.

3. For a prism of small angle prove that the dispersive power is equal to the dispersion of the extreme rays divided by the deviation of the yellow ray.

4. A composite ray consisting of light of two colours is incident on a plane surface; prove that the angular separation of the rays after refraction is $\theta = \frac{\mu - \mu_1}{\mu} \cdot \tan r$, where μ and μ_1 are the refractive indices for the rays and r is the angle of refraction for one of them. [θ is small, hence we may take $\cos \theta = 1$, $\sin \theta = \theta$.]

5. Define "dispersive power." Explain how to combine prisms so as to produce (*a*) Deviation without dispersion, (*b*) Dispersion without deviation of the mean ray. (L. '91.)

6. Find the dispersion produced by a thin prism of angle 15°, having a refractive index for red light of 1·5 and for violet light of 1·6. (L. '98.)

7. Describe with the aid of diagrams how a double prism can be constructed, (*a*) To give deviation without dispersion, and (*b*) Dispersion without deviation. (L. '08.)

8. If you were observing a small luminous object by a telescope not corrected for dispersion what appearances would present themselves on sliding the eye-piece in and out? (L. '81.)

CHAPTER XIX

PHOTOMETRY

Light as a Measurable Quantity.—Light being a form of energy is capable of physical measurement; thus if light waves fall on the blackened face of a thermopile they produce a rise of temperature proportional to the energy they carry and independent of the colour of the light, *i.e.* of the wave-length. It might appear at first sight that some such means could be used to compare the quantities of light emitted by different sources, but, unfortunately for the simplicity of such measurements, the physiological effects of light which are responsible for the sensation of sight vary greatly with the colour. The eye is most sensitive to yellowish-green; thus if a number of white screens could be illuminated with lights of different colours so that a thermopile gave equal deflexions at each, it would be found that those which were exposed to a yellow or greenish light would appear brightest to the eye. On this account the physical or calorimetric method is useless when the light emitting properties of two sources are to be compared, and it has to be replaced by a physiological or photometric test. The comparison of quantities of light under these conditions is termed "photometry."

Intensity of Illumination. Illuminating Power.—Before describing the methods of measurement some of the terms to be used must be defined. Let a screen be exposed to a source and suppose that a quantity of light Q falls in each second on a small area S, then Q/S, which is the quantity falling on unit area, is called the **intensity of illumination** of the small part S. If this quantity is the same, no matter where the area is taken, the screen is said to be uniformly illuminated. When the light is unequally distributed we must speak of the intensity of illumination at a point. Take a small area S round the point in question and find the value of the fraction Q/S when this area, and therefore Q, is made very small; this is the

intensity of illumination at the point. To render the definition of any value it must be made clear in what units Q is to be measured. The **unit of light** is the amount that falls on a screen 1 sq. cm. in area held perpendicular to the rays coming from a standard candle placed 1 cm. away. Standard candles are made from sperm, they weigh six to the pound and should burn at the rate of 120 grains per hour. Such a standard cannot be regulated easily and is unsatisfactory for other reasons; it is now replaced for practical purposes by a lamp, called the Hefner lamp, which burns amyl acetate. The flame of this is adjusted to a fixed height and the amount of light it then emits has been carefully compared with that given out by a standard candle. When a screen is placed at a distance of 1 cm. from a standard candle, so that the light falls on it normally, its intensity of illumination is unity.

The **illuminating power of a source** is the ratio of the quantity of

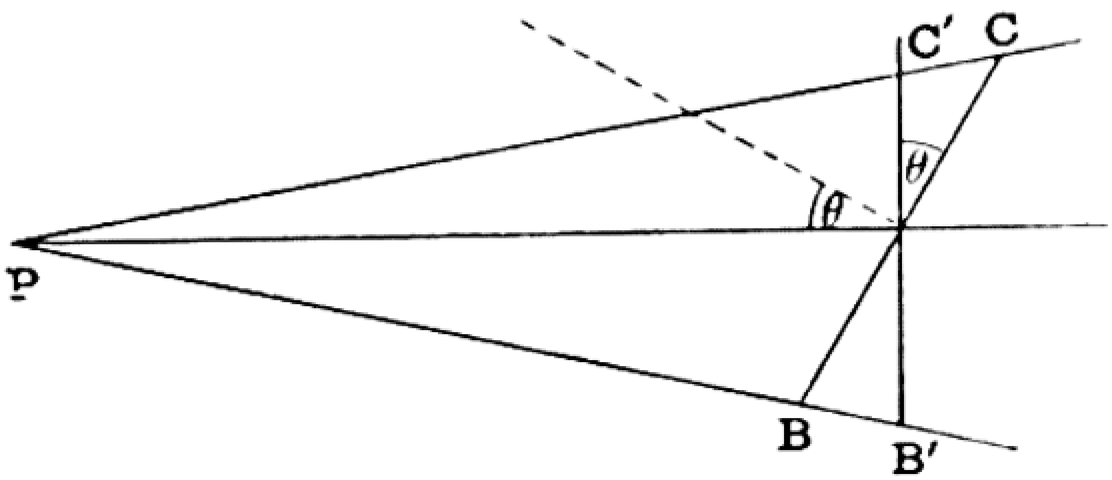


Fig. 128.

light it emits to the quantity emitted in the same time by a standard candle. Let a small screen be held at a cm. distance from a standard candle so that the light falls on it normally, its intensity of illumination is unity; the illumination will become two if we add another candle to the first, to three if three candles are used, and so on. Hence the illuminating power of a source is measured by the intensity of illumination it produces on a small screen 1 cm. away when the light is incident normally. The unit of illuminating power is the standard candle. An incandescent gas mantle when new has an illuminating power equal to about sixty candles.

The illumination of a screen varies with the angle at which the light is incident. Thus let BC (Fig. 128) be a small screen which is receiving each second a quantity of light Q from the source P; turn it round its mid-point through an angle θ to the position B′C′ where

the rays meet it normally. The quantity Q now falls on the area B′C′ and

$$\frac{\text{Intensity of illumination of BC}}{\text{Intensity of illumination of B}'\text{C}'}=\frac{\text{Q/area BC}}{\text{Q/area B}'\text{C}'}=\frac{\text{area B}'\text{C}'}{\text{area BC}}$$

But $B'C' = BC \cdot \cos\theta$, as may easily be seen by dropping perpendiculars on it from B and C, hence if I and I′ are the illuminations of BC and B′C′, $I = I' \cdot \cos\theta$. As the figure shows θ is the angle of incidence on BC, hence this result may be put in words :—If the intensity of illumination is I′ when the incidence is normal, it becomes $I' \cdot \cos\theta$ when the angle of incidence is θ.

Inverse Square Law.—It is important to determine how the illumination of a screen varies with its distance from the source of light. Consider a small source which is emitting light equally in all directions and imagine it to be placed at the centre of a sphere whose radius is R_1. If Q is the total quantity of light emitted the amount falling on unit surface of the sphere is $Q/4\pi R_1^2$; similarly if this sphere be replaced by another of radius R_2 the light received per unit area is $Q/4\pi R_2^2$. Denoting by I_1 and I_2 the intensities of illumination at the surfaces of the spheres,

$$\frac{I_1}{I_2}=\frac{Q/4\pi R_1^2}{Q/4\pi R_2^2}=\frac{R_2^2}{R_1^2}$$

that is, **the intensity of illumination of a surface varies inversely as the square of its distance from the source.** This law is the basis of most of the measurements in photometry. Suppose now the small source of light to be replaced by one which gives out so-called " heat " waves ; a like process of reasoning shows that the amount of radiant energy falling on a given area varies inversely as the square of its distance from the source. The inverse square law can be easily verified in this case with the help of a thermopile.

Experiment.—Connect the thermopile to a mirror galvanometer (p. 369), and use as source a small rose Bunsen burner. First break the circuit and read the zero of the galvanometer needle, then place the burner at distances of 50, 60, 70, etc., cms. from the pile and read the galvanometer deflexions directly they become steady. The thermopile should be screened from the radiations for a short time between each reading. The numbers in the following table were obtained in this manner; the first column gives the distance of the burner from the thermopile, the second the observed deflexion, and the third the deflexion calculated from the inverse square law assuming

the deflexion at 50 cms. to be correct. Thus if x is the deflexion that should be produced at 80 cms.

$$\frac{x}{194} = \frac{50^2}{80^2}$$

or

$$x = 76$$

The remaining numbers in the third column were calculated in the same way; it is seen that they agree very approximately with those found by experiment.

Distance in cms.	Observed deflexion.	Theoretical deflexion.
50	194	194
60	138	135
70	102	99
80	75	76
90	61	60
100	48	48

Another method of proving the inverse square law for radiant energy is shown in Fig. 129. A large metal vessel is filled with

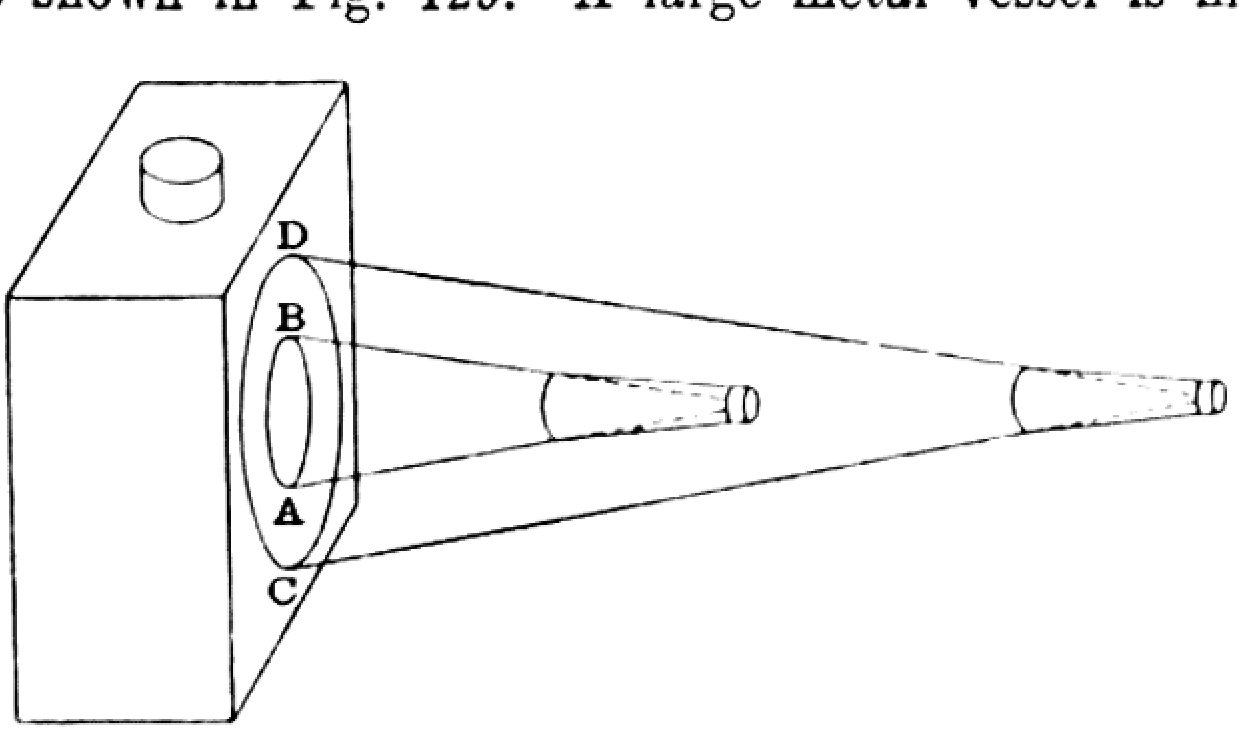


FIG. 129.—Method of proving the Inverse Square Law.

boiling water and a thermopile is placed to receive the radiation coming from one face. When the deflexion has been noted the thermopile is placed twice as far away; it is found that the deflexion is unaltered. In the first case the radiation comes from the area enclosed by the circle AB, in the second from that enclosed by CD; the diameter of CD is twice that of AB and therefore the areas are as 4 : 1. When the distance between the thermopile and

the source is doubled the quantity of energy received from a given area is reduced to one-quarter if the inverse square law is true, but this decrease is just balanced by using an area four times as large.

Photometry.—An instrument which is used to compare the candle-powers of different sources is called a photometer. It is found that the eye is incapable of judging the relative intensities of illumination of two surfaces when these are different, but two observers will agree in a fairly consistent manner in estimating when two surfaces are equally illuminated. Hence in the comparison of illuminating powers it is arranged that two neighbouring patches on a screen are illuminated, one by each source, and the distances of the sources are adjusted until the patches are equally bright. Let I_1 and I_2 be the candle powers of the two sources, R_1 and R_2 their distances from the screen when the two patches are equally bright.

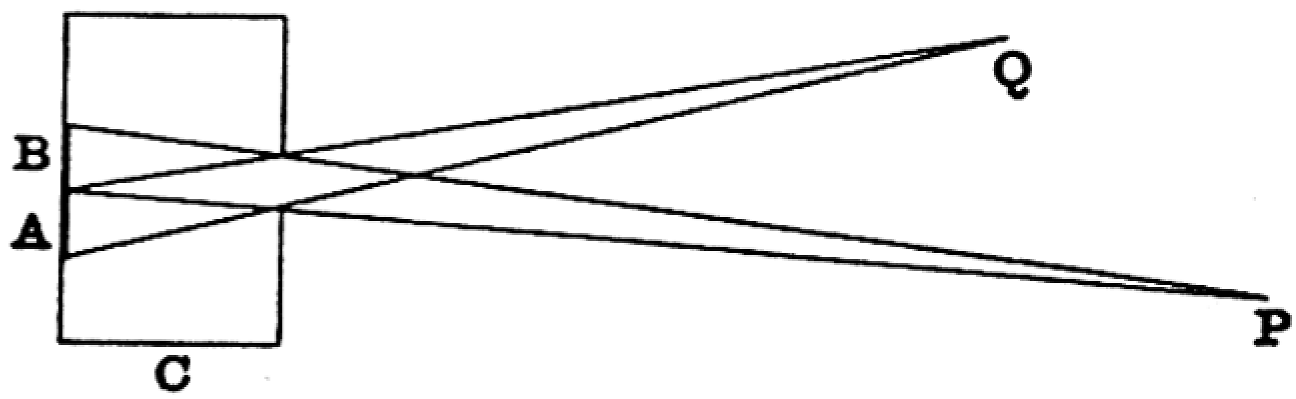


FIG. 130.—Simple Photometer.

From definition the intensity of illumination is I_1 when the first source is 1 cm. away from the screen, hence whence the distance is R_1 the illumination is I_1/R_1^2, by the inverse square law. Similarly the illumination due to the second source at a distance R_2 is I_2/R_2^2, hence

$$\frac{I_1}{R_1^2} = \frac{I_2}{R_2^2}$$

or

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2}$$

i.e. the illuminating powers are *directly* proportional to the squares of the distances from the screen when the two patches appear equally illuminated. A simple photometer can be made as follows: A piece of brass tube, C (Fig. 130), 5 cms. in diameter and 3 cms. long is closed at one end with thin tissue paper, the other end is covered with a brass plate in which has been cut a vertical slit 2 cms. high and 1 cm. broad. The two sources to be compared are arranged to

be at the same height as the slit which is turned towards them. Two rectangular patches of light are seen on the tissue paper ; by moving one of the sources it can be arranged that these patches are in contact and equally bright. (It is found easier to judge of this equality when they are in contact.) Then A receives light from Q only and B is lighted by P alone, hence

$$\frac{I_Q}{I_P} = \frac{QA^2}{PB^2}$$

Rumford's Photometer.—Another method, which is practically a reversal of the one just described, is here used to get the two patches. In front of a white screen is placed an opaque rod C (Fig. 131), P

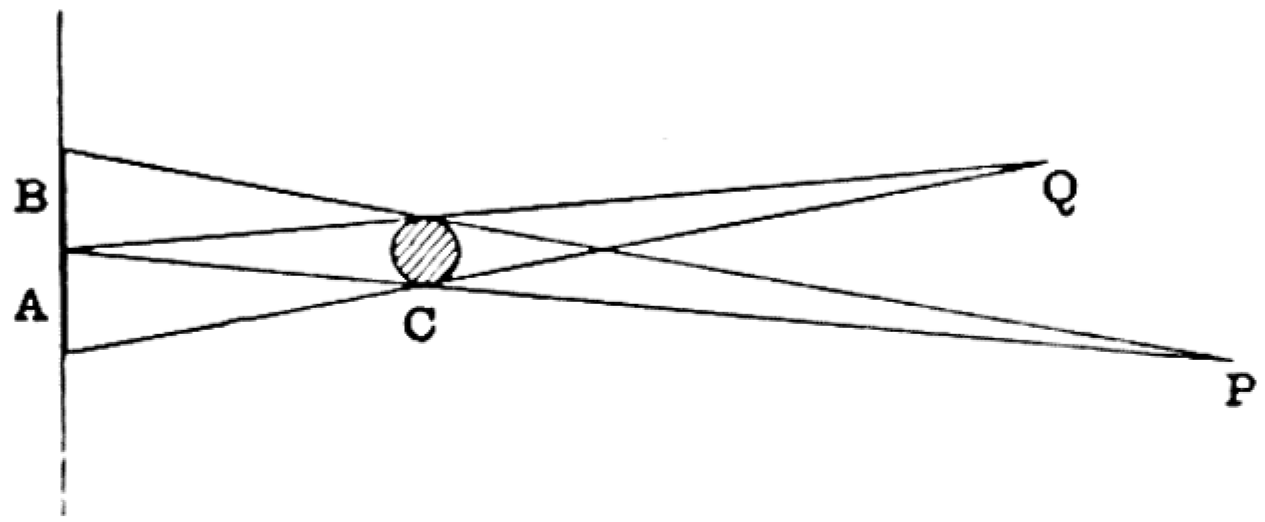


FIG. 131.—Rumford's Photometer.

and Q represent the sources of light. Two shadows of the rod are cast on the screen and, as the figure shows, A receives light from P alone and B from Q alone. One source is moved until the shadows are in contact and are equally dark, then the illumination of A is I_P/PA^2, while that of B is I_Q/QB^2,

hence
$$\frac{I_P}{I_Q} = \frac{PA^2}{QB^2}$$

Bunsen's Grease-spot Photometer.—In some form or other this is the one most frequently used.

EXPERIMENT.—Run a drop of candle grease about the size of a shilling on to a piece of filter paper and when it has set remove most of it with a knife. Hold the paper between the eye and a window ; the grease-spot is brighter than the remainder because it transmits more light. If it is viewed from the side from which the light is coming the grease-spot is darker than the surrounding paper, for as it transmits more light than its surroundings there is less remaining to be diffusely reflected to the eye.

Such a grease-spot arrangement is fixed on an optical bench and the sources to be compared are placed on opposite sides of it. When the illuminations are the same on each side the light transmitted through the grease-spot from right to left is just balanced by that coming from left to right and the spot disappears. To make the adjustment the lamps are fixed at the ends of the bench, which is two or more metres long, and the spot is moved to and fro between them. As in the preceding cases, the illuminating powers are directly proportional to the squares of the distances between the screen and the sources of light. In accurate work the measurements are made in a room with dull black walls, and the spot is placed in a box to screen it from all light except that coming directly from a source under test. A pair of mirrors are also fixed at M_1M (Fig. 132)

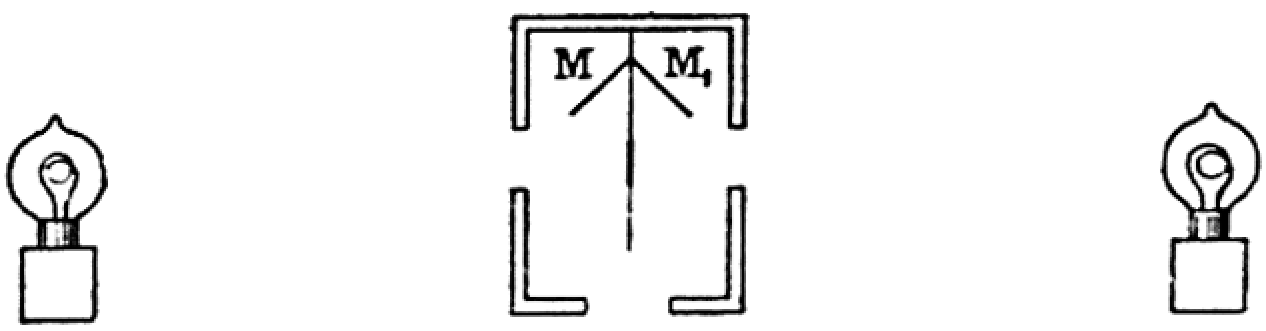


FIG. 132.—Grease-spot Photometer.

so that each side of the screen can be seen with the same eye, otherwise a difference in the eyes might cause error.

EXPERIMENT.—*Use the Bunsen photometer to prove the inverse square law.* Fix one candle on one side of the spot and a group of four, placed as close together as possible, on the other. Move the screen until the spot disappears, the group of four will then be found to be approximately twice as far from the screen as the single candle. If the inverse square law is true each candle in the group produces at the grease-spot an illumination equal to $1/2^2$ of that produced by the single candle. Hence the illuminations should be of equal intensity when four candles are used at the greater distance, and this the experiment shows to be the case. Another, and more satisfactory method, is as follows :—Compare the candle-powers of two sources, then move them closer together and compare again at various distances apart. As $I_1/I_2 = R_1^2/R_2^2$ the ratio of the squares of the distances from the grease-spot should be constant ; this equation is a direct consequence of the inverse square law, hence if it can be proved to hold the inverse square law is true.

EXAMPLES ON CHAPTER XIX

1. How would you determine experimentally the quantity of light reflected at different angles by a piece of plane glass ? (L. '98.)

2. A 10 c.p. lamp is placed 1 metre from a surface. At what distances must gas flames of 14 and 16 c.p. respectively be placed so as to produce an equal illumination of the surface ? (L. '03.)

3. Two lamps A and B are placed 60 cms. and 80 cms. respectively from a Bunsen photometer and it is found that the grease-spot disappears. Find the ratio of the candle powers. When a sheet of glass is interposed between lamp B and the photometer it is found that, to produce a balance, this lamp must be displaced 10 cms. Find what percentage of the incident light is reflected by the glass.

4. Light from a 32 c.p. lamp falls on a silvered mirror and is reflected thence to a grease-spot photometer. The distance from lamp to screen *viâ* the mirror is 150 cms. If the mirror reflects 90 per cent. of the light falling on it where must an 8 c.p. lamp be placed in order that the grease-spot shall disappear ?

5. Two lamps of 8 and 32 c.p. are fixed 120 cms. apart. Where, on the line joining them, must a screen be placed so as to be equally illuminated by each ?

CHAPTER XX

THE EYE AND OPTICAL INSTRUMENTS

The Photographic Camera.—One of the simplest applications of the principles explained in the foregoing chapters is the photographic camera. By means of a convex lens a real image of the object to be photographed is focussed on to a glass plate whose surface is coated with certain silver salts. Fig. 113 *a*, p. 191, shows the path of the rays. The blue-violet and ultra-violet rays produce chemical changes in the salts which, by treatment with various solutions, are made to produce a permanent record of the image. To obtain good definition an achromatic lens must be used. The linear size of the image is approximately proportional to the focal length of the lens. (See Astronomical Telescope, p. 235.)

The Optical Lantern.—This is an apparatus for throwing on to a screen an enlarged image of an object such as a lantern slide. The optical parts are shown in Fig. 133. AB is the slide, E the achromatic projecting lens, and A′B′ the image. Only those rays are drawn which pass through the optical centre of the lens. Owing to the magnification, which is equal to v/u (p. 193), the light which starts from the object is spread over a much larger area in the image, it is therefore necessary that the slide should be strongly illuminated. For this purpose a powerful source of light, such as an arc, is placed at D and the divergent rays are concentrated on to the slide by two large convex lenses, called the condenser, at C. Where a weaker source has to be used it is an advantage to place a concave mirror at F, then the light travelling to the left, which would otherwise be lost, is reflected back to the condenser and adds to the illumination of the slide. If a piece of apparatus is to be projected it is clear from the figure that its image will be inverted; to obtain the image upright an erecting prism is used. This is a right-angled isosceles glass prism silvered on its largest face; it is placed between the

projecting lens and the screen. Fig. 134 shows how, by reflexion, the relative positions of the rays are inverted, resulting in the formation of an upright image.

The Sextant.—This is an instrument which is used to measure the angle subtended at the eye by two distant objects. It is used by sailors to determine the sun's altitude, *i.e.* the angle which the line

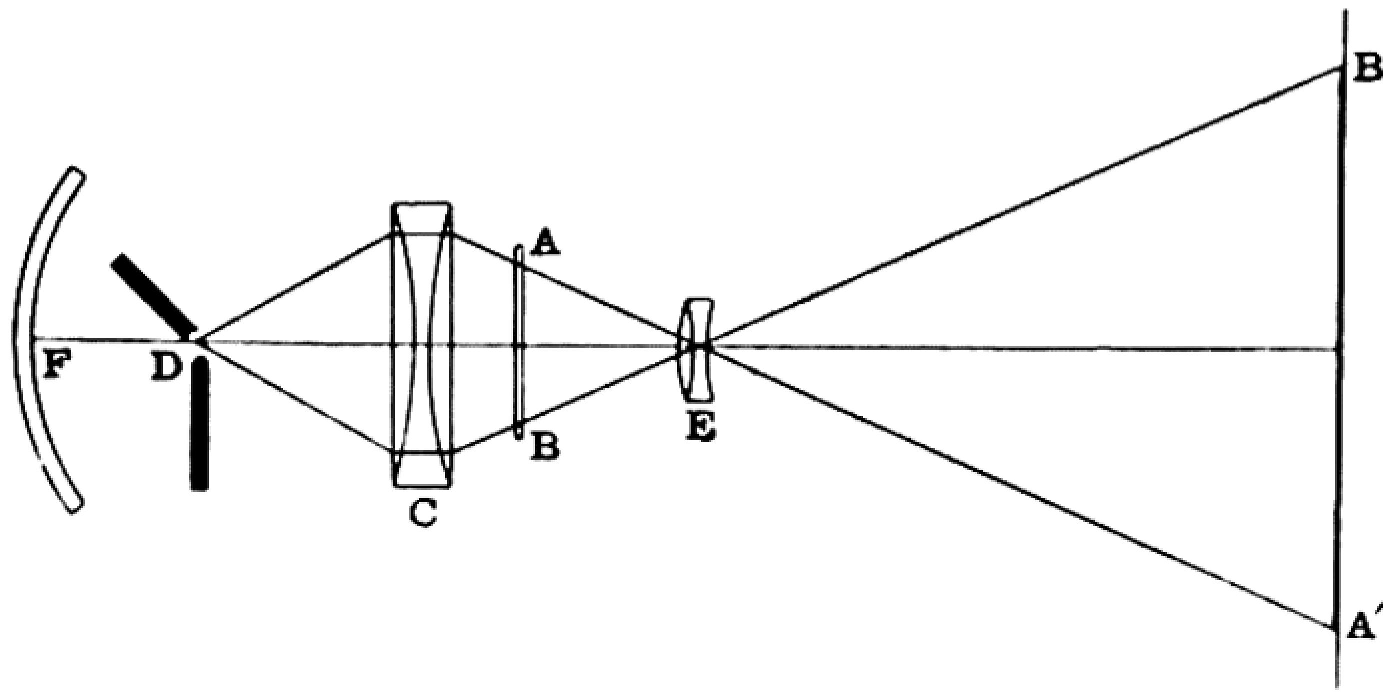


FIG. 133.—Optical Lantern.

going from the observer to the sun makes with the horizontal; this is used in calculating their position when out of sight of land. AB (Fig. 135) is a graduated arc of a circle whose centre is C. A movable radius carries a vernier at D and a small vertical mirror at C. A second vertical mirror whose lower half only is silvered is fixed at E, and at T is a telescope. When the mirrors are nearly parallel suppose the telescope to be directed to some distant object such as a star. A ray FE comes through the unsilvered part of E and enters the telescope, while a parallel ray OC is reflected from C to E and thence also to the telescope. Two images are thus seen and these will coincide when the mirrors are made exactly parallel. The vernier D should then stand at the scale zero at B; if it does not a small correction must be made in subsequent readings. Next let it be required to measure the angle subtended at the observer by two objects situated along CM and EF respectively. That along EF is viewed directly through E, and the arm CD is turned until the rays coming

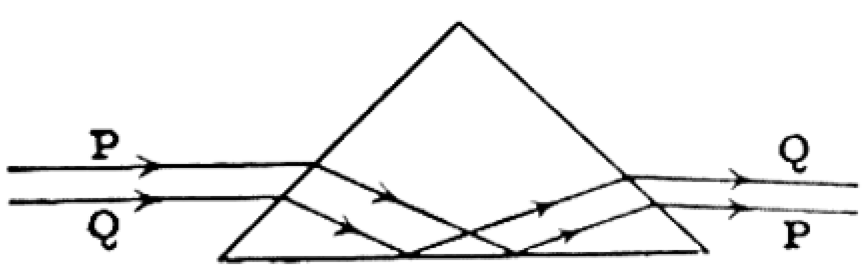


FIG. 134.—Erecting Prism.

in the direction MC also enter the telescope after reflexions at C and E. When the two images coincide the angle required is twice $\angle$BCD. For suppose the path of the light to be reversed; when the mirrors were parallel the ray TEC would be reflected along CO parallel to EF, while now it is reflected along CM. But $\angle$MCO is twice the angle through which the mirror C has been turned (p. 144), *i.e.* $\angle$MCO is twice $\angle$BCD, and $\angle$MCO is the angle between MC and EF. To save calculation each single degree on the graduated arm is marked as two so that the reading gives the angle directly.

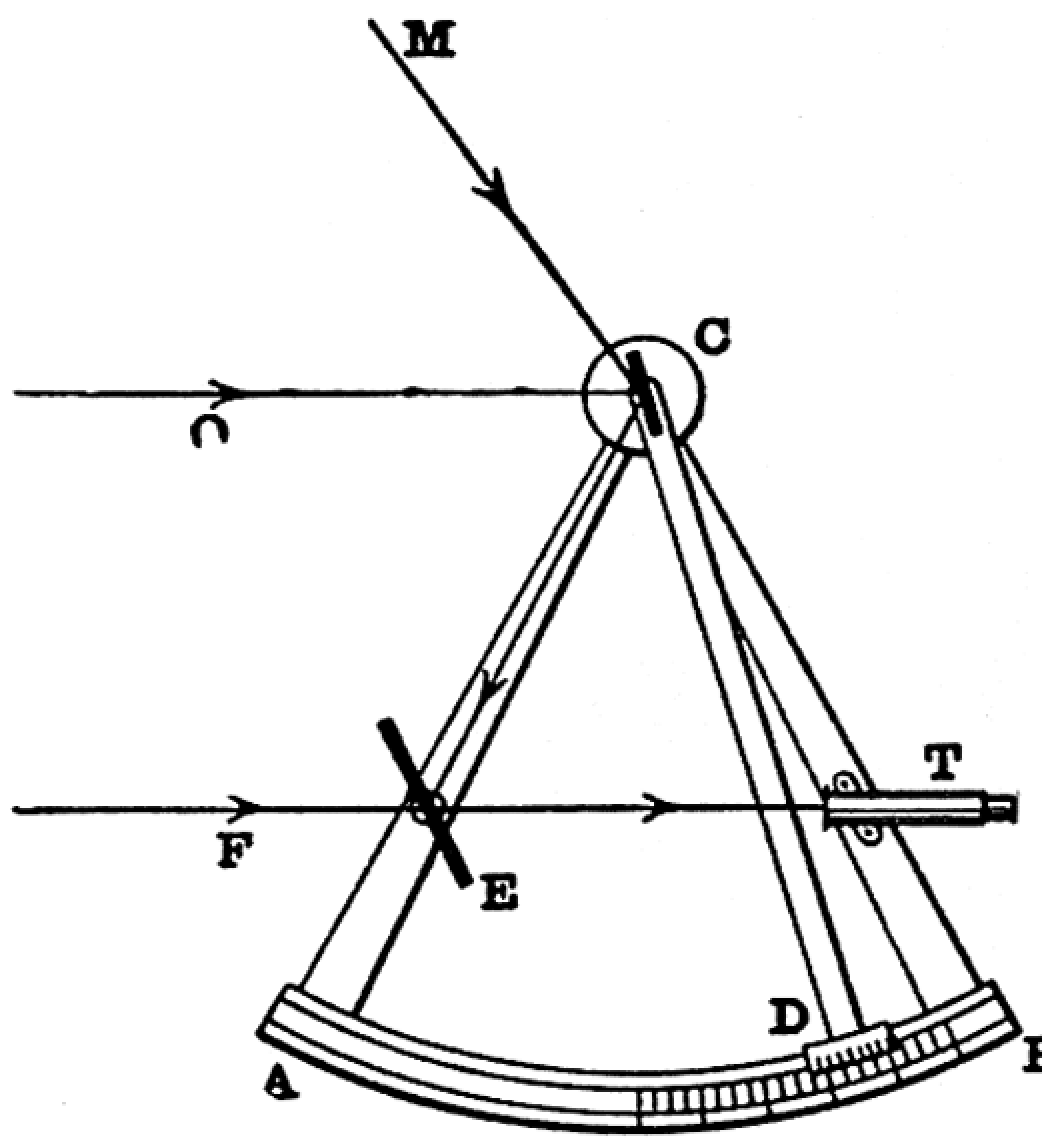


FIG. 135.—The Sextant.

The Eye.—The eye is shown in diagrammatic section in Fig. 136. It is approximately spherical in shape and is surrounded, except in front, by an opaque outer protective layer S called the sclerotic. In front the outer layer is transparent and bulges slightly to form the cornea D. Inside the sclerotic is a second layer, the choroid C. Immediately behind the cornea the choroid is coloured; this portion I is the iris, it gives the characteristic colour to the eye. The iris is pierced centrally by a circular aperture P, named the pupil, whose size varies involuntarily with the amount of light passing through it. In a weak light it expands, while in a strong light it contracts, thereby protecting the inner, sensitive, portions of the eye from injury. The

optic nerve N passes through the two outer layers at the back of the eye and spreads out into a thin tissue of nerve fibres which forms a lining to the choroid. This layer is the retina R. Behind the pupil there comes the crystalline lens L, a double convex lens whose front and back faces have radii of curvature of about 11 mm. and 8 mm. respectively. Between lens and cornea is a quantity of watery fluid called the aqueous humour, A, and between lens and retina there is more fluid, the vitreous humour, V. Optically the eye acts just like a camera ; the lens forms on the retina a real, inverted, image of external objects, the appropriate sensations are then transmitted to the brain by way of the optic nerve. For a normal eye at rest the

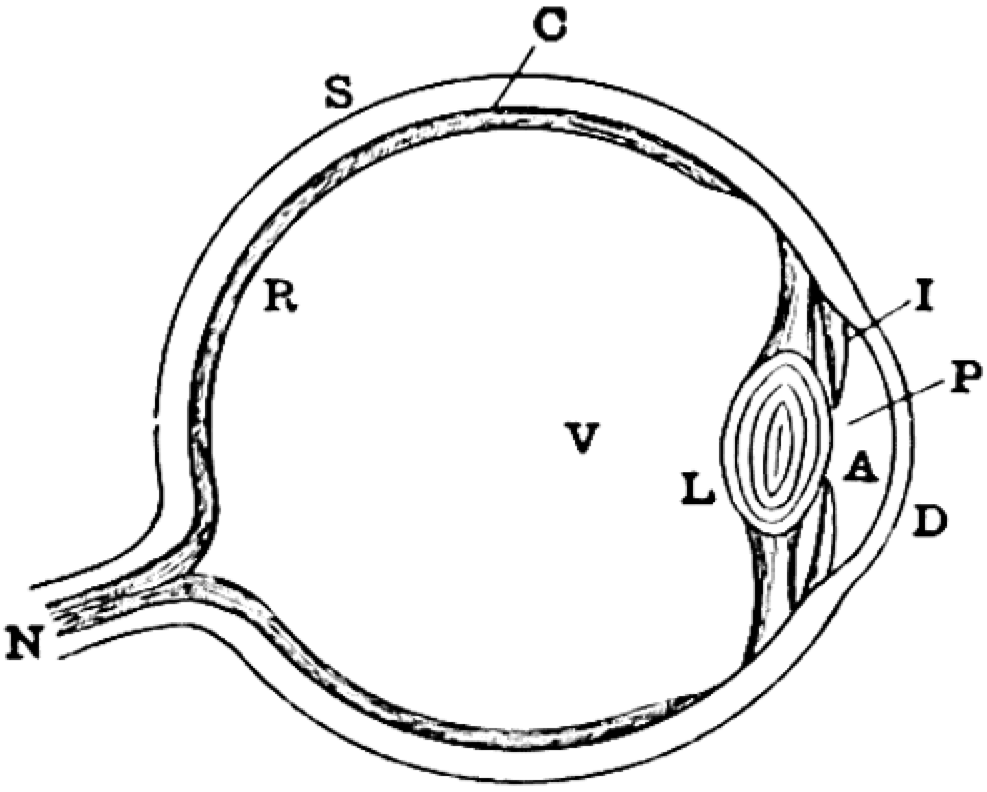


FIG. 136.—The Eye.

principal focus of the lens falls on the retina, so that distant objects are sharply focussed. It follows that the images of near objects will fall behind the retina unless a lens of shorter focal length is used. This is counteracted by the process of *accommodation*. By means of muscles attached to the lens the radii of curvature of its faces can be reduced and hence its focal length made less. As the details of neither very near nor very distant objects can be seen distinctly there is some intermediate position at which vision is most distinct. This is called the least distance of distinct vision ; for normal eyes it is about 25 cms.

EXPERIMENT.—*To prove that images on the retina are inverted.* Make a pin-hole in a piece of cardboard and hold it about 3 cms. in front of the eye facing a window. Hold a pin with its head uppermost close to the eye and move it across from right to left, it appears to cross the hole from left to right.

Also when the pin-head is opposite the hole it appears to be inverted. Since the pin is close to the eye its image must be upright (Fig. 113 *b*), hence it is clear that in interpreting our sensations we regard an inverted image on the retina as if it were upright.

Defects of Vision. Spectacles.—The most frequent defects are (1) Short sight, (2) Long sight, (3) Astigmatism.

(1) *Short sight or myopia.*—If the distance between lens and retina is too great parallel rays are brought to a focus in front of the retina and distant objects appear indistinct. Such an eye is said to be myopic or short-sighted. As an object moves nearer its image approaches and finally falls on the retina without accommodation. The distance c at which this occurs is the maximum distance at which the short-sighted person can see distinctly. At closer range the accommodating mechanism comes into play, if this is normal the person can see clearly the details of an object which is nearer than the usual 25 cms. To remedy the defect a lens is required which shall make parallel rays appear to diverge from a point c cms. in front of the eye, *i.e.* a concave lens whose focal length is c cms., distant objects can then be focussed distinctly. Suppose, for example, that the range of vision is from 8 cms. to 20 cms.; a lens of focal length 20 cms. is required. Let x be the nearest point of distinct vision through the lens; then a point x cms. away must appear to be 8 cms. distant, or $v = 8$ when $u = x$.

$$\therefore \frac{1}{8} - \frac{1}{x} = \frac{1}{20}$$

and $$x = 13 \text{ cms.}$$

i.e. the range instead of being from 8 – 20 cms. is now from 13 cms. to infinity.

(2) *Long sight or hypermetropia.*—In cases of long sight the focal length of the lens is too large and parallel rays are brought to a focus behind the retina. Consequently the accommodating mechanism must always be in use and the least distance of distinct vision is greater than 25 cms. It is only convergent pencils that can be properly focussed with the eye at rest. The glasses to be used vary with the purpose for which they are required. Suppose, for example, that the least distance of distinct vision is 40 cms., and that rays converging to a point 20 cms. behind the eye can be sharply focussed with relaxed accommodation. Then for reading purposes the least distance of distinct vision must be made normal, while for outdoor

work parallel rays must be clearly focussed with the eye at rest. In the first case if an object is 25 cms. distant its image must appear 40 cms. away on the same side of the lens, *i.e.* $v = 40$, $u = 25$,

$$\therefore \frac{1}{40} - \frac{1}{25} = \frac{1}{f}$$

and

$$f = -66 \text{ cms.}$$

or a convex lens of 66 cms. focal length is required.

For outdoor work parallel rays must be made to converge to a point 20 cms. behind the eye, *i.e.* a convex lens for which $f = 20$ cms. is required. With increasing age the accommodating mechanism becomes imperfect and the focal length of the eye lens cannot be altered sufficiently to allow of near objects being sharply focussed. This defect is called *presbyopia*. For example, suppose the nearest distance of distinct vision is 40 cms.; for reading purposes this has to be reduced to the normal, and, from the example above, it is seen that a convex lens of 66 cms. focal length is required.

(3) *Astigmatism.*—In some eyes the surfaces of the cornea or the lens do not form parts of spheres, generally a vertical section shows a stronger curvature than a horizontal one. In such cases horizontal and vertical lines are brought to a focus at different distances and the eye is said to be astigmatic. The necessary correction is obtained by the use of lenses which are portions of cylinders.

In some cases the defect may not be the same for both eyes and different lenses must be employed.

Magnifying Power.—Our estimate of the size of an object depends not only on its actual dimensions but also on its distance. Perspective is based on this fact. Thus the metals of a railway appear to approach each other as they recede in the distance, the moon appears to be as large as the sun although it is known to be much smaller, and the height of a distant church spire increases relatively to ourselves as we get nearer to it. In each case we base our estimate on the angle that the body subtends at the eye. In most cases we unconsciously correct our estimate by making allowance for the distance factor; when this is impossible or difficult our judgment may be far from the truth; for example, in the case of the sun and moon just mentioned, or in a landsman's estimate of the length of a ship at sea. Similarly when we view an object through a telescope the image may appear greatly magnified, although calculation may

show that its linear dimensions are less than those of the object. When an optical instrument is said to magnify we mean that it causes the image to subtend at the eye a greater angle than the object does. But the latter angle will vary with the observer's position; in order therefore to get an exact definition of magnifying power the circumstances must be stated under which the object is viewed directly. Any optical arrangement which apparently increases the angle subtended at the eye by a distant object is called a telescope. With such instruments, as the object is distant, the angle it subtends when seen with the naked eye will be practically independent of slight motions of the observer. Hence we get the following definition:— **The magnifying power of a telescope is the ratio of the angle subtended at the eye by the image to that subtended by the object as seen directly at its actual distance.**

With microscopes or reading lenses the object is much nearer; for the instrument to be any advantage it must cause the image to subtend a larger angle than the object would do, supposing the latter to be placed in its most favourable position, *i.e.* at the nearest distance of distinct vision. Hence the magnifying power of a microscope or reading lens is defined as **the ratio of the angle subtended at the eye by the image to that subtended by the object seen directly at the nearest distance of distinct vision.**[1]

The student must not confuse these definitions with that of the linear magnification given on p. 156. In that case we were concerned only with linear dimensions, while here we are dealing with angular magnification. It would perhaps conduce to clearness if the terms linear and angular magnification were always used to differentiate the two quantities.

The Simple Microscope or Reading Lens.—It has been seen (p. 193) that a convex lens produces a virtual, erect, and enlarged image of any object which is nearer to it than its principal focus. This is the purpose of a reading lens or magnifying glass. Other things being equal, the image subtends the largest angle at the eye when the latter is placed close to the lens. Let AB, A′B′ (Fig. 137) represent object and image respectively, and let us suppose that the latter is at the nearest distance of distinct vision D. Let $OP = u$. Then the angle subtended by the image $= A'B'/D$, and the angle

[1] These angles in practice are small, hence they are given in circular measure by dividing the linear dimensions of the image or object by the corresponding distance from the eye.

subtended by the object when it is placed at the nearest distance of distinct vision $= AB/D$.

Hence the magnifying power $= A'B'/D \div AB/D = A'B'/AB$

But
$$\frac{A'B'}{AB} = \frac{v}{u} = \frac{D}{u}$$

and
$$\frac{1}{D} - \frac{1}{u} = -\frac{1}{f}$$

$$\therefore 1 - \frac{D}{u} = -\frac{D}{f}$$

and
$$\frac{D}{u} = 1 + \frac{D}{f}$$

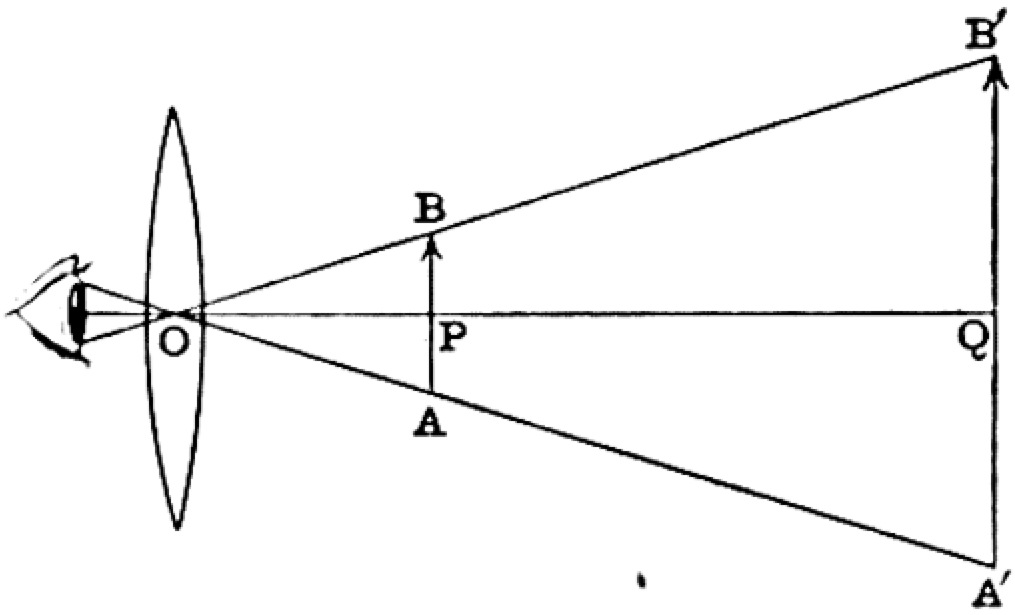


FIG. 137.—Reading Lens.

Hence the magnifying power $= 1 + \frac{D}{f}$, where f is the *numerical* value of the focal length.

EXAMPLE.—The magnifying power of a reading lens of 5 cms. focal length is $1 + \frac{25}{5} = 6$. (See also Example 5, p. 241.)

Compound Microscope.—In the compound microscope (Fig. 138) two convex lenses are used and the magnification takes place in two steps. The lens O which the light first enters is called the objective, that by which the final image is formed is the eye-piece E. The object AB, which must be well illuminated, is placed at a distance from O slightly greater than the focal length of this lens and a real, inverted, and magnified image is formed at PQ. The rays pass on through the eye-piece and form the final image A′B′ as in the last paragraph. Let U be the distance of the object and V that of

its image PQ from O, u and v the distances of PQ and A′B′ respectively from E, and f the focal length of the latter lens. Then, from definition,

$$\text{the magnifying power} = \frac{A'B'/v}{AB/D} = \frac{A'B'}{AB} \cdot \frac{D}{v} \quad . \quad . \quad (1)$$

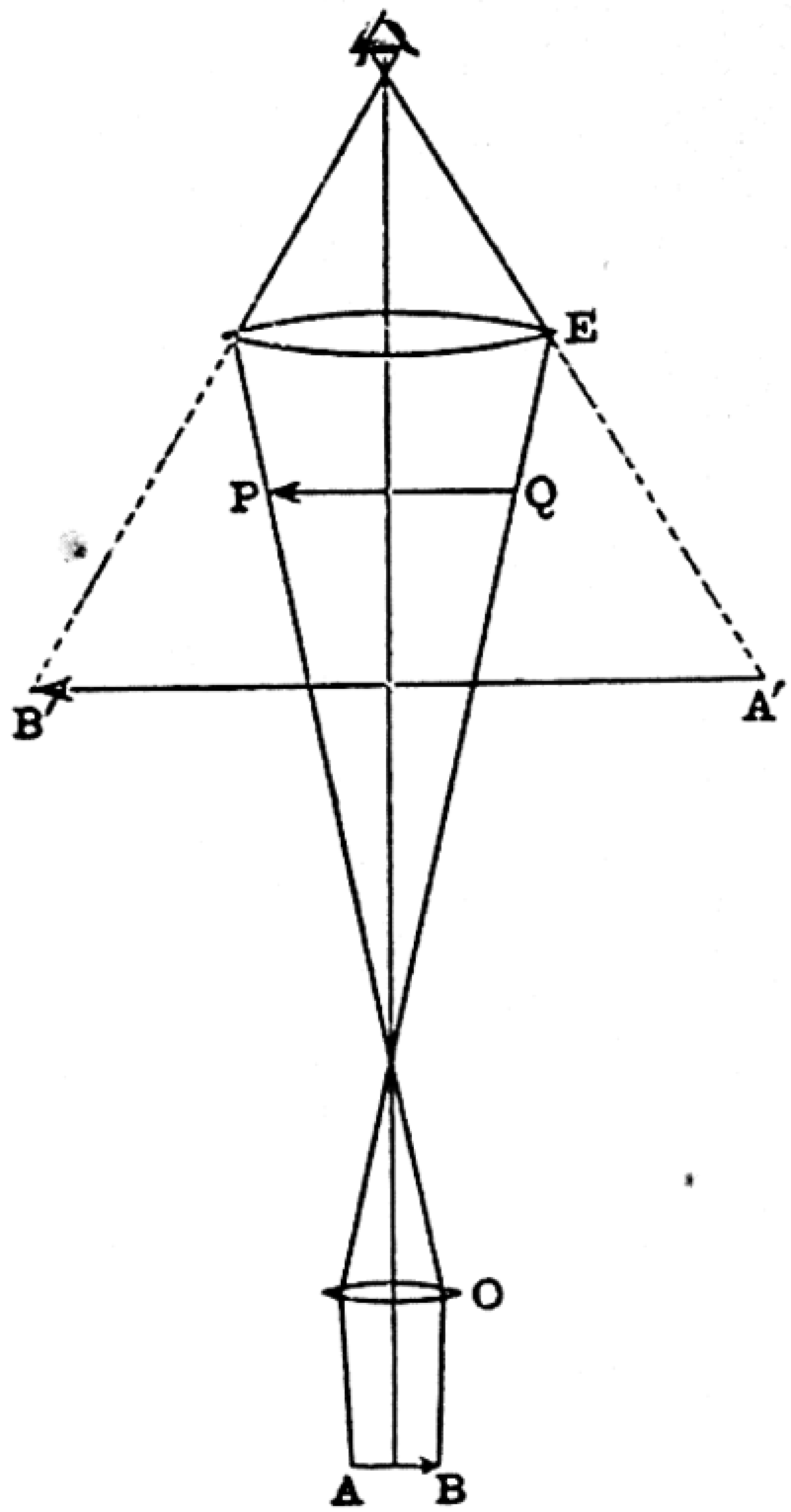


FIG. 138.—The Compound Microscope.

Also
$$\frac{PQ}{AB} = \frac{V}{U}$$

and
$$\frac{A'B'}{PQ} = \frac{v}{u}$$

$$\therefore \frac{A'B'}{AB} = \frac{V}{U} \cdot \frac{v}{u}$$

Substituting in (1) we get the magnifying power $= \dfrac{V}{U} \cdot \dfrac{D}{u}$

If the final image is at the distance D we get, as in the last paragraph,

$$\frac{D}{u} = 1 + \frac{D}{f}$$

$$\text{and magnifying power} = \left(1 + \frac{D}{f}\right)\frac{V}{U}$$

In practice there is found to be less strain on the eye if the image is viewed with relaxed accommodation, in that case A′B′ is at infinity, $u = f$,

$$\text{and the magnifying power} = \frac{V}{U} \cdot \frac{D}{f}$$

In both cases the magnifying power is increased by reducing f. If an objective of shorter focal length is used the distance U must be reduced in order that the real image PQ may be formed in its previous position, and the expressions just obtained show that the magnifying power is increased. Hence each lens must have a short focal length. If a cross-wire or scale is to be placed in the eye-piece for purposes of measurement (p. 42) it is fixed in the plane PQ

An actual microscope is a much more complicated affair than the simple form described.

The Astronomical Telescope.—Like the microscope this consists of two convex lenses called respectively the objective and eye-piece. As the object under observation is usually very distant the rays coming from any point on it are parallel, and the real, inverted, image PQ formed by the objective is situated in the focal plane of this lens, *i.e.* OP = F, where F is the focal length of the lens. This image is magnified in the usual way by the eye-piece. Fig. 139 shows the path of the rays. It is easy to see that F should be large if large magnifications are required, a difference in this respect from the microscope. For, in the notation of the last paragraph, calling the object AB,

$$\frac{PQ}{AB} = \frac{F}{U + F} \text{ (p. 193)}$$

and, as F is very small compared with U, this is practically F/U, or the linear magnification produced by the objective is proportional to its focal length. To calculate the magnifying power of the instrument notice that the angle which the object AB subtends at the eye is practically equal to that which it subtends at the objective, since OE is small compared with U.

Hence angle subtended by object $= AB/U$
and the angle subtended by image $= A'B'/v$

$$\therefore \text{ magnifying power} = \frac{A'B'}{AB} \cdot \frac{U}{v}$$

But
$$\frac{PQ}{AB} = \frac{V}{U}$$

and
$$\frac{A'B'}{PQ} = \frac{v}{u}$$

$$\therefore \frac{A'B'}{AB} = \frac{V}{U} \cdot \frac{v}{u}$$

$$\text{and the magnifying power} = \frac{A'B'}{AB} \cdot \frac{U}{v} = \frac{V}{u}$$

If the image is to be viewed with the unaccommodated eye, PQ must be at the focus of the eye-piece, *i.e.* $u = f$, also $V = F$,

$$\therefore \text{ magnifying power} = \frac{F}{f}$$

A large F means that the instrument, as sketched above, will have an unwieldy length; to overcome this difficulty the rays in some field glasses are reflected up and down the tube three times before they meet at PQ, hence a shorter tube can be used. The objective should be achromatic and it should have a large diameter in order that it may throw a considerable amount of light into the

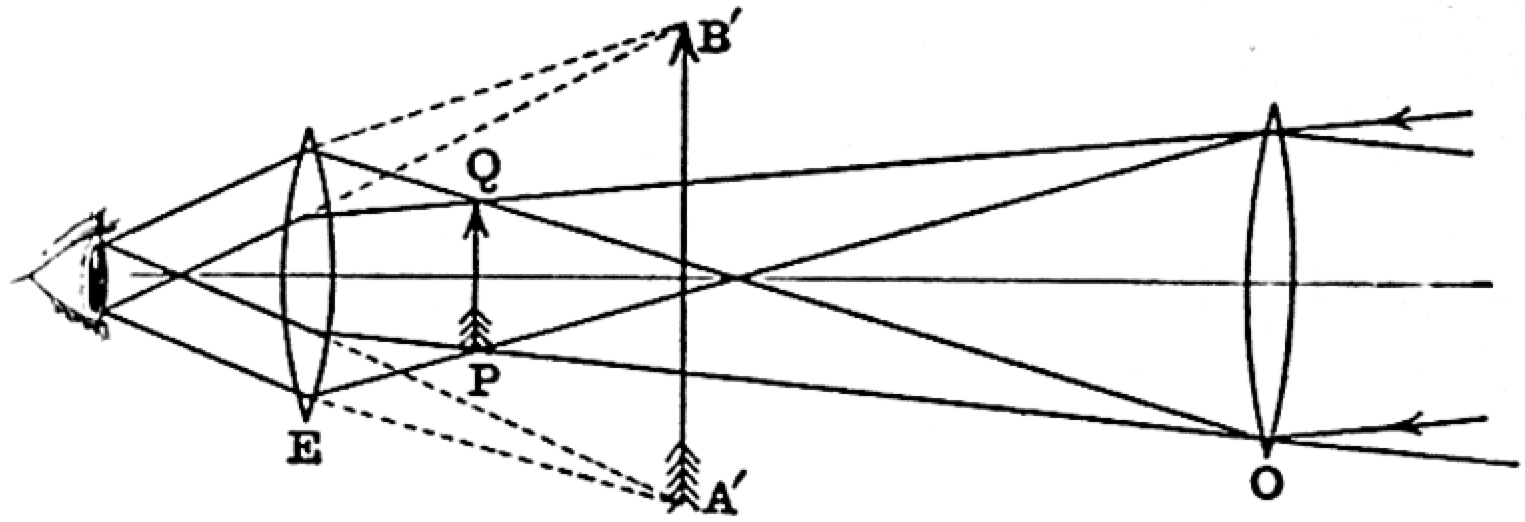


FIG. 139.—The Astronomical Telescope.

image PQ. Some telescopes used by astronomers have objectives whose diameter is 3 feet or more. Any cross-wires used in measurements are placed in the plane of PQ.

Galileo's Telescope (Opera glass).—In addition to its length the astronomical telescope has the disadvantage that the final image is

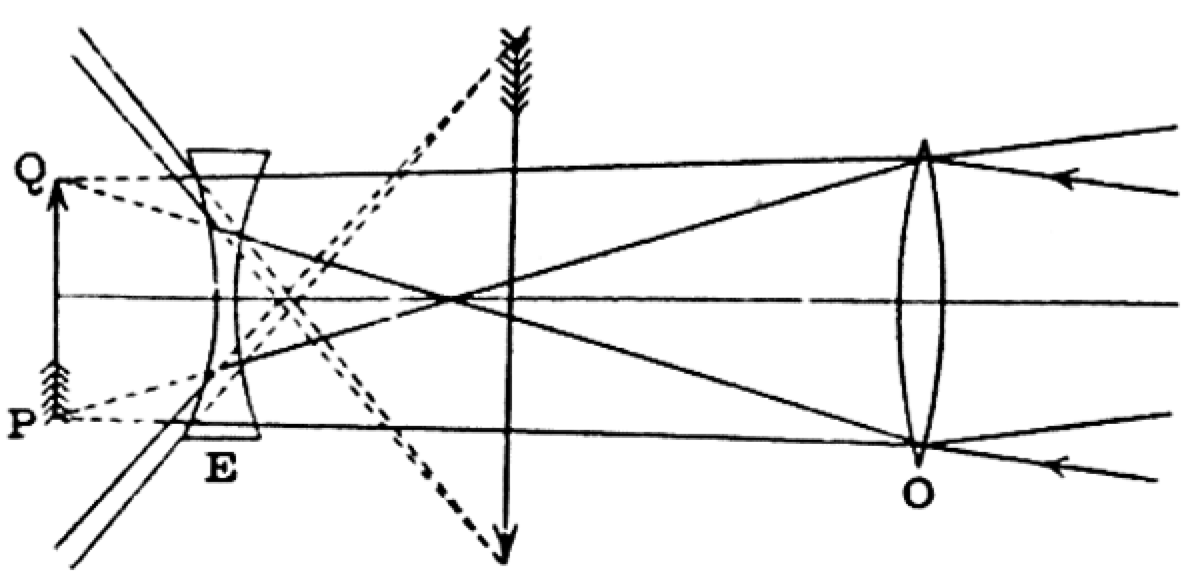


FIG. 140.—Galileo's Telescope.

inverted. Each of these inconveniences is avoided in the Galilean telescope (Fig. 140), which can be used where large magnifying power is unnecessary. The object-glass is a convex lens O, which, as in the last case, would form a real, inverted, image at PQ, but before the rays can come to a focus they pass through a concave lens E which constitutes the eye-piece. The distance EP is slightly greater than or equal to f, and the rays instead of meeting at PQ form an erect virtual image (Fig. 113 c). If EP $= f$ (the focal length of the

eye-piece), the emergent rays are parallel and the image can be viewed with relaxed accommodation. A calculation exactly the same as before shows that the magnifying power is F/f. A disadvantage of this type is the loss of light; if Figs. 139 and 140 are compared it will be seen that in the former the rays cross each other immediately behind the eye-piece, and an eye placed at this point receives all the light that has entered the objective. In the present case, owing to the divergence of the rays, only a small proportion can enter the

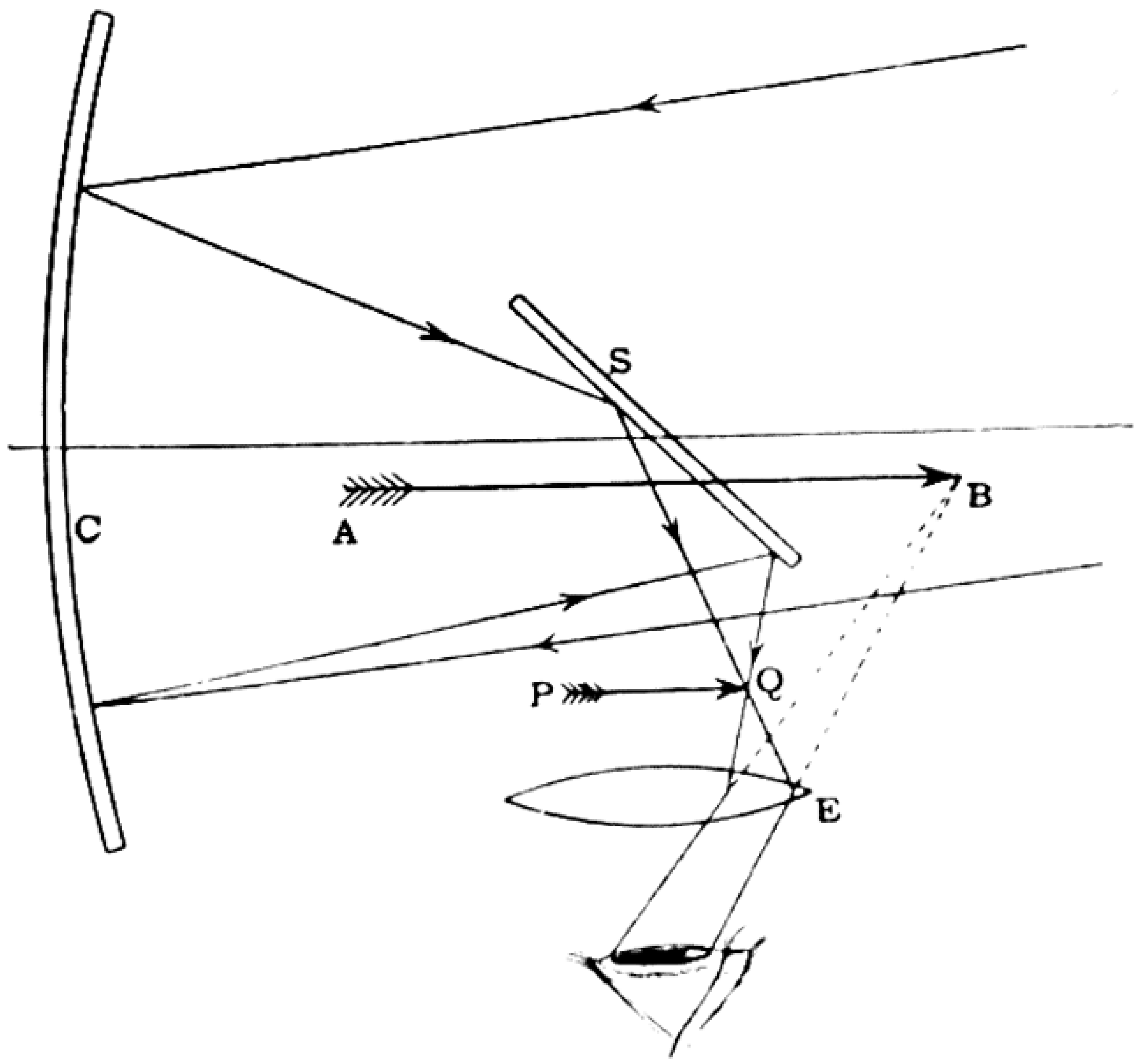


FIG. 141.—Newton's Reflecting Telescope.

eye and the image is less bright. For this reason the Galilean telescope cannot be used where large magnifying power is necessary.

Reflecting Telescope.—For astronomical observations telescopes with large objectives are required, hence their manufacture is a very difficult and costly process. Now the purpose of the objective is to form a well-illuminated real image PQ, and for many purposes this may be done as conveniently by means of a large concave mirror, which costs much less than a large lens, and is already achromatic since the rays are not refracted. An instrument constructed on these lines is called a reflecting telescope. Fig. 141 shows one form due to

Newton. The concave mirror C collects the rays and throws them into a real image PQ which is observed in the usual way through an eye-piece E. In order that the observer's head may not obstruct the incident light the rays are reflected through a right angle by a small plane mirror S. The magnifying power is F/f, where F is the focal length of the mirror.

The student should set up each type of telescope and measure their magnifying powers.

EXPERIMENT.—*To measure the magnifying power of a telescope.* Make a vertical graduated scale on a black-board, with the chalk lines about 5 cms. apart, and focus the telescope on it. Scale and telescope should be at the opposite ends of a large room. Observe the image with one eye and the scale directly with the other; this may be difficult at first, perhaps the easiest way is to look at the scale and gradually bring the head up to the eye-piece, focussing the telescope if necessary. Arrange that scale and image are clearly seen without parallax. The number of scale divisions equal to one division on the image can then be found, and this is the magnifying power since image and object are the same distance from the eye and therefore the angles they subtend are proportional to their linear dimensions.

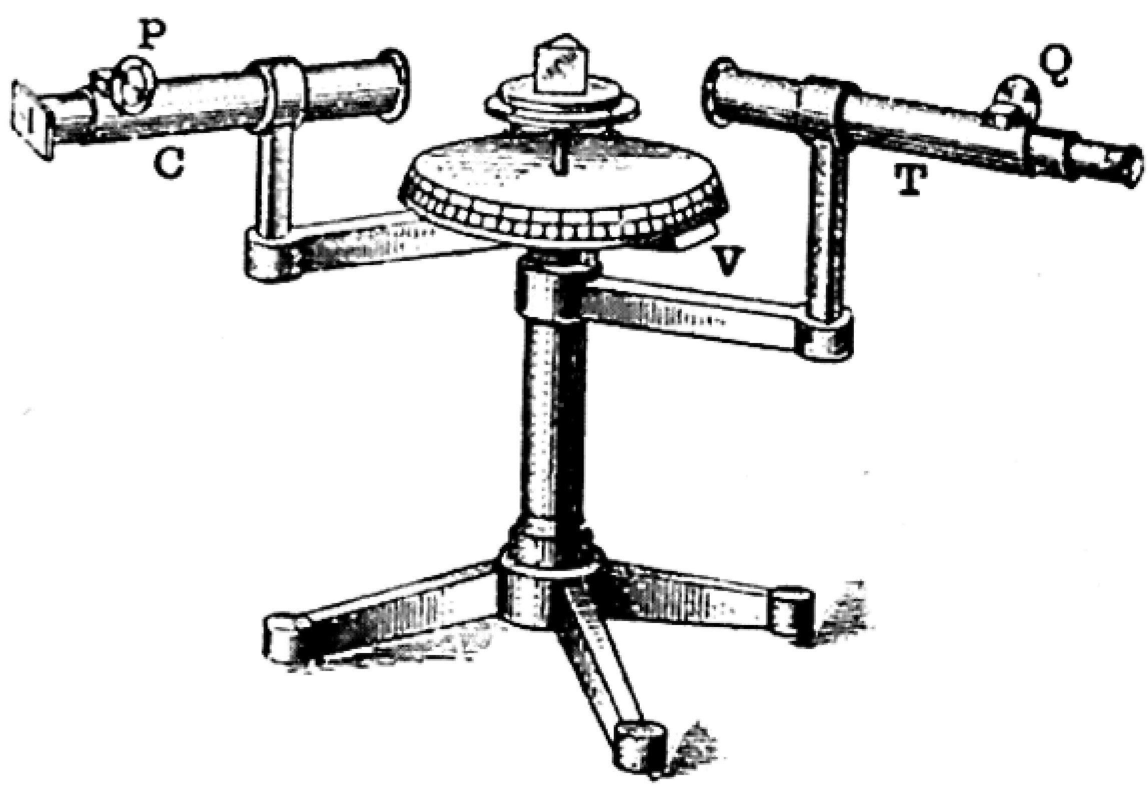


FIG. 142.—The Spectrometer.

Spectrometer.—The optical parts of the spectrometer have already been described (p. 209), the complete instrument is shown in Fig. 142. The collimator C is fixed, but the slit at its further end can be moved nearer to or further from the lens by the rack and pinion P. The astronomical telescope T turns round a vertical axis, the angle through which it rotates is measured by a vernier V which moves with it over a graduated circle; it carries cross-wires in front of the eye-piece, and eye-piece and wires can be moved together

by the rack and pinion Q. The table which carries the prism can be levelled by three screws to get the refracting edge of the prism vertical; it also rotates round the same axis as the telescope, its position with reference to the graduated circle being given by a second vernier.

EXPERIMENT.—*To adjust the collimator for parallel light.* Focus the eye-piece on the cross-wires; this adjustment must not afterwards be disturbed. Direct the telescope towards some distant object and focus by means of Q until there is no parallax between the image and cross-wires. This part of the apparatus is now adjusted for parallel light, but it is only a preliminary in the adjustment of the collimator. Put the telescope in line with the collimator and view the illuminated slit. By means of P arrange that there is no parallax between the slit image and the cross-wires. As none but parallel light is focussed by the telescope the rays from the collimator must then be parallel. The instrument may now be used for the following experiments.

EXPERIMENT.—*To show that when a mirror is turned through an angle θ the reflected ray is turned through 2θ.* As this is a direct consequence of the law of reflexion its experimental verification may be regarded as a proof of the law. Place a prism on the spectrometer table in the position indicated by Fig. 143, so as to receive on its reflecting faces the light coming from the collimator. Turn the telescope into the position RQ, where the slit image coincides with the vertical wire. Read each vernier. Turn the table through 5° and note how far the telescope must be moved to bring the wire and slit into coincidence again. This angle will be very approximately 10°. Repeat for different angles.

EXPERIMENT.—*To measure the angle at which light is incident on the prism face.* Read the vernier when the telescope is in direct line with the collimator; the prism must be removed for this observation. Next receive in the telescope the light reflected in the direction QR (Fig. 143). The angle θ between the two positions of the telescope is given by the difference of the readings. But $\angle$ PQR $= \pi - \theta$, and $\angle$ PQR is also the sum of the angles of incidence and reflexion, *i.e.* is twice the angle of incidence i.

Hence
$$2i = \pi - \theta$$
and
$$i = \frac{\pi}{2} - \frac{\theta}{2}$$

Note the reading of the prism table and calculate what it would be for normal incidence; knowing this zero reading the prism may be fixed so that the angle of incidence has any desired value.

EXPERIMENT.—*To measure the angle A of a prism.* Fix the prism in the position shown in Fig. 143, and get the image of the slit into coincidence with the cross-wires, first when the telescope is in the position RQ, and secondly when it is directed along VT. The $\angle$ ROV measures the rotation of the telescope. Now the prism faces are inclined at an angle A, hence (p. 144) the reflected rays are inclined at an $\angle$ 2A, and $\angle$ ROV $=$ twice the angle of the prism. (See also Ex. 15, p. 242.)

EXPERIMENT.—*To measure the refractive index of the prism.* Monochromatic light must be used for this experiment. First measure the prism angle as in the last experiment. Next turn the prism into the position shown in Fig. 123, and receive the refracted rays in the telescope. Turn the table in that direction which causes the deviation of the rays to diminish and follow the image with the telescope. Note the telescope reading when the minimum deviation position is reached and the image is on the cross-wires. Remove the prism and read the telescope when it is receiving light directly from the collimator. The difference of the readings is the minimum deviation δ. The refractive index can be calculated from the formula on p. 182.

By a combination of the third with the last experiment a curve showing the deviation for different angles of incidence can be plotted in a more accurate manner than that already given on p. 180.

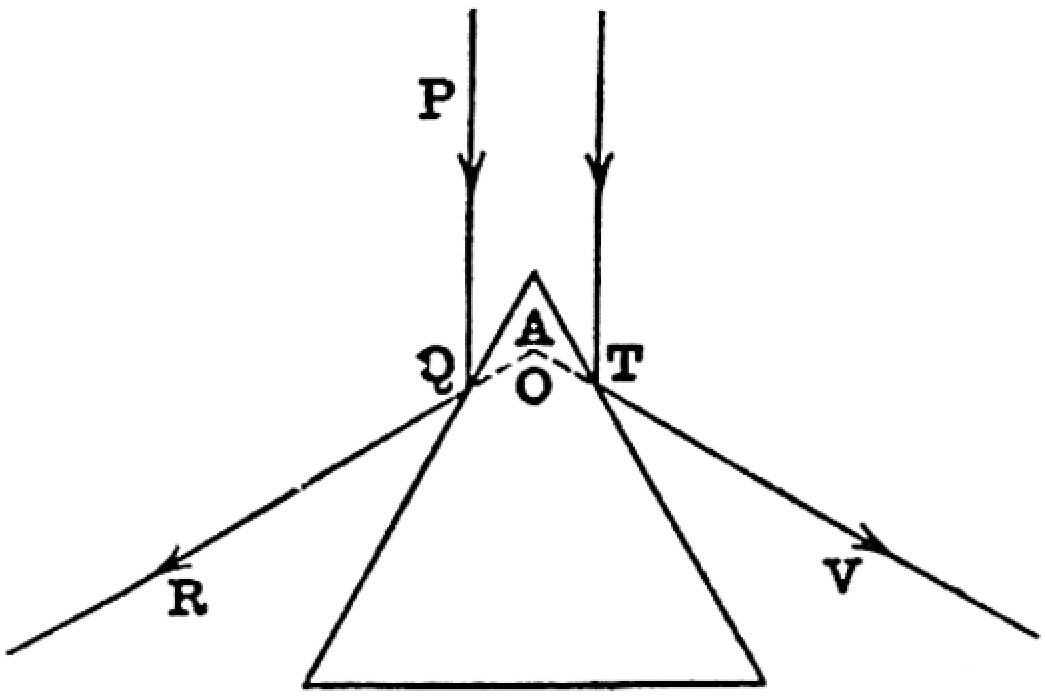


FIG. 143.—Method of measuring the Angle of a Prism.

The Spectroscope.—A spectroscope is used merely to identify lines in the spectrum of any substance; its purpose may be served by a spectrometer, for if the same prism is always placed in the minimum deviation position for sodium light the cross-wires can be made to coincide with any line, and the vernier reading will always be the same for a given line. Fig. 144 shows another method of making the observations. The apparatus is a spectrometer without a graduated circle but with the addition of a third tube R, this carries at its outer end S a small glass scale fixed at the principal focus of a convex lens which is placed at the inner extremity of the tube. C represents the collimator and T the telescope receiving the refracted rays. Light coming from R is reflected from the second face of the prism and enters the telescope along with the light under examination. The lines of the spectrum are thus seen distributed

along a graduated scale; they may easily be identified from their positions on this scale if the prism is placed in the minimum deviation position for sodium light, and the tube R is adjusted to

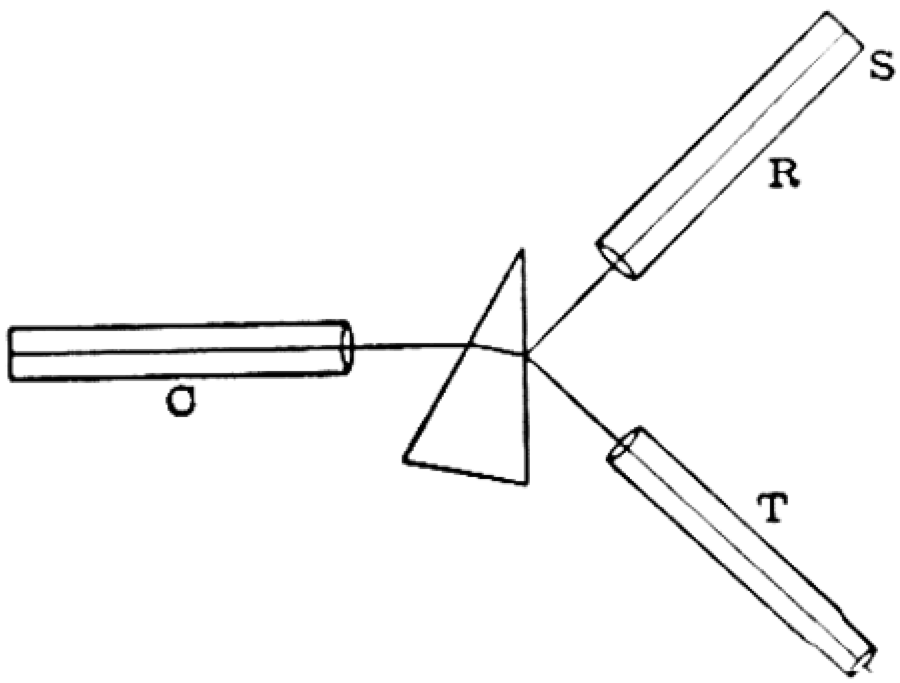


FIG. 144.—The Spectroscope.

bring the yellow rays on some standard mark. To increase the dispersion two or more prisms may be used.

EXAMPLES ON CHAPTER XX

1. A camera is used to take a photograph of a distant spire. Prove that the length of the image is proportional (nearly) to the focal length of the lens.

2. A lantern slide $3'' \times 3''$ is placed 1 ft. from a lamp of 250 c.p. By means of a lens an image of 10 ft. side is thrown on a distant screen. Find the intensity of illumination of the clear parts of the image.

3. A telescope is focussed on a distant object so that the image appears at the least distance of distinct vision. Prove, with the notation already used, that the magnifying power is $\frac{F}{f}\left(1 + \frac{f}{D}\right)$

4. Prove that the magnifying power of a reflecting telescope is F/f, where F and f are the focal lengths of the concave mirror and eye-piece respectively.

5. Show that the magnifying power of a reading lens is $\left(1 + \frac{D - a}{f}\right)$, when the lens is held at a distance a from the eye, and the image is formed at the least distance of distinct vision. What conclusion do you draw as to the best position of the eye?

6. Calculate the magnifying power of a simple magnifying lens of $\frac{1}{2}$ in. focal length held close to an eye whose least distance of distinct vision is 8 in. (L. '92.)

7. If the projecting lens of an optical lantern has a focal length of 8 in. and is 15 ft. distant from the screen, find the size of the picture if the slide is $3'' \times 3''$. Compare also the illuminations of the slide and picture. (L. '09.)

8. The object glass of a microscope has a focal length of 1 in., and the eye-piece of $1\frac{1}{9}$ in. The lenses are fixed 4 in. apart and focussed on an object so as to form a virtual image 10 in. from the eye-piece. Calculate the magnifying power and make a diagram showing the passage of two rays, coming from a point on the object not on the axis, through the microscope. (L. '09.)

9. A person whose nearest distance of distinct vision is 15 cms. uses a lens of 5 cms. focal length to magnify a small object. What is the distance of the object when in focus and what magnification is obtained? (L. '10.)

10. An object viewed through a convex lens of 5 in. focal length held close to the eye appears to be 10 in. away. Find the actual position and the magnification. Why is it necessary to state the position of the eye with respect to the lens? (L. '10.)

11. Explain the action of a lens when used as an eye-glass. A man who can see most distinctly at a distance of 5 in. from his eye wishes to read a notice at a distance of 15 ft. off. What sort of spectacles must he use, and what must be their focal length? (L. '89.)

12. A short-sighted person has distinct vision at 5 in. What kind of lens should he use, and of what focal length, to enable him to read a book 20 in. from his eyes? (L. '95.)

13. Suppose a short-sighted person can see an object clearly only when it is placed at a distance not exceeding 8 in. What kind of lens should be used, and of what power, in order that if placed close to the eye it should enable objects that are 48 in. away to be clearly seen? (L. '01.)

14. A short-sighted person can see distinctly objects at distances ranging from 10 to 20 cms. from the eye. Give the focal power or dioptric strength of suitable spectacles and calculate the new near and far points. (L. '08.)

15. A prism is placed on the table of a spectrometer and the image formed by reflexion at one face is viewed in the telescope. The prism is now turned about the refracting edge through an angle A until the reflected image is seen in the second face, the telescope remaining fixed. Prove that the angle of the prism is $(\pi - A)$.

CHAPTER XXI

VELOCITY OF LIGHT

VARIOUS common experiences show clearly that the velocities of light and sound are very different. Thus the flash of a distant gun is seen some seconds before the report is heard, and we see the lightning some time before we hear the thunder which originates at the same instant. It was first shown from astronomical observations that the velocity of light is perfectly definite, although it is so great that the most refined experiments are required to measure it. The difficulty of a direct measurement will be apparent from the following illustration. Suppose two observers furnished with stop-watches are situated due south of a gun and one mile apart. When the gun is fired let each note the instant he hears the report; the difference between the times gives the interval required by sound to travel one mile. This will be about five seconds. If now they note the instants at which they see the flash it is found that it appears to them simultaneously; in order that one observer shall see it one second after the other they must be separated by a distance of 186,000 miles!

Römer's Method.—A Danish astronomer, named Römer, first succeeded, about 1675, in measuring the velocity of light. The event he observed from different positions was the eclipse of one of the moons of the planet Jupiter. In Fig. 145 let S represent the sun, E_1 the earth, and J Jupiter. One of Jupiter's satellites M moves completely round the planet once every two days, and during part of each revolution it is eclipsed to an observer on the earth. Now when the earth is at E_1 its distance from Jupiter is changing very little from day to day and the time between successive eclipses can be determined accurately; hence the instants at which future eclipses should occur can be predicted with precision. At a later date the earth and Jupiter will be on opposite sides of the sun; let E_2 and J

be the new positions. It was found while the earth was moving to E_2 that the eclipses took place after their predicted times, but when the position E_1 was reached observation and prediction were in agreement once more. The maximum discrepancy occurred when the earth was at E_2; in this position the eclipse was 16·5 minutes late. This can be explained if it be supposed that light takes 16·5 minutes to travel over the distance E_1E_2. Now, the distance of the earth from the sun is 92,000,000 miles, hence E_1E_2 is 184,000,000 miles, and it is traversed by light in 990 seconds. Thus the velocity of light $= \dfrac{184{,}000{,}000}{990} = 186{,}000$ miles per second.

The Aberration Method.—About half a century later the English astronomer Bradley noticed that the apparent positions even of very

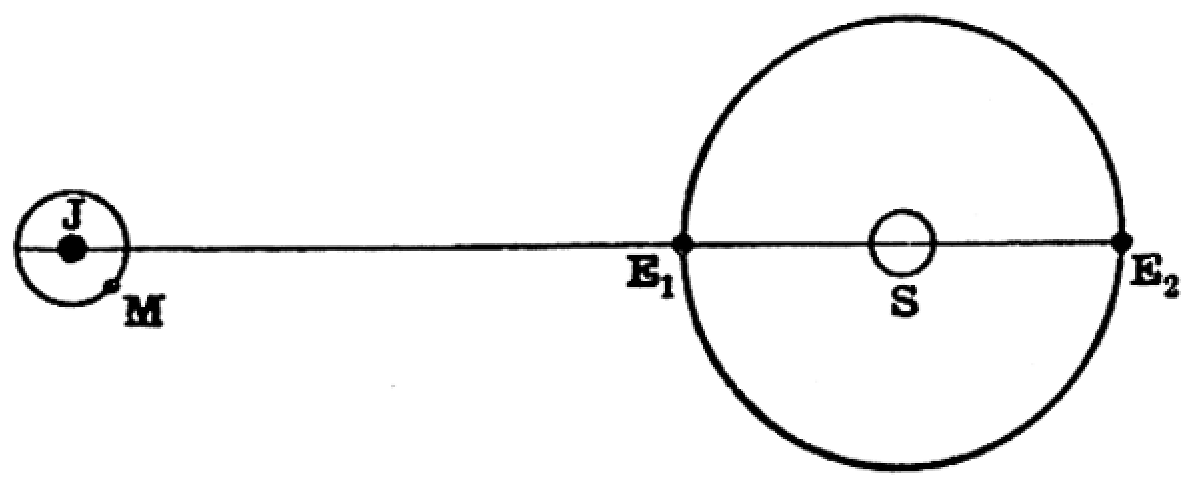


FIG. 145.—Römer's Method of measuring the Velocity of Light.

distant stars varied slightly with the direction in which the earth was moving in its orbit. He showed that this was due to the velocity of light. Let BA (Fig. 146) represent the direction in which light is travelling with a velocity V, and suppose a telescope is pointed along this line so that the image of a star should come on the cross-wires. Let the velocity of the earth in its orbit be v in the direction AC. While light is travelling from B to A the telescope moves to the parallel line CD and the star image does not fall on the wires at C but appears a distance CA to the left. In order that the image shall fall on the wires the telescope must be tilted initially in the direction AD, then when light enters the objective at B the cross-wires are at E, and by the time the light reaches A the wires have been moved to this position to receive it. Evidently

$$\frac{v}{V} = \frac{AC}{DC} = \frac{\sin ADC}{\sin DAC}$$

The angle between the apparent and true directions of a star is

called its aberration, this is $\angle$ ADC in the figure. These angles and v can be found by astronomical means and hence V can be calculated. Neither of these methods is susceptible of great accuracy; the two following do not depend on astronomical observations and the second one possesses the further advantage that it enables us to measure the velocity in different media.

Fizeau's Method.—The principle of this method is shown in Fig. 147. Let S be a source of light, M a plane mirror, and T a toothed wheel which can be rotated very quickly. Light starts from S, passes through a space between the teeth, and falls normally on the mirror M whence it is reflected back along its path. An

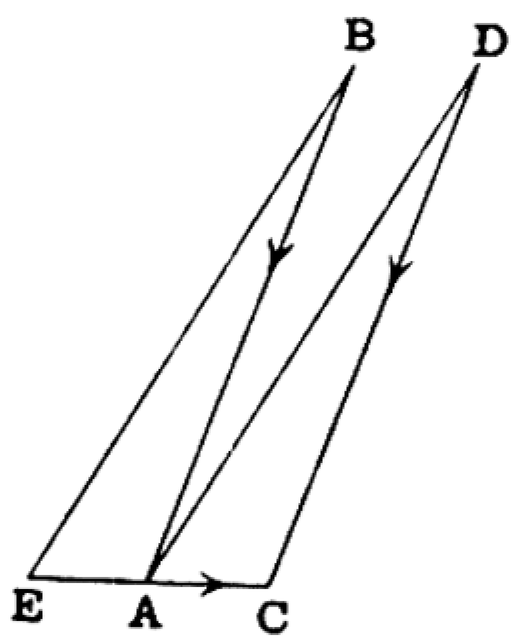


Fig. 146.—To Illustrate the Aberration of a Star.

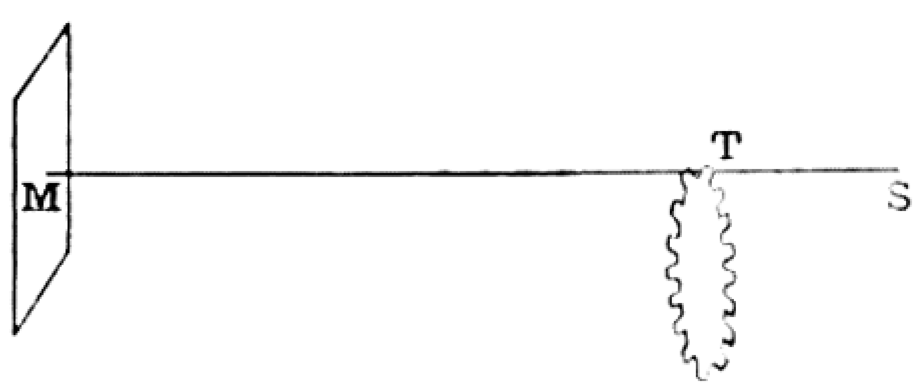


Fig. 147.—Showing the Principle of Fizeau's Method.

observer at S sees an image of the source in the mirror. Suppose now the wheel is put in motion; it may happen while the light is travelling from T to M and back again that a tooth has moved up to a position previously occupied by a space, the returning rays will then be cut off and the image will be eclipsed. If the speed of the wheel is doubled the light which passes through one space returns through the next and the image reappears. In Fizeau's experiments the wheel had 720 teeth and the distance TM was several miles. Let V be the velocity of light, l the distance TM, and T the time in seconds of one revolution of the wheel when the first eclipse takes place. Then the time required for a tooth to move into the position previously occupied by a space is $\dfrac{T}{720 \times 2}$ secs. $= t$, since the circumference is divided into (720×2) equal parts; during

this time light travels from T to M and back again, *i.e.* over a distance $2l$.

Hence $$V = \frac{2l}{t} = \frac{2l \times 720 \times 2}{T}$$

In one experiment Fizeau found that the wheel made 12·6 revolutions per second when the first eclipse took place and $l = 8633$ metres, hence $T = 1/12{\cdot}6$. Substituting these values $V = 313{,}000{,}000$ metres/sec.

This is rather larger than the usually accepted value, which is about 300,000,000 metres/sec.

Fig. 148 shows the apparatus used. Light from the source Q

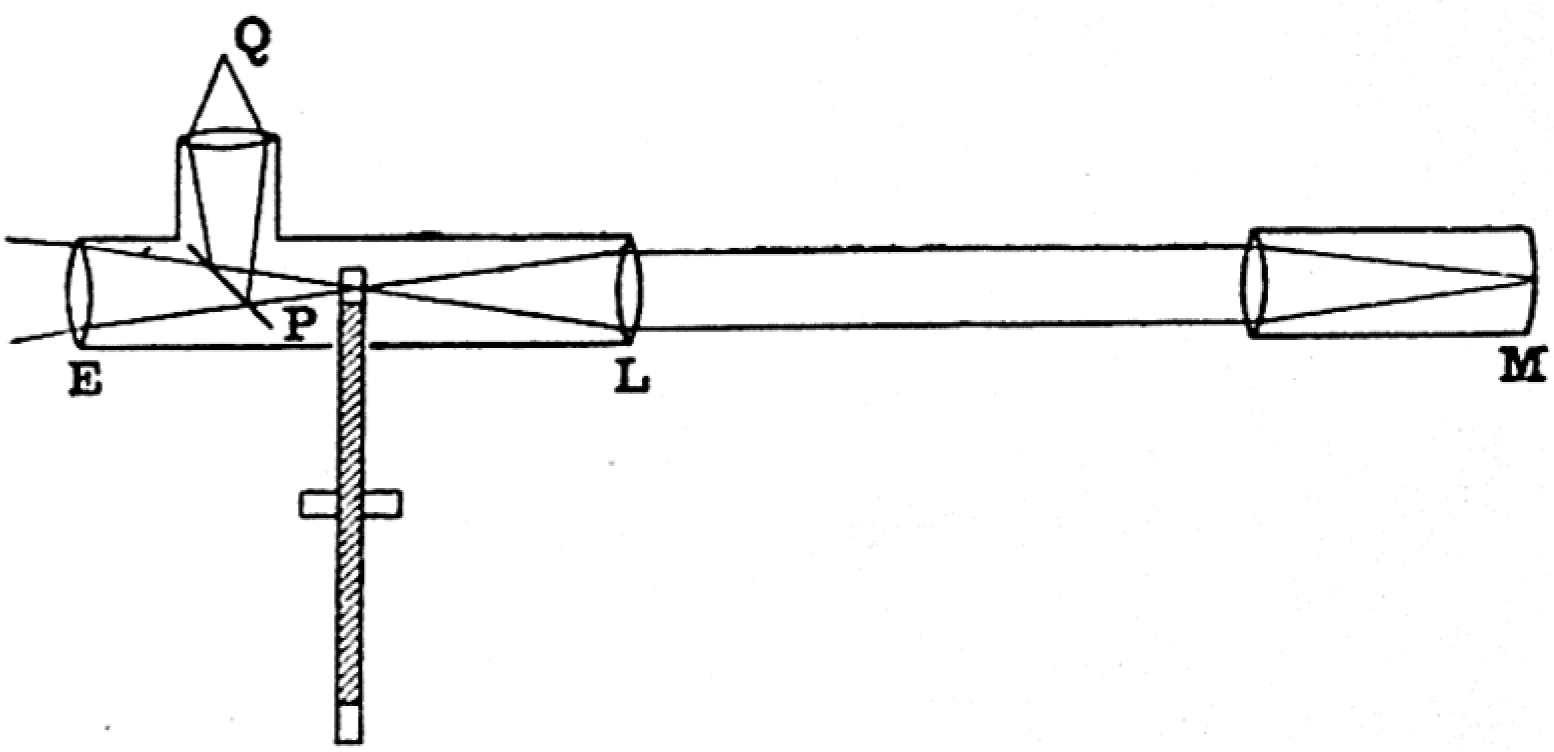


Fig. 148.—Fizeau's Apparatus.

passed through a lens and fell on a sheet of glass P inclined at 45°, from this it was reflected to a lens L and emerged in parallel rays. About 8000 m. away these rays fell on a second lens which caused them to converge on to the mirror M. From this they retraced their path to the plate P. Some of the returning light passed directly to the lens E and thence to the eye, hence the eclipses could be observed. It will be seen that EL really forms a telescope with the rim of the wheel T at the common focus of the two lenses.

Foucault's Method.—This method can be worked in a room of moderate size. Its principle is shown in Fig. 149. S is a well-illuminated narrow slit, P a plane mirror which can rotate rapidly round a vertical axis at O, M a concave mirror whose centre of curvature is at O. Suppose the mirror is at rest in the position OP; light from S travels to O, is reflected to M, and, meeting this mirror

normally, retraces its path to S. Suppose next that the mirror OP is rotating. While the light is going from O to M and back again the mirror turns through a small angle θ into the position OP′. The

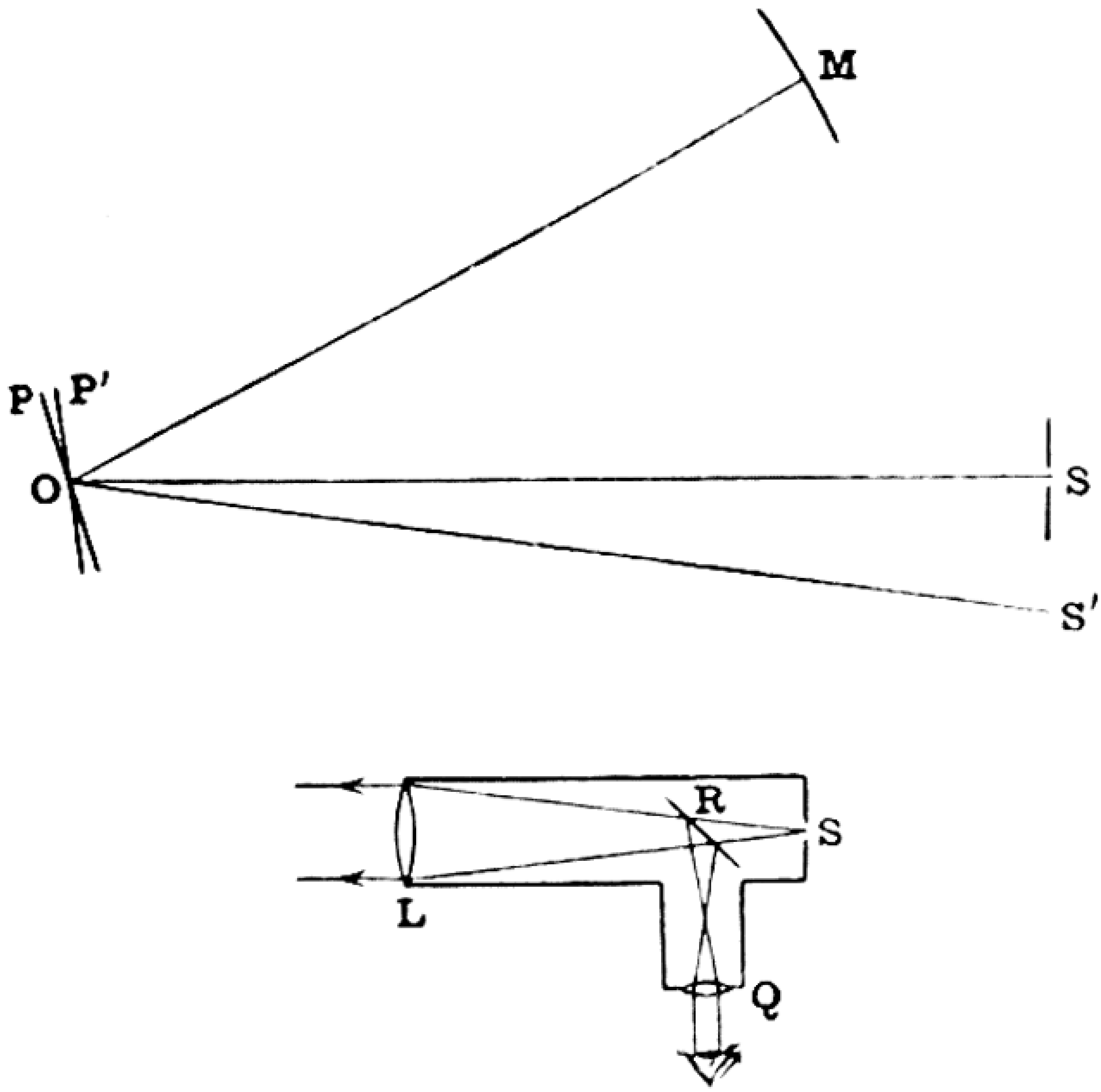


Fig. 149.—Foucault's Method of measuring the Velocity of Light.

rays are now reflected along OS′ and the image of the slit is at S′.

Also
$$\angle\, SOS' = 2\theta \text{ (p. 144)}$$
$$= \frac{SS'}{OS}$$
or
$$\theta = \frac{SS'}{2OS}$$

Hence by measuring these two distances θ can be found. Let t be the time taken by light to travel from O to M and back again, T the period of revolution of the mirror, and V the light velocity.

Then $V = 2OM/t$, and t is also the time that the revolving mirror takes to turn through an angle θ.

To turn through an angle 2π the mirror requires T secs.

$\therefore$ to turn through an angle θ the mirror requires $\frac{\theta}{2\pi}$. T. secs.

$$\therefore t = \frac{\theta . T}{2\pi} = \frac{SS'}{2OS} \cdot \frac{T}{2\pi}$$

Hence
$$V = \frac{2OM}{t} = \frac{8\pi}{T} \cdot \frac{OS}{SS'} \cdot OM$$

and all the quantities on the right can be found by experiment. The lighting and observing parts of the apparatus are shown in Fig. 149 (below). S is the slit, R a plane glass plate, L a lens which makes the light converge on M. If the lens is not used the rays from the slit diverge and only a small proportion of them reach the concave mirror, in that case the image S′ is very faint. Part of the returning rays are reflected by R into an eye-piece Q; the slit S′ is formed by these rays and can be viewed by an eye placed immediately behind the lens. The displacement when the mirror rotates is measured by cross-wires moved by a micrometer screw. If the length OM is occupied by a long tube filled with liquid the experiment determines the velocity of light in the liquid. When monochromatic light is used it is found that

$$\frac{\text{velocity of light in air}}{\text{velocity of light in the liquid}} = \text{refractive index of the liquid (p. 168).}$$

This result shows why it is that the refractive index of a substance is constant when the angle of incidence is varied.

EXAMPLES ON CHAPTER XXI

1. Describe Foucault's method of measuring the velocity of light. If red and blue light travelled with different velocities in air how would the appearance presented to the observer be modified? (L. '87.)

2. How do we know that red light travels more quickly than blue light inside glass? (L. '06.)

3. A beam of light is reflected by a rotating mirror on to a fixed mirror which sends it back to the rotating mirror from which it is again reflected and makes an angle of 18° with its original direction. The distance between the two mirrors is 10^6 cms., and the rotating mirror is making 375 revs. per sec. Calculate the velocity of light. (L. '07.)

4. In an experiment for measuring the velocity of light by Foucault's method the fixed mirror was distant 3 km. from the revolving mirror, which made 500 revs./sec. The angular deviation of the return ray was 7° 12′. Calculate the velocity of light from these data. (L. '08.)

CHAPTER XXII

SIMPLE HARMONIC MOTION

Introduction.—The term *sound* is used, like the term *light*, in two senses, to denote (1) The sensation we receive through our organs of hearing; (2) The physical cause of this sensation. Using the term in its latter sense it is a common experience that a source of sound is in a state of vibration. For example, the prongs of a tuning fork, a bell, the strings of a piano, and, as we shall see later, the air in an organ pipe are all in a state of vibration when they are producing sound. In the present chapter we will consider the conditions of the vibrating body, and will then proceed in succeeding chapters to explain how these vibrations affect the ear, and to study their results under different conditions.

Study of a Vibrating Rod.—Let us consider what conditions must be fulfilled in order that a body may be capable of vibration, and to have a definite example, let us take the case of a metre stick clamped in a horizontal position at one end so that its other extremity can oscillate in a vertical plane. Whenever it is deflected there must be a tendency for it to resume its original position or shape; in other words it must be elastic. On account of the elasticity a restoring force, named the force of restitution, is called into play whenever the free end of the stick is deflected, and work has to be done to overcome this force. Such work is stored up as potential energy. Suppose now the stick is released from its strained position; its potential energy is converted into the kinetic form, and when it reaches its initial position all its energy is kinetic, consequently it moves past its point of rest, and a restoring force in the opposite direction is called into play; the moving body does work against this until all its kinetic energy is reconverted into potential energy. The cycle of changes is then repeated in the opposite direction. Hence the conditions for a vibration are (1) a restoring force must be

called into play when the body is deflected—in most cases this arises from the elasticity, (2) the body must be capable of storing potential energy, (3) capable of possessing kinetic energy. A pendulum provides another illustration of a vibrating body; here the force of restitution arises from the weight of the bob, and the potential energy is the weight of the bob multiplied by the vertical height through which it has been raised. If the end of the metre stick above is deflected by hanging weights to it a few simple measurements will show that the deflexions are proportional to the weights applied, *provided the deflexions are small*. But in equilibrium the hanging weight is balanced by the force of restitution; it follows that this force is proportional to the displacement of the end of the stick, and is in a direction tending to restore it to its undisturbed position. For small displacements this is true of all vibrating bodies, the force of restitution is proportional to the displacement. Now the acceleration of a body is proportional to the force which acts on it, hence when it is performing *small* vibrations its acceleration is directed towards its mean position and is proportional to the force of restitution, *i.e.* to its displacement. When a particle moves along a straight line so that its acceleration is always directed towards a point in this line and is proportional to the displacement therefrom the particle is said to move **harmonically** or with a **simple harmonic motion.** This term is usually abbreviated to the letters S.H.M. In most cases of sounding bodies the displacements are small and the motion is harmonic, it will therefore be convenient if we study first some of the features of simple harmonic motion.

Simple Harmonic Motion.—In Fig. 150 APB is a circle of radius a; XX′ and YY′ are two perpendicular axes which meet at the centre O, and P is a point which is moving round the circle in the direction shown by the arrow with a uniform velocity v. Draw a perpendicular PM on to OX; we will show that as P moves round the circle the point M moves to and fro along the diameter AB with a simple harmonic motion. Let us denote by x the displacement of M from O at any instant, and by T the time that P takes to move completely round the circle. T is called **the period** of the vibration; in this time the point M performs a complete to and fro motion along AB. The number of complete vibrations performed each second is called the **frequency**; if N is the frequency the period $T = 1/N$. Displacements to the right of YY′ are to be taken as positive, those

to the left as negative. Let us take as the zero of time the instant when M is passing through its mean position O in the positive direction, at this moment the radius OP is crossing OY. After a time t OP makes an angle θ with OY; this angle is called the **phase** of the vibration. P is called the generating point and the circle APB the generating circle. In a time T the point P moves round the circle, *i.e.* through a distance $2\pi a$, hence

$$2\pi a = v\mathrm{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let ω be the angle through which the radius OP revolves in 1 sec., this is called the angular velocity of P. In a time T the radius

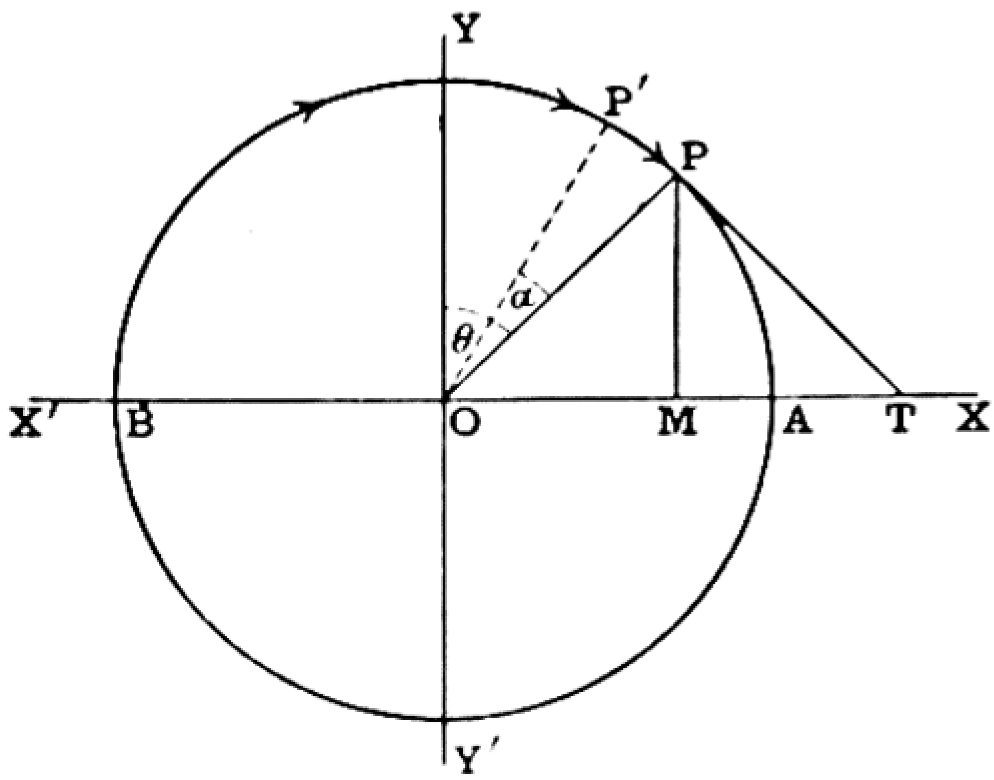


FIG. 150.—Simple Harmonic Motion.

turns through an angle $\omega\mathrm{T}$, but this is the angle described in one complete revolution,

therefore $$\omega\mathrm{T} = 2\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2) $$v = \frac{2\pi}{\mathrm{T}} \cdot a = \omega a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation gives an important relation between the linear velocity of P and its angular velocity round O.

It is shown in books on mechanics that the acceleration of P is directed towards O and is equal to v^2/a. To find the acceleration of M we have to resolve the acceleration of P along OA. Hence the acceleration f of M is directed towards O;

if the acceleration of P is represented by OP the component f is represented by OM,

$$\therefore f = \frac{v^2}{a} \cdot \frac{\mathrm{OM}}{\mathrm{OP}}$$

or substituting for v from (3)

$$f = \frac{\omega^2 a^2}{a} \cdot \frac{\mathrm{OM}}{\mathrm{OP}} = \frac{\omega^2 a^2}{a} \cdot \frac{x}{a}$$

$$\therefore f = \omega^2 x \quad \ldots\ldots\ldots\ldots \quad (4)$$

Hence the acceleration of M is towards O and is proportional to the displacement, *i.e.* its motion is a S.H.M.

From equation (2) it is seen that the period $\mathrm{T} = 2\pi/\omega$, hence whenever the acceleration of a point moving along a line is given by (displacement $\times$ constant) we conclude that the motion is harmonic, and from (2) its period is 2π divided by the square root of the constant. Also the angle θ is described in the time t,

hence $\qquad \theta = \omega t$
and the displacement $\qquad x = \mathrm{OP} \, . \sin \theta$
or $\qquad x = a \sin \omega t \quad \ldots\ldots\ldots \quad (5)$

Whenever the displacement is given by an equation like (5) it may be concluded that the motion is a S.H.M., and the period is obtained by dividing 2π by the coefficient of t. If a perpendicular PN is drawn to OY it can be shown in a similar manner that N moves with a simple harmonic motion along YY′. The displacement is given by $y = a \cos \omega t$; this equation therefore represents a S.H.M. The maximum displacement of M from its mean position is called **the amplitude** of the vibration; it is the radius a in the figure. Let a second point P′ move round the circle with the same velocity as P; the angle $\alpha = \mathrm{POP'}$ is called the phase difference of P and P′. The displacement of M′, the foot of the perpendicular from P′ on ox_1 is

$$x' = a \sin \mathrm{P'OY} = a \sin (\theta - \alpha)$$
$$\therefore x' = a \sin (\omega t - \alpha)$$

This equation also represents a simple harmonic motion.

The velocity of M along XX′ is the velocity of P parallel to this line. The velocity of P at any instant is along the tangent PT, and the component parallel to OX is $v \cos \mathrm{PTO} = v \sin \mathrm{POT} = v \cos \theta$.

Hence the velocity of M is $v \cos \theta$, or, in terms of the displacement and angular velocity,

$$\text{vel. of M} = a\omega \cdot \frac{\text{PM}}{\text{OP}} = \omega \,.\, \text{PM}$$

and

$$\text{PM}^2 = a^2 - x^2$$

$$\therefore \text{vel. of M} = \omega\sqrt{a^2 - x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Suppose now that M is a material particle of mass m; since its acceleration is towards O there must be a force F acting upon it tending to bring it back to this point, and F = mass $\times$ acceleration $= m\omega^2 x$. The potential energy of the particle in any position is the work done against this force of restitution. At A it is momentarily at rest and all its energy is potential; let us calculate the energy at this instant. The force is proportional to the displacement, therefore the average force is that which acts when the displacement is $a/2$.

But

$$\text{F} = m\omega^2 x$$

$$\therefore \text{average force} = m\omega^2 \frac{a}{2}$$

and the displacement at A is a,

$\therefore$ work done in displacing M from O to A

$$= \text{average force} \times \text{total displacement}$$
$$= \tfrac{1}{2}m\omega^2 a \times a$$
$$= \tfrac{1}{2}m\omega^2 a^2$$

This is the potential energy at A. At O all the energy is kinetic and must have the value just given. This follows also directly, for

$$\text{kinetic energy at O} = \tfrac{1}{2}m \times \text{vel.}^2$$

and the velocity at O is equal to the velocity of P, *i.e.* is ωa. We get the same result by putting $x = o$ in Equation (6), then

$$(\text{velocity})^2 = \omega^2 a^2$$
$$\therefore \text{kinetic energy at O} = \tfrac{1}{2}m\omega^2 a^2$$

This result shows that the energy of the vibrating particle is proportional to the square of the amplitude. It should be noticed that since $\text{T} = 2\pi/\omega$ the period is independent of the amplitude.

Graphical Representation of a S.H.M.—It is very convenient to be able to represent graphically the displacement of M at any instant.

Draw a line OX (Fig. 151) and mark on it equal lengths to represent equal intervals of time. In the figure these are shown as fractions of the period. Let positive and negative displacements be represented by ordinates drawn above and below this line. At the beginning of time the displacement is zero; after T/4 M is at A (Fig. 150) and the displacement is a. Hence at the point marked T/4, (Fig. 151), an ordinate of length a is drawn. After half a period the displacement is again zero, and so on. The thin curve in the figure is the result of joining all the points so found. Since the displacement $x = a \sin \theta$ it follows that the displacement curve in the figure is a sine curve. When the amplitude and the phase are known the displacement can be calculated from a table of sines and as many points on the curve as we choose can be found. The velocity at any instant can be similarly represented by a velocity curve. The velocity, from the last paragraph, is $v \cos \theta$, and the curve of velocities is a cosine curve. This, of course, is the same shape as a curve of sines, but its maximum ordinate is v when $\theta = 0$ and $\cos \theta = 1$. The two curves are displaced relatively to each other along the axis of time by a distance T/4. These results can also be seen by considering the velocity in Fig. 150. When the displacement is zero the velocity is v, when the displacement is a the velocity is zero, and so on. The dotted curve in Fig. 151 is the curve of velocities. It should be remembered that the curves do not mean that the displacements or velocities are parallel to OY but merely that their magnitudes are proportional to the corresponding ordinates.

Simple Pendulum as an Illustration of S.H.M.—As an instance of a motion which is nearly simple harmonic the simple pendulum may be taken. In Fig. 152 O is the point of support, P is the bob of mass m, and OP $= l$ is the length of the string. Draw the vertical line PR to represent the weight mg of the bob. This can be resolved into components along and perpendicular to OP. Draw PS perpendicular to OP and RS perpendicular to PS, then RS and PS represent the two components. The component RS is balanced by the tension in the string, while PS represents the force which tends to bring the pendulum back to its mean position OQ.

Hence restoring force
$$\text{PS} = \text{PR} \sin \theta$$
$$= mg \sin \theta$$

and the acceleration along the arc PQ $= \dfrac{\text{force}}{\text{mass}} = g \sin \theta$

If θ is very small $\sin\theta = \theta$ and the arc $PQ = l\theta$, or $\theta =$ arc PQ/l.

In this case the acceleration $= g . \theta$

$$= g . \frac{\text{arc PQ}}{l}$$

The acceleration along the arc is therefore proportional to the displacement measured along the same curve and the motion is simple harmonic. The acceleration is equal to (displacement $\times$ const.),

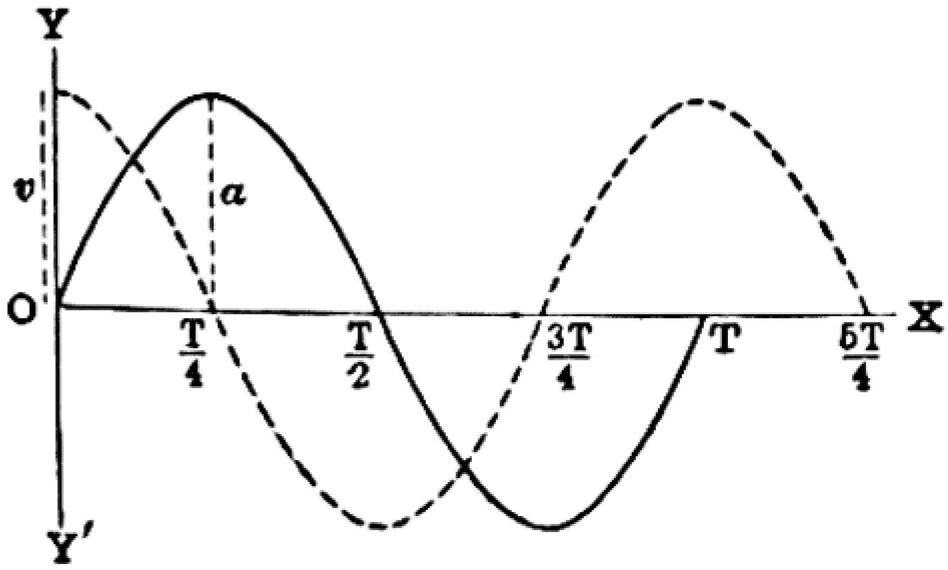


FIG. 151.—Graphical Representation of a S.H.M.

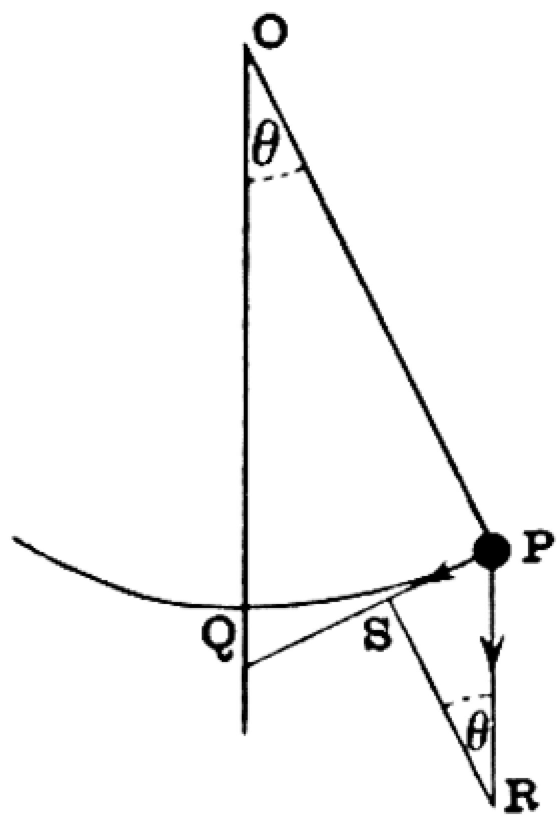


FIG. 152.—Simple Pendulum.

where the constant is g/l, hence the period of a complete to and fro vibration is

$$T = \frac{2\pi}{\sqrt{\text{constant}}} = 2\pi\sqrt{\frac{l}{g}}$$

From the mode of derivation it is clear that this formula holds only when the amplitude of the vibrations is small. The angular amplitude θ should not be greater than 5°, then, as previously, T is independent of the amplitude.

EXPERIMENT.—Note the time of vibration of a simple pendulum for different lengths of the string and plot a curve shewing T^2 and l; this should be a straight line. Calculate from your observations the value of g.

Resultant of Two S.H.M.'s in the same Straight Line.—Let the point M in Fig. 150 be performing S.H.M. along the line X'X, and suppose

the book itself is moving parallel to this line with a S.H.M. of the same period but of different amplitude and phase. The actual displacement of M is the resultant of the two motions. This displacement can be obtained graphically. Let the thin curve in Fig. 153 represent the displacement of M at different times if the book were at rest, and let the dotted curve represent the displacement of the book alone at corresponding instants. In drawing the figure it has been supposed that the phase of the second motion is exactly a quarter of a period in advance of the first. Then at the instant represented by A if M alone were moving its displacement would be AB, while if the book were moving and M were at rest on it the displacement would be AC. Hence an ordinate $AD = AB + AC$ represents the actual displacement, and similarly at any other instant the displacement of M is the algebraic sum of the ordinates

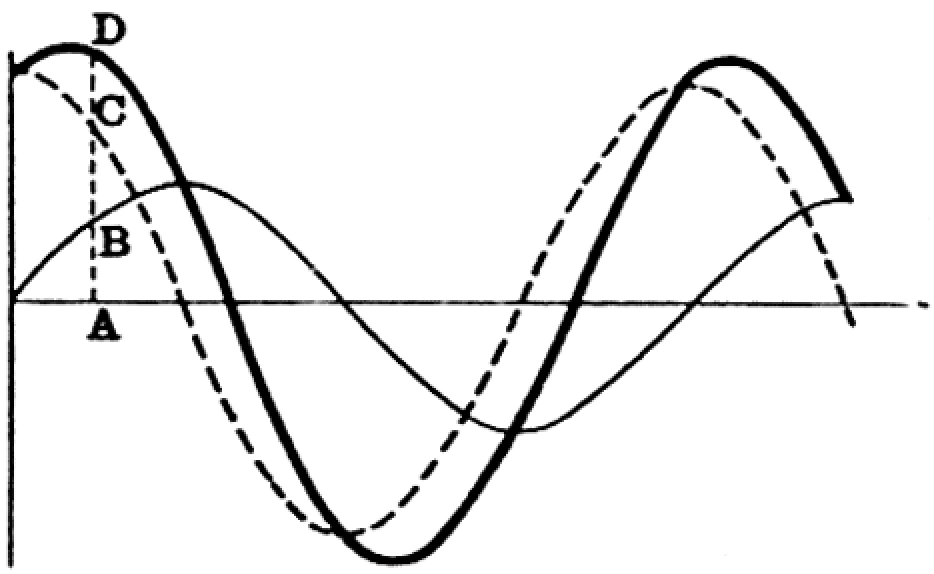


FIG. 153.—Composition of Two S.H.M.'s in the same Straight Line.

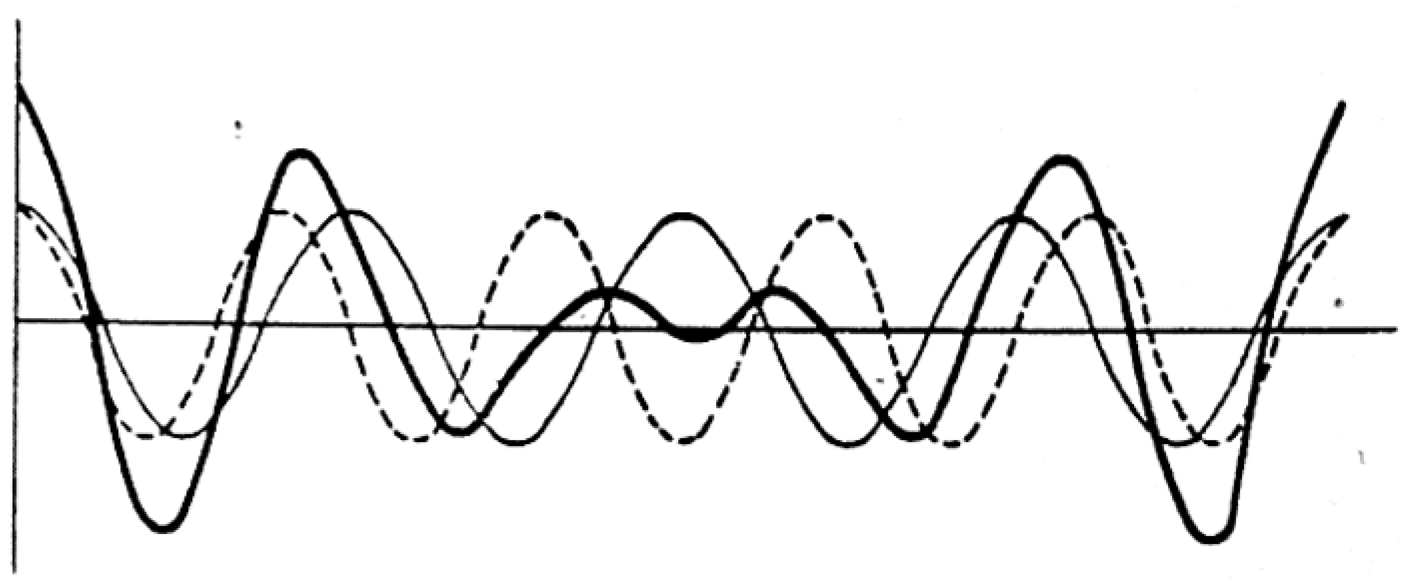


FIG. 154.—Resultant of Two S.H.M.'s in the same Straight Line when the Periods are as 5 : 4.

of the two curves. The thick curve represents the resultant displacement curve obtained in this manner. It is seen that the resultant of two S.H.M.'s of equal periods in the same straight line is itself a simple harmonic motion. The same method can be followed if the periods are unequal. Fig. 154 represents the case of two S.H.M.'s whose periods are as 5 : 4; the dotted curve represents the vibration of shorter period, and the two have been supposed to

have their maximum displacements at the beginning of the time of observation. It is clear that the resultant is no longer simple harmonic, and that it alternates between relatively very large and very small amplitudes. At the beginning, when the two have the same phase, the resultant is large, but when the quicker has made $2\frac{1}{2}$ vibrations the slower has made only 2 and the phases are exactly opposite; the resultant at this instant is therefore small. When the quicker has made 5 vibrations the slower has made 4, the phases are again equal and the resultant is large. It is evident that the

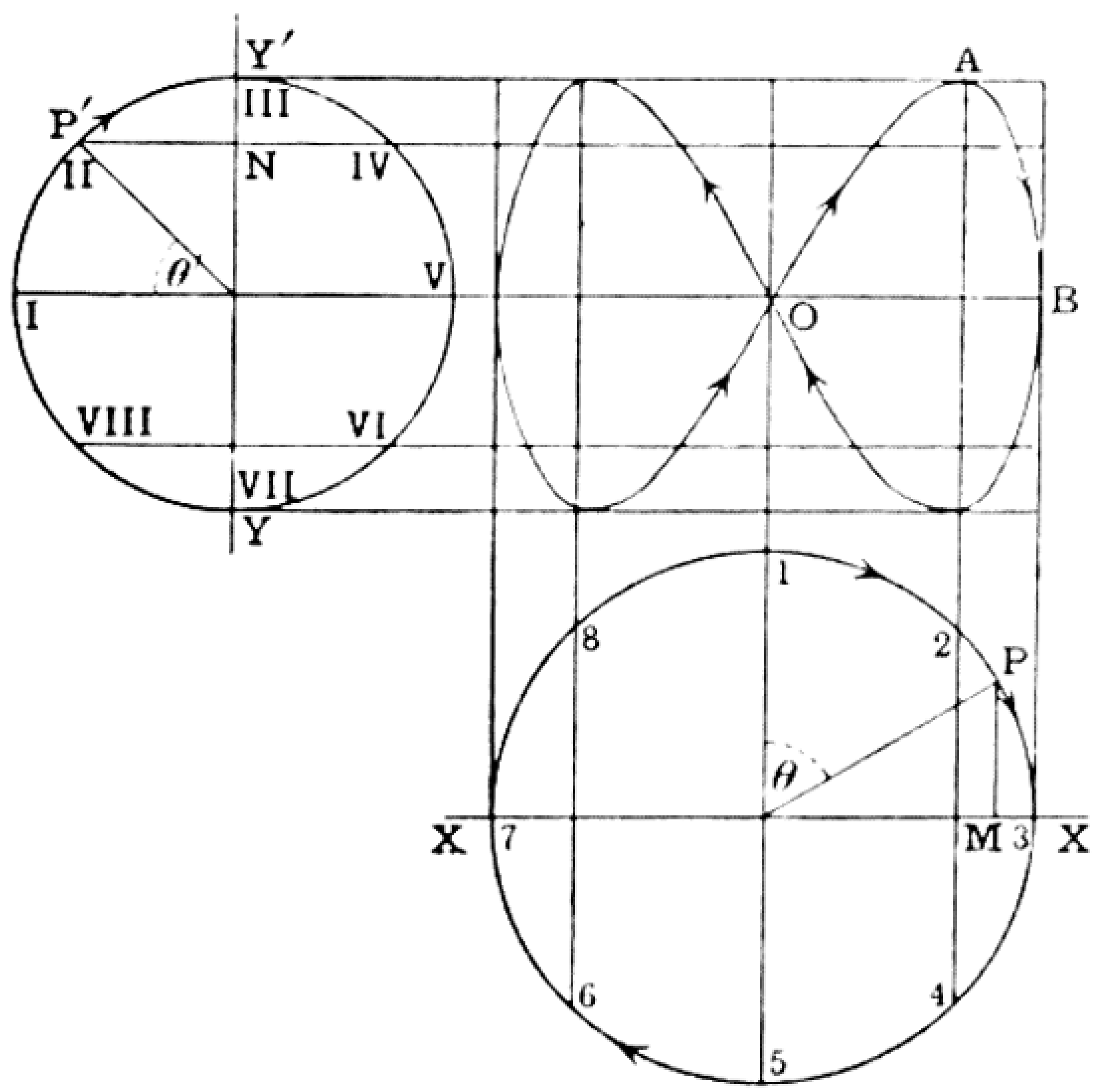


Fig. 155.—Composition of Two Rectangular S.H.M.'s.

maximum displacement occurs every fourth vibration of the slower body. This result will be useful later.

Resultant of Two S.H.M.'s at Right Angles to each other.—The method of finding the resultant in such cases will be best understood from an example. Draw two generating circles P and P′ as in Fig. 155 with their radii proportional to the amplitudes of the motions. Let us suppose the frequency of P′ is twice that of P. Draw two diameters XX′, YY′, at right angles to each other and drop the perpendiculars PM, P′N. The points M and N then perform S.H.M.'s. Divide the circumferences into a number of equal parts, number them as in the figure, and draw lines through these points

to form a network on the right. The points are numbered so that I, 1, correspond to the instants of zero phase, *i.e.* to the instants when N and M are moving through their mean positions in the positive directions. If a point is subject to the two S.H.M.'s its position at any instant is determined by the intersection of the two lines drawn through the corresponding positions of the generating points. Thus suppose P is at 1 when P′ is at I, and the point in question is at O. When P′ has reached III P has arrived at 2 and the point is at A on the intersection of the lines through III and 2. When P′ reaches V P is at 3 and the point is at B, and so on. The curve shows the complete path of the point. The curves in Fig. 156 have been obtained

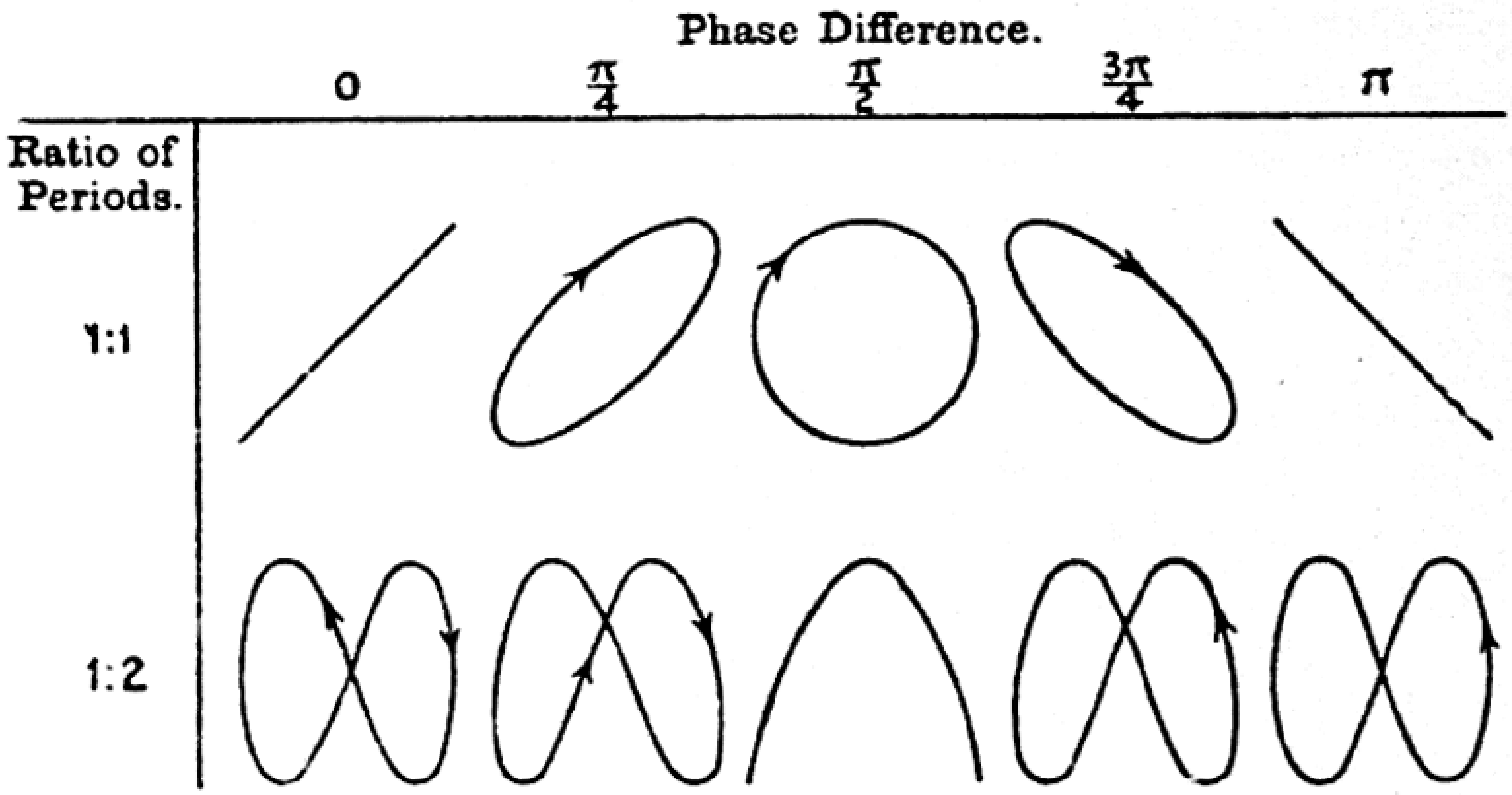


FIG. 156.—Lissajous' Figures.

by a similar construction; the ratio of the periods is shown on the left, while above is shown the phase of the faster vibration when the slower is at zero phase, *e.g.* a phase difference $3\pi/4$ means that P′ has revolved through this angle and is therefore at IV when P is at 1. These figures can be obtained experimentally by various devices of which only two need be described.

Blackburn's Pendulum.[1]—A thin string about 7 ft. long has its two ends tied to a rod E which is fixed horizontally (Fig. 157). At the end of the loop a heavy lead ring B is fastened; this carries a glass funnel whose exit tube is fairly narrow. The string can be caught up as shown in the figure by a clip at A. The whole arrangement forms a pendulum whose length is EB for vibrations perpendicular to the plane of the figure, but for vibrations in the plane of

[1] Barton and Black "Practical Physics," p. 17. Other figures are there given.

the figure it behaves like a pendulum of length AB. Hence if the bob is pulled outwards in a slanting direction and then released the two motions are combined; their relative periods can be adjusted by moving the clip A. A record of the motion can be obtained by putting fine, dry, sand in the funnel and allowing it to run out on a sheet of paper placed immediately below. The purpose of the lead ring is to keep the height of the centre of gravity of the bob constant as the sand escapes, otherwise the periods would vary.

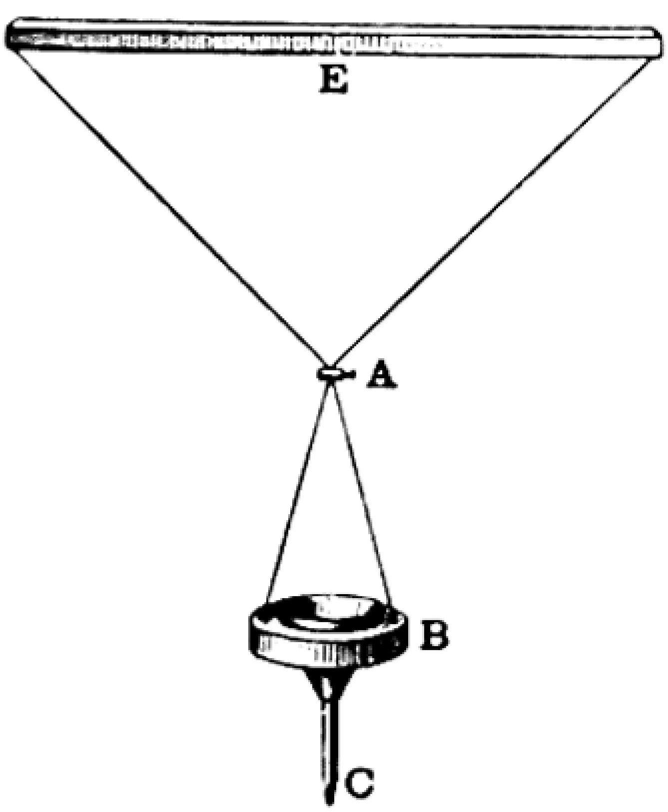


FIG. 157.—Blackburn's Pendulum.

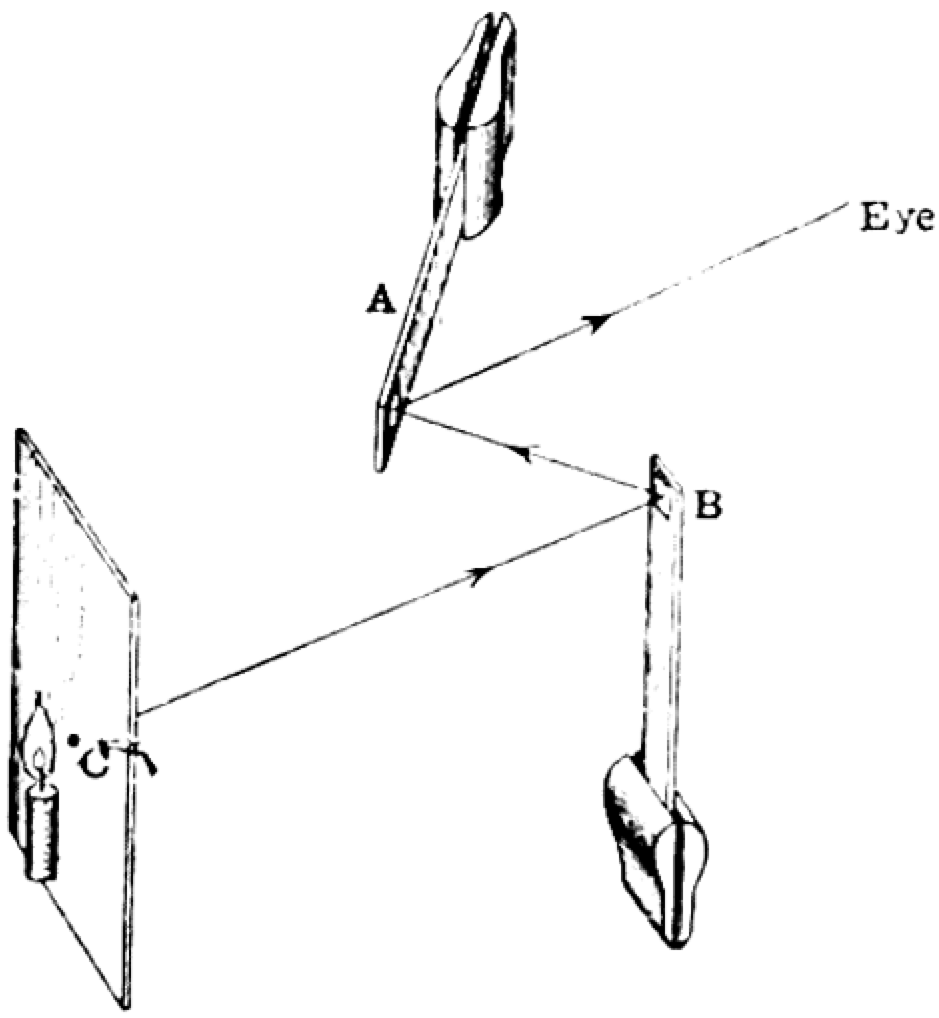


FIG. 158.—Apparatus to produce Lissajous' Figures.

Lissajous' Figures.—Two thin metal strips are supported as shown in Fig. 158, so that one, A, can oscillate in a horizontal and the other, B, in a vertical plane. Each strip carries at its free end a small piece of plane mirror. Light from a small hole C in a card-board screen falls on the mirror B, whence it is reflected to A and thence to the eye. If the strip B alone vibrates the spot of light is drawn out into a vertical line, while if A only is in motion the line is horizontal. When both are oscillating together the two S.H.M.'s are combined. The figures can be projected on a screen if a convex lens is placed between C and B. When produced optically in some such manner the curves are usually called Lissajous' figures.

They have an important application in the comparison of the

frequencies of two vibrating sources. Thus let A and B of the last figure represent the prongs of two tuning forks, and suppose the frequency of B is known to be exactly 100 per second while the frequency of A is very approximately 100. At a certain instant there may be a phase difference of, say, $\pi/2$ and the corresponding circular curve shown in Fig. 156 is seen. If one frequency is exactly equal to the other this curve persists as long as the oscillations last; but if the ratio is slightly different from 1 : 1 the phase difference gradually alters and the curves shown in Fig. 156 appear in succession, until the slower fork has lost one vibration on the other when the phase difference is again $\pi/2$. Let the time that elapses before the circle reappears be 5 secs.; in this interval the one fork has made 5×100 vibrations while the other has performed $\{(5 \times 100) \pm 1\}$. A small piece of wax is now attached to fork A which causes it to oscillate more slowly; if the circle then takes more than 5 secs. to reappear it shows that the frequency of A is more nearly equal to 100, and must therefore initially have been slightly in excess of this. Hence while B made 500 vibrations, A made 501, and the frequency of the latter fork is $\frac{501}{500} \times 100$.

EXPERIMENT.—Use the Lissajous figures to determine when the strips A and B (Fig. 158) have the same period, and measure the length of A. Then alter the length of this strip until its frequency is doubled as shown by the curve produced. Compare the lengths of A in the two cases, the ratio should be $l_1/l_2 = \sqrt{2}$.

EXAMPLES ON CHAPTER XXII

1. Two tuning forks are vibrating so as to give Lissajous' figures. The frequency of the slowest fork is 90 per second. It is found that the same figure, a straight line sloping upwards to the right, recurs at intervals of 5 seconds. Find the frequency of the second fork.

2. Two forks are producing Lissajous' figures and it is found that the same 8-shaped figure occurs at intervals of 6 seconds. If one fork has a frequency of 200 per second find the possible frequencies of the other.

3. Two S.H.M.'s in the same straight line are represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t - \beta)$. Find the resultant trigonometrically and show that it is a S.H.M. of the same period as each of the components.

CHAPTER XXIII

WAVE MOTION. VELOCITY OF SOUND

In the preceding chapter it has been shown that the motion of an oscillating body is simple harmonic provided the amplitude is small; we will now examine what effects it produces in the surrounding medium and how these are transmitted from one point to another.

Wave Motion.—The two possible alternative modes of propagation are illustrated by the case of a toy boat which is being sailed on a pond. If the boat grounds on some obstruction a short distance from the shore it can be moved by two methods (wading in the water being ruled out), either a stone may be thrown at it, or waves may be set up near the bank, and these, travelling along the surface of the water, float it off. Similarly a tuning fork may shoot off particles towards the ear or it may set up waves in the surrounding medium which, falling on the ear, produce the sensation of sound. The evidence is overwhelmingly in favour of the latter alternative. For example, if sound is propagated by waves we should expect that:—

(1) *Time would be necessary for the disturbance to be propagated.*—This agrees with common experience, for the report of a distant gun is heard some seconds after the flash is seen. Also sound travels with different velocities in different media. Thus if a person taps a telegraph pole a listener with his ear to a neighbouring pole hears two sounds, the first louder than the second. The louder sound travels along the wires while the other goes through the air. As the observer recedes from the pole the first sound is greatly weakened.

(2) *A medium is necessary.*—Light waves, as we have seen, can travel through vacuo, but sound waves require a material medium. This is shown by the following experiment.

Experiment.—Hang a small electric bell by its wires in a large jar which can be exhausted and place the whole on a thick piece of felt. When the bell

is rung by an electric current the sound can be heard distinctly, but if air is pumped out it gradually decreases in loudness and is at last heard with difficulty. The felt is to hinder the transmission of the vibrations through the wires and walls of the jar to the table top.

In order that the medium may transmit vibrations it must possess elasticity and be capable of storing potential and kinetic energy (p. 294).

(3) *Sound waves should be capable of reflexion and refraction.*—Experiments given later show that this also is true.

(4) *The waves should be diffracted, i.e.* they should bend round obstacles just like ripples on water bend round a stone and meet

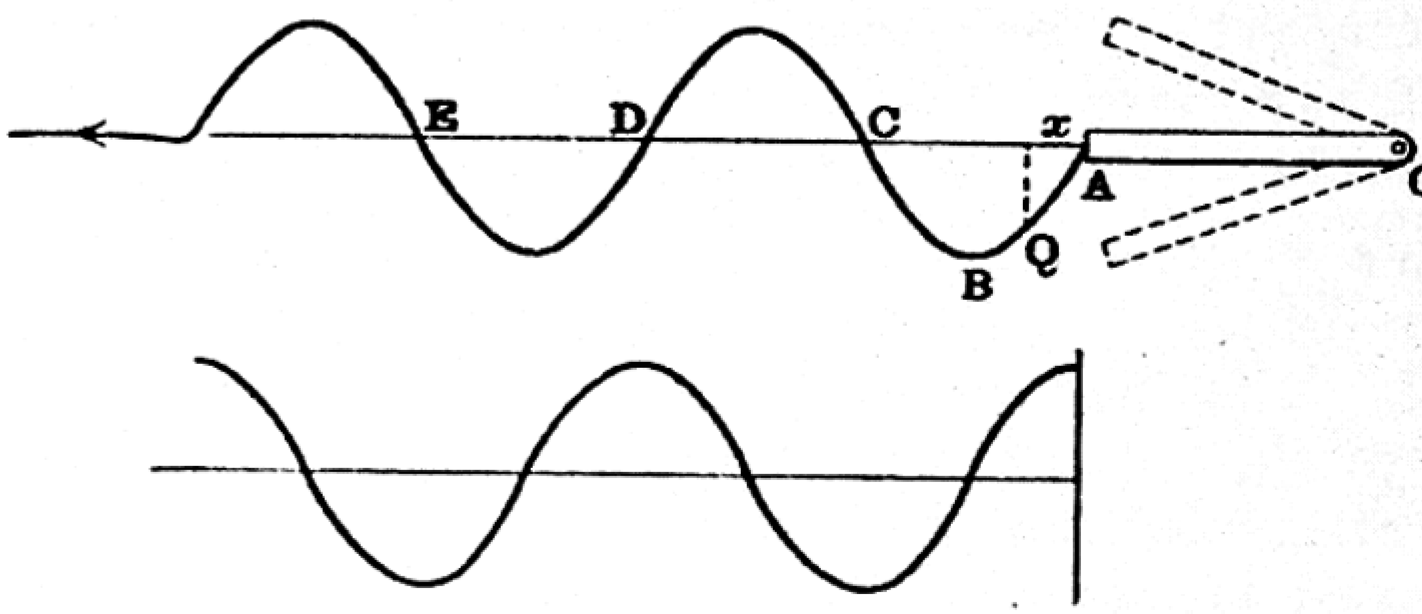


FIG. 159.—Waves along a Wire.

again behind it. It is a common experience that we can hear a person's voice round the corner of a house.

(5) *The waves should interfere.*—If the medium at a certain point is acted upon by two sets of waves, so that the displacements due to each are equal and opposite at every instant, then the medium should be at rest. This also is established by experiment.

Transverse Waves.—Let OA (Fig. 159) represent a rod with one end fixed at O and the other fastened to a long, loosely stretched, string. If the rod is made to vibrate a number of loops travel along the string as shown in the figure. The particles vibrate up and down while the disturbance travels to the left. In the figure the particle B has reached its maximum displacement downwards and is on the point of being pulled up again by the tension. Each part of the string undergoes in succession the same motion as the end of the rod, but the instant at which this occurs depends on its distance from A and the velocity with which the disturbance is propagated. Such a progressive disturbance is called a **wave**. The point to be

noticed is that although the particles only oscillate about a mean position their energy is carried along by the wave, matter does not move to the left but energy does. The distance between successive particles in the same phase of vibration is called the **wave length.** In the figure the wave length is CE, for each of these points is in its mean position and is about to move downwards. D is also in its mean position but is on the point of moving upwards, CD is therefore half a wave-length. During one complete vibration of the rod the wave travels a distance CE or AD, the wave-length is therefore the distance the disturbance travels in the periodic time T. The velocity of the wave is the distance it travels in one second. Let V be the velocity, n the frequency of the vibration, and λ the wave-length. Then in 1 sec. the rod makes n vibrations and the wave travels a distance $n\lambda$, hence $V = n\lambda$. Also $T = 1/n$, therefore $VT = \lambda$. It can be shown that $V = \sqrt{\frac{F}{m}}$ cms./sec. if F is the tension of the string in dynes, and m the mass in gms. of 1 cm. length.

Waves of this type are called **transverse,** because the displacements of the individual particles are transverse to the direction in which the wave advances. If the displacement of A is simple harmonic and is represented by $a \sin \omega t$ or $a \sin \frac{2\pi}{T} \cdot t$, then the displacement of a point Q at the same instant is given by $a \sin \left(\frac{2\pi}{T} \cdot t - \alpha\right)$, where α is the phase difference between A and Q. But the phase difference going from A to D increases by 2π, since one is a period behind the other, hence the phase of Q is $\alpha = \frac{x}{AD} \cdot 2\pi = \frac{x}{\lambda} \cdot 2\pi$ behind A. Thus the displacement of Q at any instant is given by $a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$. By putting in appropriate values of x and t this expression gives the displacement of any particle of the string; it therefore represents a simple harmonic wave of wave-length λ and period T advancing in the direction in which x increases. The velocity of all the particles at any instant can be represented by a velocity curve. In Fig. 159 A is moving upwards with its maximum velocity while B is momentarily at rest. The lower curve in the figure is the curve of velocities.

Longitudinal Waves.—Let us suppose now the rod OA is vibrating

to and fro at the ends of a tube containing air (Fig. 160). The air at B will be alternately compressed and rarefied as the rod moves to the right or left and carries the adjacent particles with it. When it is compressed it tends to release itself from the strain on account of its elasticity; to do this it compresses the layers immediately to the right, and these in turn hand on the compression. An instant later A moves to the left, the air at B is rarefied, and the neighbouring layers move slightly in the same direction. Thus the molecules move alternately to the right and left and a series of compressions and rarefactions travels along the tube. These are represented in the figure by lines drawn closer together or further apart. A wave of this type is called a **longitudinal** wave; the particles oscillate along the direction of the advancing wave instead of at right angles to it as in the last case. As before it is energy and not matter which is

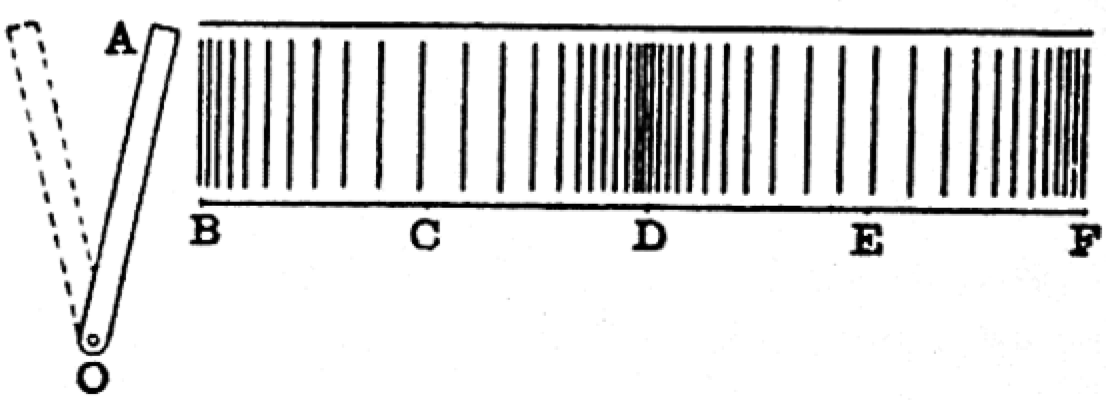


Fig. 160.—Longitudinal Waves.

transmitted. A continuous surface which passes through all particles in the same phase of vibration is called the **wave front**; it may be drawn through the particles having their maximum compression or any other phase. The definition of wave-length already given holds generally. We have supposed that the tube is filled with air, but a similar description would apply if it contained liquid or solid matter, and the tube, of course, is not necessary for the propagation of the waves. Newton showed that the velocity of longitudinal waves is given by $V=\sqrt{\frac{E}{\rho}}$, where E is a modulus of elasticity and ρ is the density of the medium. In the case of solid rods the compressions and extensions produce momentary changes of length at the different parts, the elastic forces called into play therefore depend on Young's modulus, and the velocity is $V=\sqrt{\frac{Y}{\rho}}$. A rod may be thrown into longitudinal vibration by clamping it at some point and rubbing it lengthwise with a resined cloth. In fluids the compressions bring

about volume changes, and it is owing to the bulk elasticity that the waves are transmitted ; in such cases E represents the bulk modulus. For gases it has been shown (p. 8) that the bulk modulus is equal to the pressure P, hence the velocity of sound in gases should be $V = \sqrt{\frac{P}{\rho}}$. This result is due to Newton.

Laplace's Correction.—Let us calculate from this formula the velocity of sound in air at N.T.P. From p. 8 P = 1,013,000 dynes, and a litre of air weighs 1·293 gms., hence $\rho = 0{\cdot}001293$. Substituting these values we get V = 280 metres/sec. Actual measurement, however, shows that V = 332 m./sec., a difference much too large to be explained by experimental errors. It was Laplace in 1816 who pointed out the source of the discrepancy. Suppose some gas is compressed in a cylinder by suddenly pushing in a piston, its temperature rises (p. 114) and makes it more difficult to compress. Similarly if the piston is suddenly withdrawn by a small amount the temperature falls, the inside pressure decreases, and more force must be applied to keep the piston moving outwards against the atmospheric pressure. In other words, the adiabatic elasticity (p. 113) of the gas is greater than the isothermal elasticity (p. 8). Laplace pointed out that the compressions and rarefactions in a sound wave occur so rapidly that heat has no opportunity of flowing out of or into the gas—that is, the volume changes take place adiabatically. It is therefore the adiabatic elasticity which must be used for E in the formula $V = \sqrt{\frac{E}{\rho}}$. We will calculate the adiabatic elasticity. Let P be the pressure and V_1 the volume of a certain mass of gas ; suppose the pressure is increased adiabatically by a small amount p and the volume decreased by a small amount v. Then the

$$\text{adiabatic elasticity} = \frac{\text{stress}}{\text{strain}}$$

$$= p \bigg/ \frac{v}{V_1} = \frac{pV_1}{v}$$

Also from p. 113 $PV_1^{\gamma} = (P + p)(V_1 - v)^{\gamma}$

But $$(V_1 - v)^{\gamma} = V_1^{\gamma}\left(1 - \frac{v}{V_1}\right)^{\gamma}$$

$$= V_1^{\gamma}\left(1 - \gamma \cdot \frac{v}{V_1} + \text{higher powers of } \frac{v}{V_1}\right)$$

by the binomial theorem.

Neglecting these higher powers since v/V_1 is very small

$$(V_1 - v)^\gamma = V_1^\gamma\left(1 - \gamma \cdot \frac{v}{V_1}\right)$$

Hence
$$PV_1^\gamma = (P + p)V_1^\gamma\left(1 - \gamma \cdot \frac{v}{V_1}\right)$$

whence
$$p = \gamma\frac{Pv}{V_1} + \gamma\frac{pv}{V_1}$$

The last term is very small since p and v are each small, hence it may be neglected, and

$$p = \gamma \cdot \frac{Pv}{V_1}$$

or the adiabatic elasticity $\dfrac{pV_1}{v} = \gamma P$

This shows that the adiabatic is γ times the isothermal elasticity P. Putting this in place of E we get for the velocity $V = \sqrt{\dfrac{\gamma P}{\rho}}$

For air $\gamma = 1{\cdot}40$, the previous result must therefore be multiplied by $\sqrt{1{\cdot}4} = 1{\cdot}18$, and $V = 331$ metres/sec., in close agreement with experiment.

Effect of Pressure and Humidity on the Velocity in a Gas.—From Boyle's law (p. 6) it is seen that P/ρ is constant if the temperature does not alter, hence the velocity of sound in a gas, $V = \sqrt{\dfrac{\gamma P}{\rho}}$, is independent of the pressure. This has been verified by measuring the velocity from one mountain peak to another, the result, reduced to 0° by the formula of the next paragraph, agrees with the measurements made at ordinary altitudes. If there is a large amount of water vapour in the air the velocity of sound will be increased, for the vapour is less dense than air and hence ρ of the above formula is reduced.

Effect of Temperature.—A change in temperature may affect both the pressure and the density of a gas, to see what influence this has on the velocity the gas equation (p. 69)

$$\frac{Pv}{1 + at} = \frac{P_1v_1}{1 + at_1} \text{ is employed}$$

If v is the volume of 1 gm. and ρ the density, since mass $=$ (vol. $\times$ density), we have $1 = v\rho$, or $v = 1/\rho$. Hence the equation may be written

$$\frac{P}{\rho(1 + at)} = \frac{P_1}{\rho_1(1 + at_1)} = \frac{P_0}{\rho_0}$$

where P and ρ are the pressure and density at $t°$ Cent. and P_0, ρ_0 the corresponding quantities at 0° Cent.

Thus $$\frac{P}{\rho} = \frac{P_0}{\rho_0}(1 + at)$$

also $$V_0 = \sqrt{\frac{\gamma P_0}{\rho_0}}$$

and $$V = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma \, . \, P_0}{\rho_0}(1 + at)}$$

or $$V = V_0\sqrt{1 + at}$$

If we put $a = 1/273$ we get

$$\frac{V}{V_0} = \sqrt{\frac{273 + t}{273}}$$

thus the velocities are proportional to the square roots of the absolute temperatures.

Also $$V = V_0(1 + at)^{\frac{1}{2}}$$
$$= V_0(1 + \tfrac{1}{2}at) \text{ (by the binomial theorem)}$$

if squares and higher powers of the small quantity at are neglected.

Taking $V_0 = 332$ metres/sec.
and $a = 0{\cdot}00366$
this becomes $V = 332 + {\cdot}6t$.

This shows that the velocity increases by 0·6 m. or 60 cms. for every degree rise in temperature. The molecular theory (p. 8) shows that the velocity should increase with the temperature, for the compressions have to be handed on by collisions between the molecules, and these occur at shorter intervals as the molecular velocities increase. If a wind is blowing with a velocity v in the same direction as the sound is travelling the molecular velocities are increased by the same amount and the velocity of sound is $(V + v)$. Similarly for an opposing wind the velocity is $(V - v)$.

Measurement of the Velocity of Sound in Open Air.—In some experiments by a French Commission two cannon were situated at stations in sight of each other and about 12 kilometers apart. One cannon was fired and the observers at the other station noted the times at which the flash was seen and the report heard. The flash was seen practically instantaneously owing to the great velocity of light, hence the recorded times gave the interval required by the sound to travel over 12 kms. The velocity of sound in the opposite direction was then measured in a similar manner and by taking the mean of the two results the effect of the wind was eliminated. Their result reduced to 0° was 332 metres/sec. in dry air. There are several sources of error in such experiments which it is difficult either to remove or allow for. Thus the mean taken above will not be independent of wind velocity unless this is constant, and the temperature and humidity may vary from place to place. Another error arises from personal causes. A certain short interval elapses between the hearing of the sound and the recording of it by suitable timing apparatus such as a stop-watch, and this may be different with different observers. Mr. Stone endeavoured to eliminate this error by taking the observations as follows:—One observer was about 600 ft. from the gun and a second about 15,000 ft. The difference between the times they recorded gave the time required by sound to travel from one to the other, but slightly in error on account of personal differences. If the personal equation, as it is called, were the same for each the recorded interval would have been correct, otherwise it was affected by the difference of the two. Thus if the first observer was 0·3 sec. late and the other 0·1 sec. late the recorded interval would be 0·2 sec. too small. The two persons were next made to time the arrival of a sound at the same station, the difference between the times they recorded was the difference of their personal equations; this was used to correct their first observations. Greely has made a number of measurements in the Arctic regions over a wide range of temperature and finds the velocity agrees practically with that given by the equation $V = 332 + 0{\cdot}6t$.

Velocity in Water.—Colladon and Sturm measured the velocity of sound in the Lake of Geneva in 1827. The experiments were carried out at night. A bell was hung in the water from a boat and was struck by a lever worked from above. The same blow fired a charge of gunpowder, causing a flash which could be seen at a distant station. The sound travelling through the water was received on a

kind of large ear-trumpet whose lower end, sunk in the water, was closed by a flexible membrane while the upper end was applied to the ear. The time taken by the sound to travel over a known distance in the water could thus be found. The velocity was found to be 1435 metres/sec. Methods of measuring the velocity in solids are given later.

EXAMPLES ON CHAPTER XXIII

1. Explain why the rise of temperature due to compression and the fall of temperature due to rarefaction in a sound wave *both* tend to raise the velocity of propagation of the wave. (L. '84.)

2. How does the velocity of propagation of sound through a gas vary with the specific gravity and temperature of the gas? The specific gravities of oxygen and nitrogen gases are as 16 : 14. At what temperature will the velocity of propagation of sound through oxygen be the same as that through nitrogen at 15° C.? (L. '93.)

3. Upon what properties of a solid does the speed of sound in the solid depend? Are all kinds of waves in a solid propagated with the same speed? Why does sound travel faster in steel than in air? (L. '97.)

4. Show that the expression $y = a \sin \frac{2\pi}{\lambda}(x - vt)$ represents a train of waves of amplitude a and wave-length λ moving along the x axis with velocity v. Draw curves showing the variation of the displacement y (1) with the time at a point $x = \frac{\lambda}{2}$, and (2) with x at a time $t = \lambda/v$. (L. '09.)

5. A litre of hydrogen at N.T.P. weighs 0·0896 gm. Find the velocity of sound in hydrogen at a temperature 16° when the pressure is 750 mm., the ratio of the specific heats being 1·4, density of mercury 13·6, $g = 980$.

CHAPTER XXIV

REFLEXION, REFRACTION, AND INTERFERENCE OF SOUND WAVES

Loudness. Pitch. Quality.—If different regular sounds or musical notes are compared it is found that they differ from each other in three characteristics which are called **loudness, pitch, and quality.**

Loudness.—This corresponds to brightness in optics and like it depends on the amount of energy carried by the incident waves. The amount of energy in ergs which passes in one second through an area of 1 cm.^{2} is called the **intensity** of the sound. The unit area is supposed perpendicular to the direction of propagation. Erect on this area a column of length V and section unity, where V is the velocity of sound. In one second all the energy in the column passes through the given area. To calculate its amount we note that the average energy W of a mass m performing S.H.M. is proportional to $m\omega^2\omega^2$, hence

$$\begin{aligned} W &\propto m\omega^2 a^2 \text{ (p. 253)} \\ &\propto \frac{4\pi^2}{T^2} \cdot a^2 m \end{aligned}$$

If ρ is the density of the medium the mass of the column is $V\rho$ and its energy varies as $\frac{4\pi^2}{T^2} a^2 V\rho$. This expression is proportional to the intensity of the sound. Loudness is a physiological effect and depends on the ear, but the greater the intensity the louder will the sound be. By the method of p. 220 it can be shown that the intensity varies inversely as the square of the distance from the source. From the above expression it is seen that the intensity I is proportional to the square of the amplitude of the vibrations, hence combining the two results we have, if R is the distance from the source,

$$I \propto a^2 \propto \frac{1}{R^2}$$

Therefore $a \propto \frac{1}{R}$, or the amplitude varies inversely as the distance from the source.

Pitch.—We speak in everyday language of the pitch of a musical note, thus we talk of the high (pitched) notes of a soprano or the low notes of a bass singer. The pitch of a note is determined by the frequency of the vibrations, the greater the frequency the higher the pitch. All musical notes have a definite frequency; this distinguishes them from mere noises, such as the sound of wheels on a macadamised road. In this book we are concerned only with sounds of definite frequency.

EXPERIMENT.—*Savart's wheel.* Clamp a short strip of steel so that its free end rests on the edge of a circular saw or toothed wheel. When the wheel rotates the steel spring vibrates and a note is emitted if the frequency is high enough. It will be found that the pitch rises as the velocity of the wheel is increased.

EXPERIMENT. *Cardboard siren.* A cardboard circle pierced by a ring of small, equally spaced, holes is mounted on an axis perpendicular to its plane and rotated rapidly. If a jet of air is directed on the holes the successive puffs succeed each other quickly and a note is produced. As in the last experiment the pitch rises as the disc is turned more rapidly.

EXPERIMENT.—Strike a tuning fork to make it emit a note; as the amplitude decreases the loudness dies away but the pitch remains the same. This shows that the pitch does not depend on the amplitude while the loudness does.

Quality.—Even when two notes are of the same pitch and loudness they may differ from each other. Thus the notes of an organ and a violin are quite distinctive, and we recognise the voices of our acquaintances. This characteristic which differentiates one note from another of the same pitch is called the quality of the sound. It will be seen later to depend on the presence of other notes in addition to the main or fundamental note.

Doppler's Principle.—When a source, *e.g.* a whistle, is emitting a note of frequency n, a person at a distance receives in each second n sound waves, but if the distance between the observer and the source is being altered the pitch of the note is apparently changed. This was first pointed out by Doppler and is called **Doppler's Principle.** Suppose the whistle is on the left, the observer on the right, and let each be moving to the right, the former with velocity b the latter with velocity c. Let V be the velocity of sound. During one second the whistle emits n waves, if it were stationary these

would occupy a length V, but as in this time it moves to the right a distance b the n waves are now contained in a length $(V - b)$. Hence the distance apart of the waves, *i.e.* their wave-length, is $(V - b)/n$. The length of the block of waves passing a stationary observer per second is V, hence the length passing the moving observer is $(V - c)$.

As the length of a wave is $\frac{V-b}{n}$ the number he receives per sec.

$$= (V - c) \div \frac{V-b}{n}$$

$$= \frac{n(V-c)}{V-b}$$

This is the apparent frequency of the whistle. If either motion is reversed the sign of the corresponding quantity must be altered. It should be noted that the physical effect of the two motions is different, the movement of the source alters the wave-length while that of the observer varies the number of waves received. The effect is frequently noticed; thus the pitch of a train whistle is higher when the train is moving towards the observer than when it is receding, and the same occurs with a bicyclist's bell.

Sensitive Flames.—It has been shown that when a beam of light falls on the surface of separation of two media part is reflected and part refracted. Exactly the same thing happens with sound waves. In the study of these effects it is convenient to have some means of detecting the waves other than the ear, we will therefore describe some sensitive flames which can be used for the purpose. The first form can be set up by the student in a few minutes.

Experiment.—Draw off a piece of glass tubing 0·5 cm. in diameter to form a jet about 0·5 mm. in diameter. Connect it to the gas supply and fix it about 3 cms. below a horizontal piece of fine copper gauze. Light the gas above the gauze, as we have seen (p. 117) it does not burn below. If the glass jet is too low the flame flickers badly, adjust the height until it is just on the point of flickering. When a high note is sounded in its vicinity the flame ducks violently; it is especially sensitive to the sound of the letter *s*. It is an improvement to protect the flame with a glass chimney.

The second form, shown in Fig. 161, is due to Lord Rayleigh.[1] A is a brass cylinder about 4 cms. long · one end is closed while the

[1] I am indebted to Prof. S. P. Thompson for the actual dimensions. The apparatus is, I believe, supplied by Messrs. Gallenkamp according to Prof. Thompson's specifications.

other is covered with a piece of thin tissue paper. Gas from the mains enters at the lower side and escapes through the burning tube above where it is lighted. If the supply is properly adjusted the flame appears to be detached from the exit tube; it is in this condition that it is sensitive. When sound waves of suitable frequency fall on the tissue paper the escaping gas is disturbed and the flame flickers. It is especially sensitive to explosive sounds like the letters *p*, *b*, and responds violently if "Peter Piper picked, etc." is recited near it. It can be made more sensitive for high notes by placing over the orifice a plate pierced with a hole of small diameter.

The third form is most useful but has the disadvantage that it requires gas at a higher pressure than the usual supply. Gas under pressure, *e.g.* from a steel cylinder of compressed gas, is led through a fine pin-hole jet and is there burnt. A flame about 25 cms. high is obtained which flares badly if the pressure is too large; it is adjusted until it is just on the point of flaring. When a high note is sounded the flame shortens to a length of a few cms. and flickers violently. The most convenient source of sound when sensitive flames are used is a Galton's whistle, this gives a very high-pitched note.

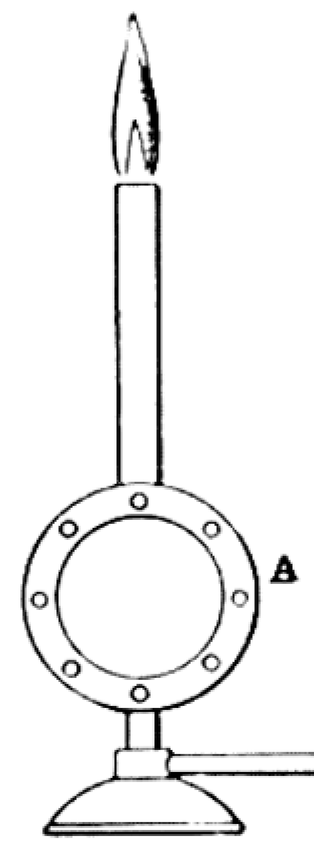


FIG. 161.—Rayleigh's Sensitive Flame.

Reflexion of Sound.—The apparatus used to show the reflexion of "heat waves" (p. 160) may be used for the present purpose, except that the source is a Galton's whistle and the receiver a sensitive flame.

EXPERIMENT.—Incline the two glass tubes (p. 160) to each other at an angle of about 120°, place the whistle at the outer end of one and the flame at the corresponding end of the other. The latter must be protected by a wooden screen from direct waves. It should not respond when the whistle is blown, but if a piece of cardboard is equally inclined to the tubes at their further ends it flares violently, showing that the angles of incidence and reflexion are equal. The cardboard may be replaced by a flat bat's-wing gas flame when the sensitive flame responds as before. The waves are reflected from the hot, and therefore less dense, layer of burning gas. The experiment with the concave mirrors (p. 159) may also be performed with success; the whistle is placed at the focus of one mirror and the sensitive flame, screened from direct waves, at the focus of the other.

Echoes are due to reflexion; sound travels from the source to a

surface, which may be a wall or mountain side, and is there reflected back again. Whispering galleries act in a similar way. The sounding board above a pulpit is put there to reflect the sound of the preacher's voice down to the audience. In some buildings it is very difficult to hear a speaker on account of the numerous reflexions which set up a series of echoes; for this reason sharp angles and large flat walls should be avoided in any room intended for public speaking.

Refraction of Sound.—Tyndall showed that sound could be refracted and brought to a focus by a convex lens just like light. He employed a Galton whistle and sensitive flame, and for a lens a soap bubble filled with a dense gas like carbon dioxide was used. It is a well-known fact that sounds can be heard much more distinctly on a cold frosty morning than on a warm summer's day. In the first instance the air is homogeneous and sounds travel in straight lines, while in the latter the temperature varies from point to point and the waves encounter layers of air of varying density. This causes them to be reflected and refracted in different directions. The effect can be imitated by an experiment of Tyndall's.

Experiment.—Set up a whistle and sensitive flame a few yards apart and screen the latter from all waves except those travelling direct. Place a large ring burner between the two; when the gas is lighted the response of the flame is greatly enfeebled. The effect is analogous to the flickering of objects frequently noticed on hot days; this is due to the varying refraction of the light rays by the unequally heated air. If the whistle and flame are placed on the same side of the burner the reflexion of the sound waves can readily be shown.

Refraction by Wind.—It is commonly observed that sound travels better with the wind than against it; the reason for this was first given by Sir G. Stokes. Let AB (Fig. 162 *a*) represent a wave front advancing towards the left and suppose the wind to be blowing in the same direction. The layer of air next to the ground is at rest and the upper layers slide over it, hence, owing to viscosity, the velocity of the wind gradually increases with the height. This causes the higher parts of the front to advance more quickly than the lower, and, at some later instant, instead of occupying the position A_1B_1 it is swung round to A_1B_2. An observer at P therefore receives the advancing wave. Similarly if the wind is blowing to the right the upper portions of the wave front are more delayed than the lower and it assumes the direction A_1B_2 (Fig. 162 *b*). In this case the

wave passes over the head of the observer at P. Combining these results it is clear that he will hear the sound best when it is travelling with the wind.

Reflexion of Sound by a Wall or a Dense Medium.—In Fig. 163 let PQ represent a wall; suppose sound waves travelling in air to be approaching from the right and falling on it at perpendicular incidence. Consider what happens when a compression reaches PQ. The compressed layer in contact with the wall tries to relieve itself from its strained condition, the only way in which it can succeed is to push back the neighbouring layers, *i.e.* compress them from the left. These layers compress those still further away and so a wave of compression is reflected backwards. The displacements of the molecules due to the incident and reflected waves are in opposite directions. Similarly when the layer next to the wall is rarefied adjacent particles move towards it from the right and a rarefaction travels outwards. The waves are said to be reflected **with change of sign** because the motions of the particles are reversed by reflexion. To find the state of the air in the region on the right we must draw the velocity or displacement curves of the incident and reflected waves and find their resultant as on p. 256. The reflected waves can be represented by a wave train moving from the left and crossing the wall to the right; to get their phase we notice that the air close to the wall is permanently at rest, hence at this point the velocities due to the two sets of waves must be equal and opposite at every instant. In Fig. 163 the thin and dotted lines are the velocity curves of the incident and reflected waves respectively, while the thick line represents the resultant of the two. In the first line, for $t = 0$, the curves have been drawn so that the velocity at the wall due to the incident wave is a maximum, that arising from the reflected wave is, from what has just been said, equal and opposite. The next line represents the state of affairs $\frac{1}{8}$th of a period later. To get this we have merely to move the thin curve $\frac{1}{8}$th of a wave-length

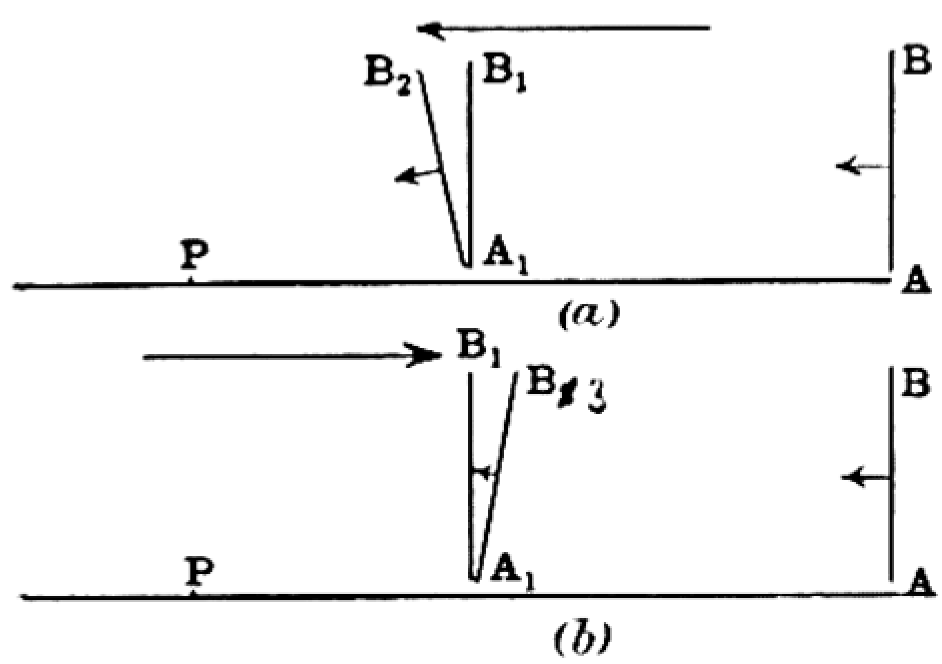


FIG. 162.—Refraction by Wind.

to the left, and advance the dotted curve the same distance to the right. The remaining figures are constructed in a similar manner.

Remembering that the velocity of a particle is zero when its displacement is a maximum and *vice versâ*, (see Fig. 159), the curves of displacement can easily be constructed ; they are like the velocity

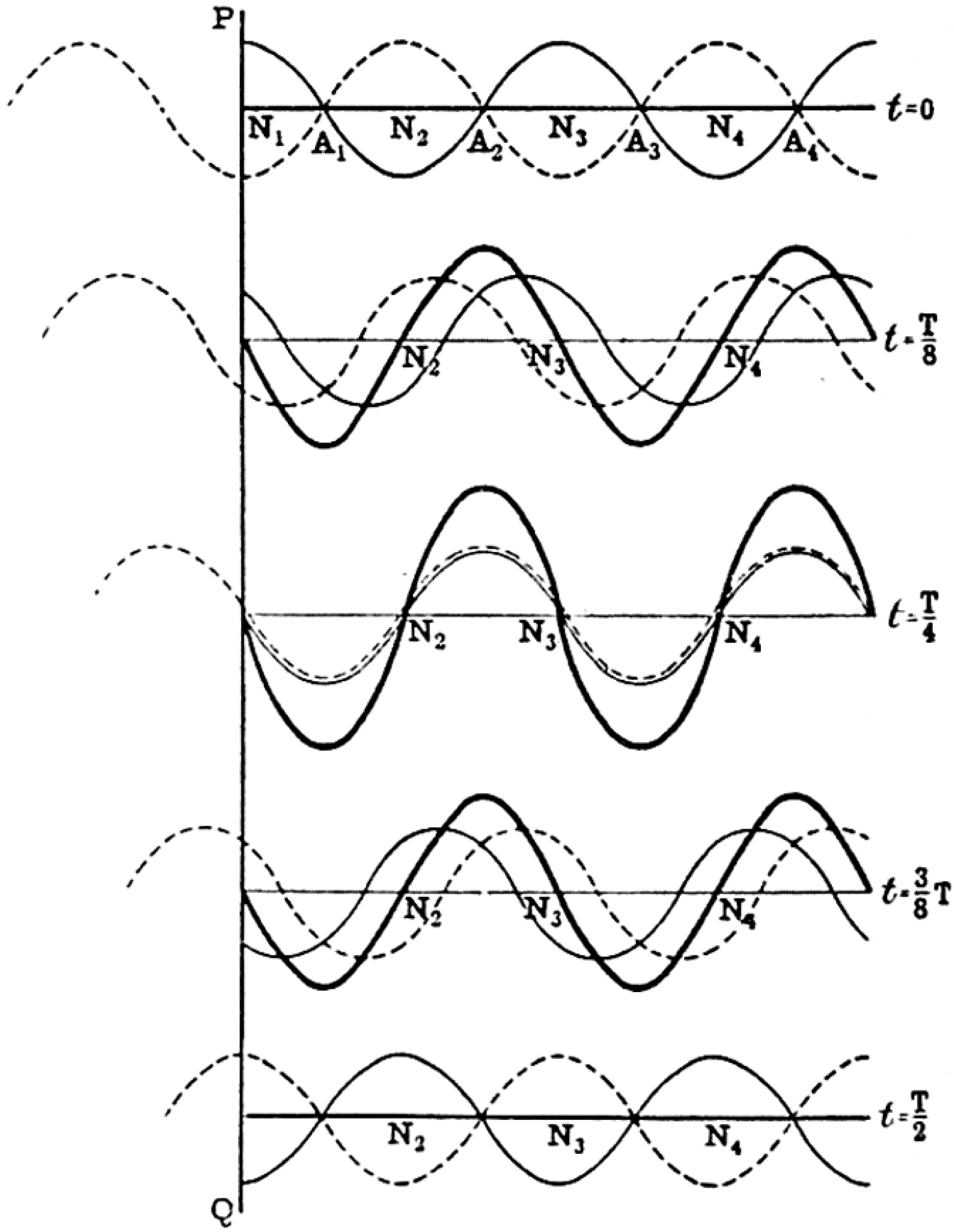


FIG. 163.—Reflexion of Waves from a Wall.

curves but come T/4 later. Some important conclusions can be drawn from the figure which show that the motion of the air differs considerably from that in an ordinary wave. In the first place the air particles at N_1, N_2, etc. are always at rest, as is shown by the resultant always crossing the axis at these points. N_1, N_2, etc. are called **nodes**, they are $\lambda/2$ apart, where λ is the wave-length. Another

important difference is that twice during every period all the molecules are momentarily at rest together, this is shown at $t = 0$ and $t = T/2$. Further, in a single wave train such as the incident waves the amplitude of the motion is the same for each particle and the maximum displacement is reached at different times; here the amplitude varies from point to point, being zero at the nodes and a maximum midway between them at A_1, A_2, etc. The latter points are called **antinodes.** Also all the particles situated between adjacent nodes reach their maximum positive velocities (or displacements) at the same instant, while between the next pair on either side the velocities (or displacements) at the same moment have their greatest negative values. In the figure the velocity between N_2 and N_3 is a maximum at $t = T/4$, in the neighbouring sections N_1 N_2 and N_3 N_4 it is equally large but in the opposite direction. Hence the velocity changes sign when we cross a node. Consider next how the pressure varies. We will suppose that a positive velocity means that the particles are moving to the right. In the figure from $t = 0$ to $t = T/2$ all the molecules between N_2 and N_3 are moving to the right, between N_3 and N_4 they are moving to the left, in each case the motion is towards N_3, hence the air at this point becomes compressed while at N_2 and N_4 it is rarefied. Similarly for the other nodes the density is alternately greater and less than the normal. Half a period later the air at N_3 is rarefied, that at N_2 and N_4 is compressed, and so on. The layers on opposite sides of an antinode and at equal distances from it are moving in the same direction with equal velocities, hence no variation in density ensues at A_1, A_2, etc. Thus the nodes are the points at which the density changes are the greatest but the velocity and displacement of the particles are a minimum, at the antinodes these conditions are reversed. Vibrations of this character are called **stationary vibrations** or **standing waves.** Generally the amplitude is slightly reduced by reflexion, in which event the velocity at the nodes is not exactly zero. We have supposed that PQ is unyielding, if it represents the surface of a medium on the left which is denser than air reflexion will occur as before, but the amplitude of the reflected wave will be diminished as some of the incident energy will be transmitted into the second medium.

The position of the nodes and antinodes can be found experimentally. The source of sound should be a Galton whistle and the detector a sensitive flame. If the latter is placed at a node it does

not flare as the molecules are not in vibration, at an antinode it is greatly disturbed. The position of the antinodes can be fixed directly by ear. One end of a piece of tubing is held close to the ear while the other end is moved in a direction normal to the wall; the pressure changes at the nodes affect the ear drum and a sound is heard, at the antinodes the density does not vary and there is silence. As the distance between successive nodes or antinodes is $\lambda/2$ the wave-length can be found directly by such an experiment. Also $V = n\lambda$, and V being known from the last chapter the frequency n can be calculated. By an experiment like this the frequency of the whistle can be found even when it is so high that the ear cannot detect a sound, hence the frequency at the limit of audibility can be fixed.

Reflexion without Change of Sign.—Let us examine now what takes place when the wall of Fig. 163 is replaced by a medium less dense than air. Imagine a wave advancing to the left in Fig. 163 with an amplitude a. Each layer of air receives momentum from the wave which it expends in setting the succeeding layer in motion. But when the final layer at PQ has moved through a distance a it has not lost all its momentum as the lighter medium is easier to move, it therefore advances a further distance b to the left, causing the air behind it to be rarefied. The next layer to the right moves towards this space, and so on; thus the wave of compression sets up a reflected wave of rarefaction in which the amplitude is b. As the molecules in the incident and reflected waves move in the same direction this is called **reflexion without change of sign.** The student can show in the same way that a rarefaction is reflected as a compression. At the surface of separation of the two media it is evident that the displacements are large, hence this is an antinode; the first node occurs one-quarter of a wave-length to the right. The curves analogous to those in Fig. 163 can easily be constructed; all that has to be remembered is that the velocities at PQ due to the two sets of waves are in the same direction. For example, at $t = 0$ the curve representing the reflected wave must be moved $\lambda/2$ to the left from the position shown in the figure.

Interference.—The last two paragraphs show that the motion of the medium is greatly modified when a second system of waves is superposed on the first, at some points the amplitudes are greatly exaggerated, at others reduced to zero. When these conditions are

produced the two sets of waves are said to **interfere.** Interference may also occur between waves originating from different sources. Let A, B (Fig. 164) represent two sources which are emitting sounds of the same period and amplitude. If the waves arrive at P in the same phase they will produce a large displacement and an observer at this point will hear a loud sound. If, however, the distances AP, BP differ by $\lambda/2$, or any odd multiple of this, the waves arrive in opposite phases, the displacements they produce are equal and opposite and the medium is undisturbed. As we have supposed that the sources have the same period the same phase difference will persist at P while the vibrations last. These effects can easily be shown with a tuning fork.

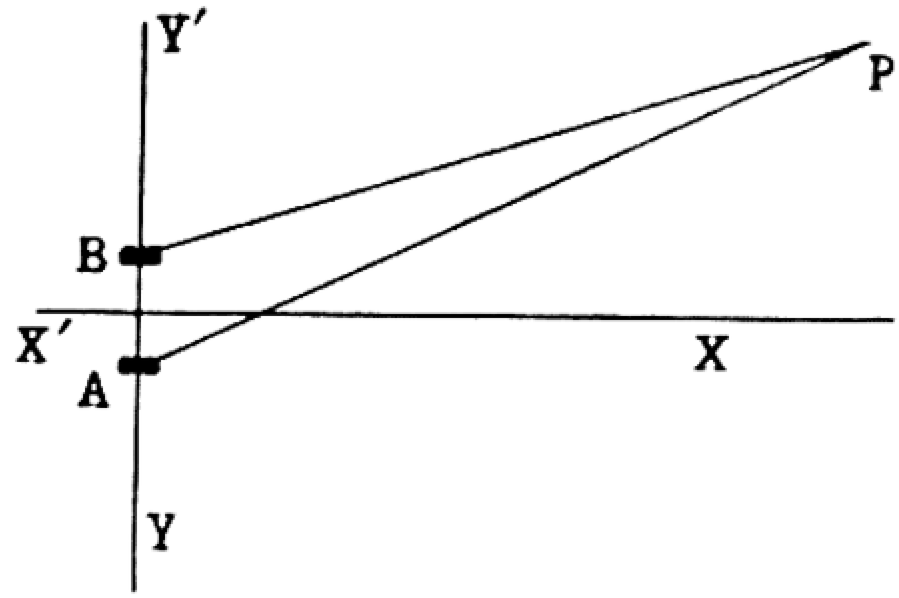


FIG. 164.—Interference of Two Sources.

EXPERIMENT.—Let A, B represent the prongs which are vibrating to and fro along YY′ (Fig. 164). An observer at Y is affected only by the prong A, as this screens off the effect of B, accordingly a loud sound is heard. Similarly when the prongs are approaching or receding from each other the air between them is compressed or rarefied, and their joint effect is to send out strong waves along XX′. But to P, midway between these lines, when the prongs are approaching each other B is sending a rarefaction and A a compression, hence the two effects annul each other and the observer at P hears only a faint sound. Strike the tuning fork and rotate it round a vertical axis a short distance from the ear, the sound alternately waxes and wanes as the ear comes into the positions, relatively to the fork, shown at X, P, Y′.

Conditions for Interference.—For interference to take place it is necessary that—

(1) The two sets of waves should have the same period, otherwise their phase difference at P would not constantly be π or $\lambda/2$; at one moment they would destroy each other but at the next they would render mutual assistance and a large displacement would be produced.

(2) The displacements they produce should be in the same straight line. If they are not so the particles of the medium would not be reduced to rest but would describe a kind of Lissajous figure.

(3) The amplitudes should be nearly equal, then they can nearly annul each other's effect.

It will be noticed that these conditions are fulfilled in the two cases of stationary vibration described above.

Further Examples of Interference.—The wave-length of a sound can be determined by means of Quincke's tube (Fig. 165). A and D are brass or glass tubes about 3 cms. in diameter, B and C are slightly narrower. The part C can be pushed in or out through a distance of several cms. A Galton whistle is sounded near A; at D a sensitive flame is fixed which is screened from direct waves by a piece of cardboard S. The waves in the tube can reach D by either of the two paths ABD or ACD; if these are of equal length the two wave trains arrive in the same phase and produce a maximum disturbance of the flame. When the slider C is pulled out a distance $\lambda/4$ the path along this route is increased by $\lambda/2$; the waves then interfere at D and the flame is undisturbed. If C is withdrawn a further $\lambda/4$ the path difference is λ, the waves are again in phase and the flame flares. Suppose that between successive states of rest for the flame C has been withdrawn 2 cms., then the path difference has been increased by one wave-length and $\lambda = 4$ cms.

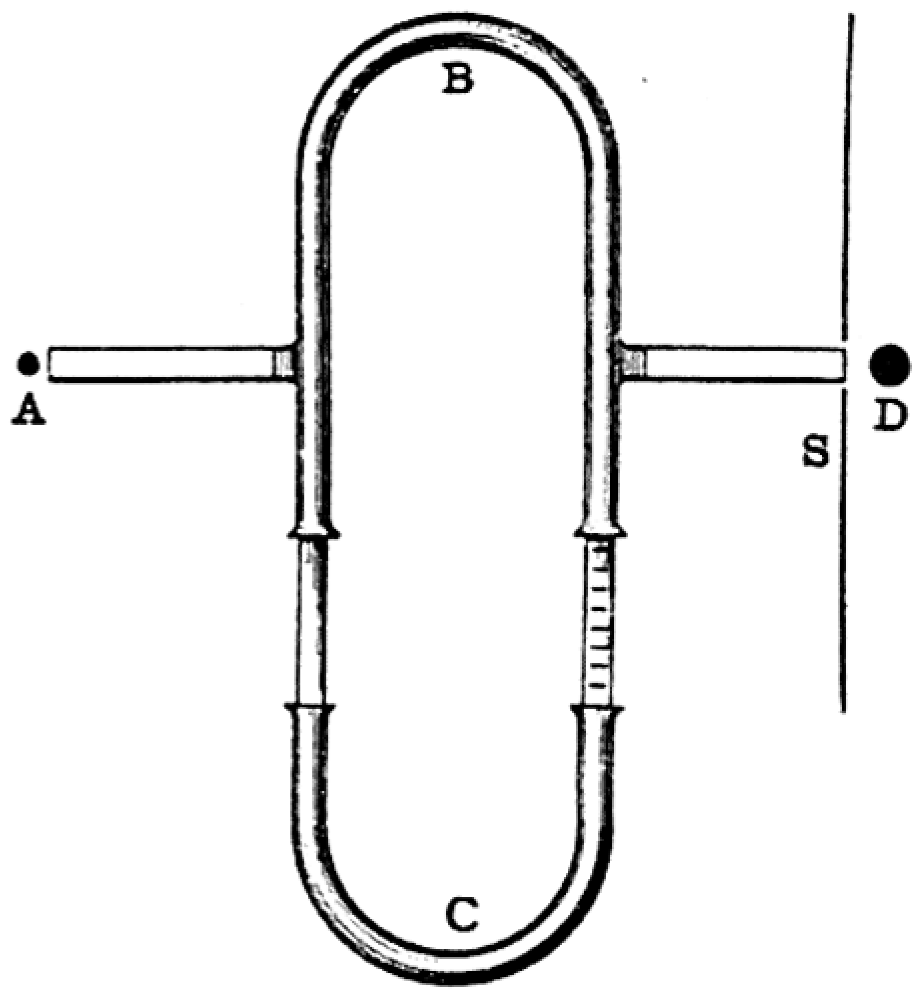


Fig. 165.—Quincke's Tube.

Interference of waves in the form of ripples can easily be shown.

Experiment.—Fix a tuning fork above a large dish of mercury so that its prongs vibrate in a vertical plane. Attach to each prong a thin bit of wire with their lower ends just in the liquid. When the fork vibrates the wires create two sets of ripples; if at some point the crest of one set always arrives at the same instant as the trough of the other the two interfere and the surface is undisturbed. The interference pattern may be projected on a screen by a suitable convex lens if the surface is well illuminated.

Experiment.—Replace the wires of the last experiment by a light plate fastened to one prong, arrange that this is parallel to one side of the mercury vessel and about 4 cms. away from it. When the fork vibrates standing waves are produced between the plate and the side and the nodes can easily be located. In this case it is the incident and reflected waves which interfere.

Beats.—Closely connected with interference is the following. When two sources whose frequencies are nearly equal are sounding together the loudness of the resultant sound waxes and wanes. These alternations of strong and weak sounds are called **beats.** They may be readily observed if the two lowest notes on a piano are struck together; if any difficulty is found in hearing them the ear should be placed in contact with the piano frame, a throbbing sound can then be distinguished if the notes are held down. Similarly if two stretched strings on the sonometer (p. 289) are tuned to nearly the same note the beats can readily be heard. Suppose the two sources are tuning forks of frequencies 100 and 104, the beats will then occur 4 times each second. The reason for this is clear from Fig. 154. On account of their different frequencies the faster fork gains on the slower 4 vibrations per second, hence their phases will be in agreement this number of times per second and each time the resultant will be large. These effects can evidently be used to find the frequency of one source when that of the other is known, the number of beats per second is equal to their frequency difference. In the above example the frequency of the second fork, if unknown, might be 104 or 96. To determine which of these is the true value the fork is loaded with a small lump of wax, the mass to be moved is now bigger while the restoring force on the deflected prong is unaltered, hence its period is increased. If the beats now occur at longer intervals the frequencies of the forks are more nearly equal, hence the original frequency was 104 and not 96. Beats are frequently used by piano tuners to enable them to bring two strings into unison, when they are properly tuned the beats should occur at relatively long intervals.

EXAMPLES ON CHAPTER XXIV

1. Fifty-six tuning forks can be arranged in a series so that each gives four beats per second with the previous one. The last fork is the octave of the first. Calculate the frequencies of the forks. (L. '07.)

2. Explain beats in sound, and calculate the velocity of sound in a gas in which two waves of lengths 1 and 1·01 metres produce ten beats in three seconds. (L. '08.)

3. Two expresses travelling at sixty miles per hour are meeting each other when one sounds its whistle. Given that the frequency of the note is 800 per sec., find its apparent frequency to an observer in the other train, (1) before the trains meet, (2) after they have passed each other. What will be the apparent frequencies to a stationary observer near the line? (Vel. of sound = 1100 ft./sec.)

CHAPTER XXV

MEASUREMENT OF FREQUENCY

Siren.[1]—In theory the siren is perhaps the simplest method of finding the frequency of a source. One form of the apparatus is shown in Fig. 166. A is a cylindrical wind-chest into which air can be forced by a bellows through a tube in its lower face. Its upper end is closed by a brass plate pierced with a circular row of holes N in number. A movable disc B, which rests on this at its centre, has also N holes. In the simplest form the two sets of holes are inclined to each other as shown at F, the escaping air then forces the disc round in the direction of the arrow. Each time the holes come opposite each other jets of gas escape and the air above B is momentarily condensed. If this happens n times per second a note of corresponding frequency is created. The rate at which the disc revolves is measured by a speed counter D, connected to the axle through a worm gear; it moves forward one division for every 100 revolutions. To find the frequency of a note, *e.g.* that of a tuning fork, the wind pressure is regulated until the two notes are in unison, *i.e.* are of the same pitch. It is most convenient to blow rather too hard and then press lightly on the axle with a flexible card until the proper speed is obtained. The number of

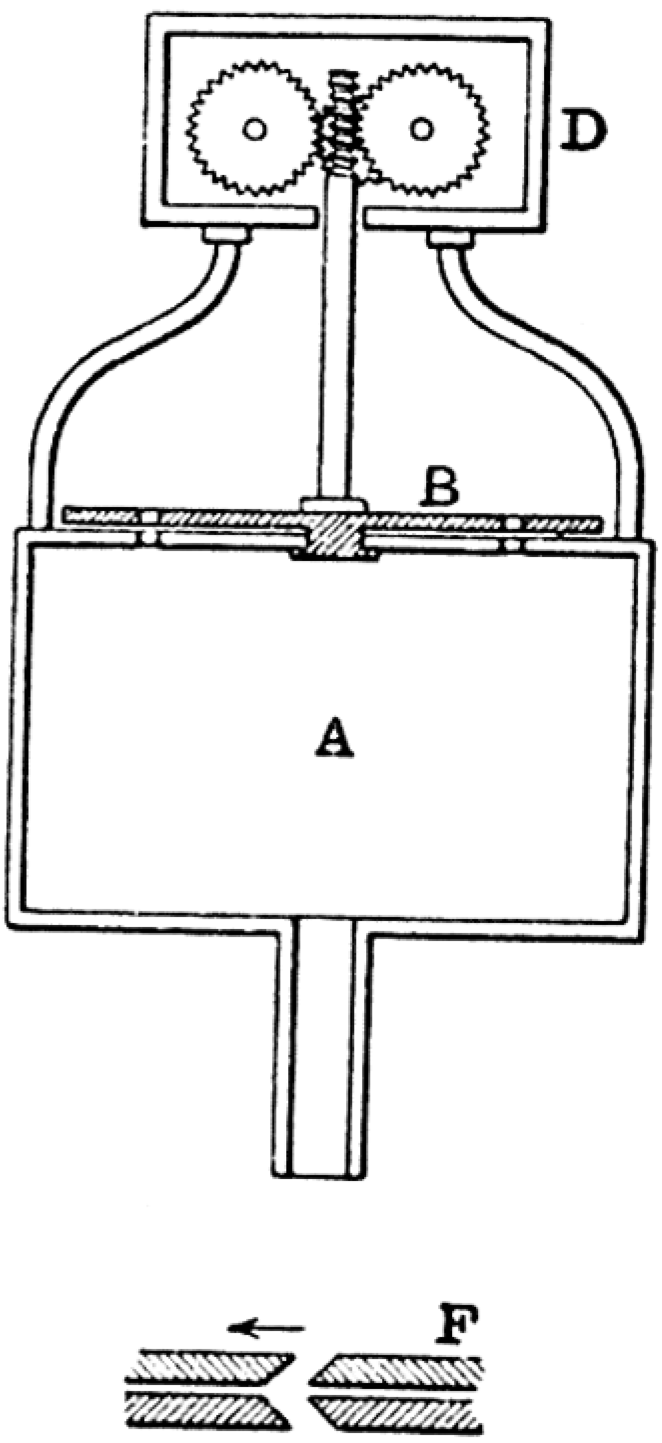


Fig. 166.—The Siren.

[1] Barton and Black, "Practical Physics," p. 113.

revolutions in a given time is noted with a stop-watch, the rate being maintained constant during the interval. In one turn the holes come opposite each other N times, hence if the disc makes m turns per second the frequency of the note is mN. The method is not a satisfactory one, as it is difficult to keep the speed constant for more than a few seconds and the note is very poor in quality, something like the sound of a steam whistle, so that it is difficult to say when the two have the same pitch. In better forms the holes are bored vertically and the disc is driven by a small

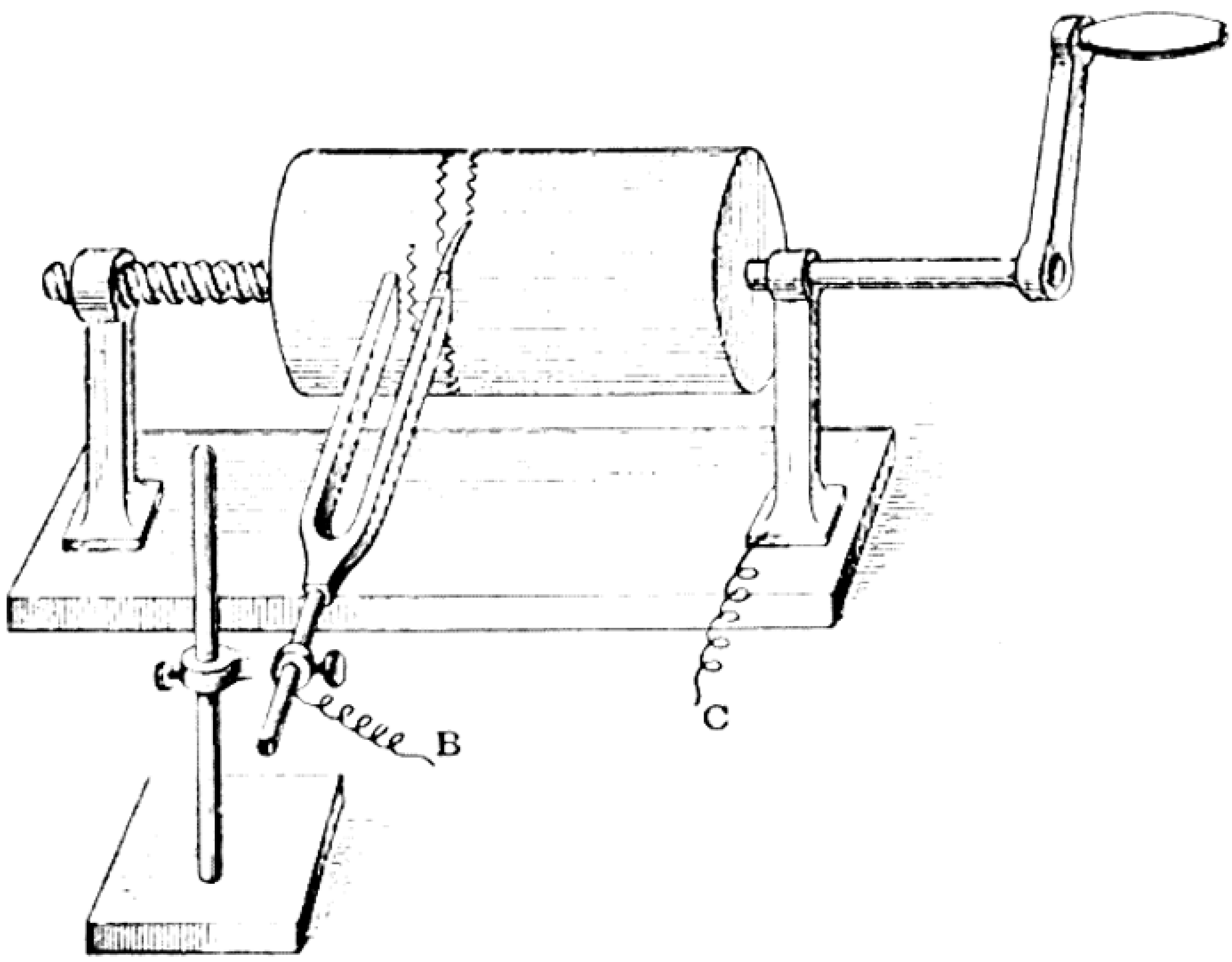


FIG. 167.—Revolving Drum for comparing the Frequencies of Forks.

motor, the speed is then independent of the pressure in the wind-chest.

Revolving Drum.—This method may be used either to compare the frequencies of two forks or to measure that of one. The surface of a brass cylinder (Fig. 167) about 5 inches in diameter is covered with smooth paper, and on this a thin film of soot is deposited by holding it over a smoky flame, such as a bit of burning camphor. To one prong of each of the forks to be compared a stiff bit of bristle is attached and the two are mounted so that the bristles are just in contact with the drum. Having thrown the forks into vibration by bowing with a 'cello bow the cylinder is rotated rapidly, the light pointers then trace out vibration curves on the paper. The number

of each in a given length is counted, this gives at once the ratio of the frequencies. In order that the trace in one revolution shall not be destroyed in the next, the axis of the cylinder consists of a screw which causes it to advance to the left as it turns. Only one fork is shown in the figure. The method can be modified to give the frequency of a single fork. The fork in question is connected to a wire B which forms one end of an electrical circuit of which a second wire C, the screw axis, and the drum are the other extremity. The bristle is replaced by a very thin wire. A clock pendulum beating seconds carries at its lower end a thin metallic strip which touches a small bead of mercury when the bob is at its lowest point. By suitable electrical arrangements this causes a spark to pass once every second between the cylinder and the pointer, then imposed on each other we have the trace of the vibrations and the dots caused by the sparks. The number of vibrations between each dot can thus be found directly.

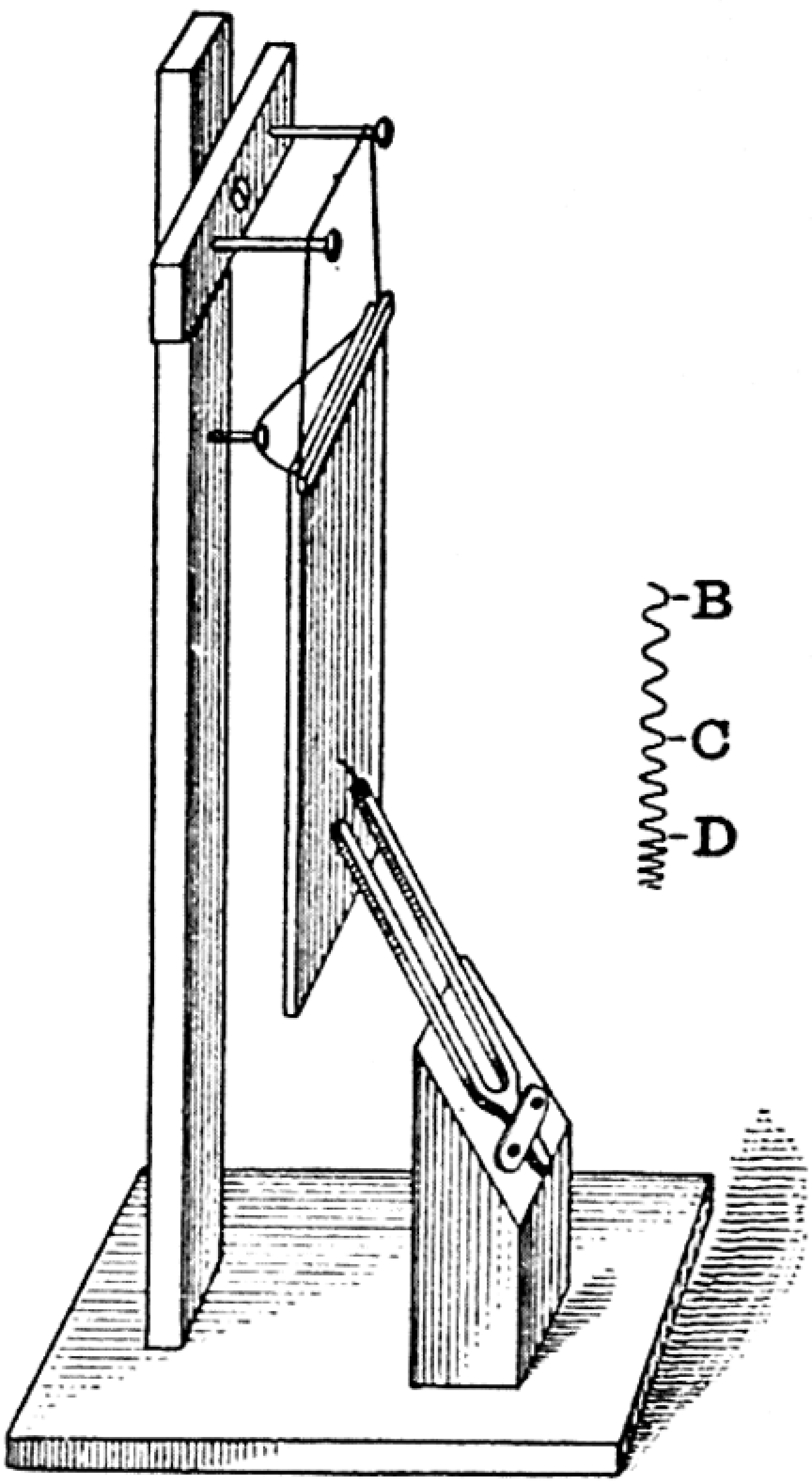


FIG. 168.—Falling Plate Method of finding the Frequency of a Fork.

Falling Plate.—A variation of the above method can be used if the acceleration due to gravity is assumed to be known; it cannot be regarded as an accurate method and it is rather troublesome to work, but it is not without physical interest. A glass plate about 10 cms. long is blackened on one face and is hung by a cotton thread from a peg (Fig. 168). The strings are fastened to the back of the plate so that it tilts slightly forward. A tuning fork with bristle attached is fixed in contact with the smoked surface, the fork is bowed and the plate caused to fall by burning the thread when a vibration curve is traced out as before. To hinder breakage or destruction of the trace

another thread, passing over a lower peg, is fixed to the plate, this is slack at the beginning of the fall but becomes taut before the plate reaches the table. At the lower end the curve is too crowded to see clearly, hence two lengths l_1 and l_2, which contain an equal number of vibrations, are measured off; let these be DC, CB, in the figure. Suppose u is the velocity of the plate when the fork is at D, and t the time taken to move over the distance l_1 or l_2.

Then
$$l_1 = ut + \tfrac{1}{2}gt^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If v is the velocity at C

$$v = u + gt$$
and
$$l_2 = vt + \tfrac{1}{2}gt^2$$
or
$$l_2 = (u + gt)t + \tfrac{1}{2}gt^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Subtracting (1) from (2) we have

$$l_2 - l_1 = gt^2$$

Each vibration takes $\frac{1}{n}$th of a second; hence if l_1 and l_2 contain m vibrations

$$t = m \, . \, \frac{1}{n} \text{ secs.}$$

and
$$l_2 - l_1 = g\frac{m^2}{n^2}$$

hence n is found.

Electrically driven Tuning Fork.—For many purposes it is convenient to have a fork which shall maintain its vibrations continuously without bowing. One form is shown diagrammatically in Fig. 169. The student will understand its action better after he has read the chapters on electricity. The fork is mounted on a heavy metal stand, and one prong carries a spring C which is just in contact with an adjustable screw D, E represents a small electromagnet placed between the prongs, B is a battery. The arrows show how the current can flow when the fork is at rest. Directly the circuit is completed by a key the current passes and makes E a magnet; this attracts the prongs and breaks the contact at DC. As the current is stopped E loses its magnetism, and the prongs spring back to complete tho

circuit again. The cycle of changes is thus repeated continuously and the fork maintains its vibrations.

Stroboscopic Method of finding the Frequency of a Fork.—For this method it is convenient to use an electrically maintained fork. To each prong a light metal plate is attached; these are perforated near their centres by a narrow rectangular slit (Fig. 170). When the fork is at rest the slits are opposite each other and a beam of light can pass through them, but when it is in vibration the rays are interrupted except when the prongs are in their mean position,

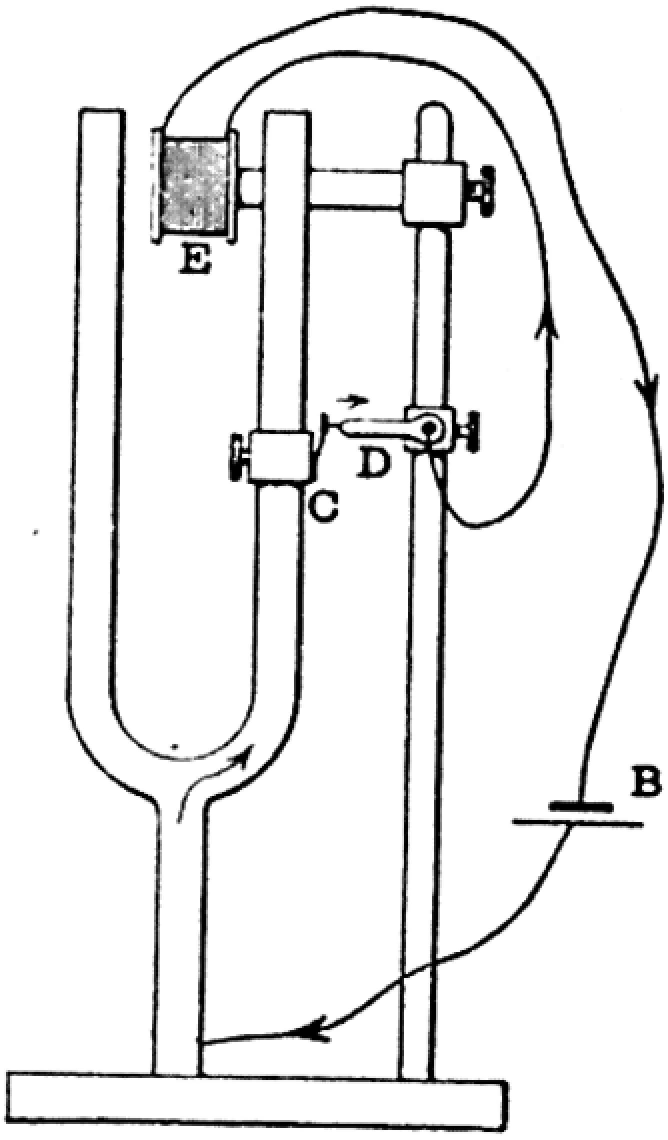


FIG. 169.—Electrically driven Tuning Fork.

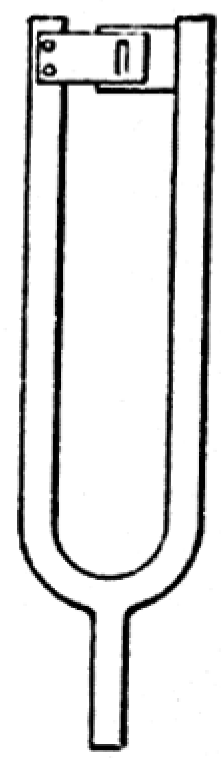


FIG. 170.—Tuning Fork with Slits attached.

this occurs twice during each period. A white circular disc, upon which a ring of equally spaced black dots has been marked, is placed in a vertical position behind the fork and is observed through the slits. It is kept in steady rotation round a horizontal axis by a small motor, its average speed over any interval being given by a speed counter. When both disc and prongs are moving the dots will generally appear to be in motion, but if the rate of revolution is properly adjusted it can be arranged that, while the light is cut off, one dot moves into the position previously occupied by its neighbour, when this occurs the disc appears to be at rest. Suppose the disc make N revolutions per second when this condition is reached,

let m be the number of dots on it, and T the period of the fork. Then since the disc is seen twice in each vibration it makes $\frac{1}{m}$th of a revolution in a time T/2.

But in 1 sec. it makes N revs.

$$\therefore \text{ in } T/2 \text{ it makes } \frac{NT}{2} \text{ revs.}$$

hence
$$\frac{1}{m} = \frac{NT}{2}$$

and
$$T = \frac{2}{Nm}$$

Therefore the frequency $n = \frac{1}{T} = \frac{Nm}{2}$

The effect just described is frequently noticed in kinematograph pictures of moving vehicles. When the first film is made it is exposed in the camera a number of times each second. Suppose during the interval between two exposures that the spoke of a wheel has moved into the position previously occupied by its neighbour, then the wheel appears not to revolve although the vehicle is moving. Similarly, if the exposures are made at shorter intervals, the spoke will not have advanced so far and the wheel appears to revolve the wrong way.

We have already described (pp. 260 and 281) how Lissajous' figures and the occurrence of beats can be applied in the determination of frequency. Another method is given on p. 291.

EXAMPLES ON CHAPTER XXV

1. Describe experiments by which, in the case of a given sound, the frequency and length of the waves can be ascertained. (L. '97.)

2. Describe any method of counting the number of vibrations made by a tuning fork. In what way would you expect temperature to affect the number and why? (L. '08.)

3. A stroboscopic disc has 20 dots round its circumference. When observed through slits carried by the prongs of a vibrating fork the dots appear to be at rest. Given that the frequency of the fork is 100/sec. calculate the smallest possible number of revolutions per minute made by the disc.

CHAPTER XXVI

STANDING WAVES ON WIRES AND IN TUBES

Transverse Waves along a Wire.—It has already been stated that transverse waves are propagated along a wire with a velocity $V = \sqrt{\frac{T}{m}}$, where T is the tension in dynes, and m the mass in grams of unit length of the wire (p. 263). We must now see what happens when the wire is limited in length and fixed at each end. Consider a short bit of the wire near its further end. Suppose the advancing wave has moved it upwards against the force exerted by the tension, it possesses potential energy; shortly afterwards the tension pulls it down again, its potential energy assumes the kinetic form and this carries it past its position of rest. Thus a reflected wave is set up and we have conditions similar to those on p. 276, except that the motions are transverse. The fixed ends of the string are nodes, hence the Fig. 163 gives the proper phases of the incident and reflected waves. In the simplest case there are only two nodes, one at each end; if l is the length of the wire, λ the wave-length of the disturbance, and n the frequency with which the wire oscillates, $\lambda/2 = l$, since nodes are half a wave-length apart, or $\lambda = 2l$. Also

$$V = n\lambda = \sqrt{\frac{T}{m}}$$

$$\therefore n = \frac{1}{\lambda} \cdot \sqrt{\frac{T}{m}}$$

or

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The tension may be produced by a hanging weight of P gms.,

then $T = Pg$. If r is the radius of the wire and ρ the density of its material $m = \pi r^2 \rho$,

then
$$n = \frac{1}{2l}\sqrt{\frac{Pg}{\pi r^2 \rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Either of these formulæ gives the frequency of the note emitted by the wire when it is caused to vibrate under the given conditions.

Experimental Verifications. The Sonometer.[1]—These results can be verified by the sonometer (Fig. 171). This consists simply of a wooden box about one metre long upon which two or more wires can be stretched. The necessary tension is obtained either by a weight and pulley arrangement, as shown in the figure, or by wrapping the ends of the wires round pegs after the manner used in a violin. Two

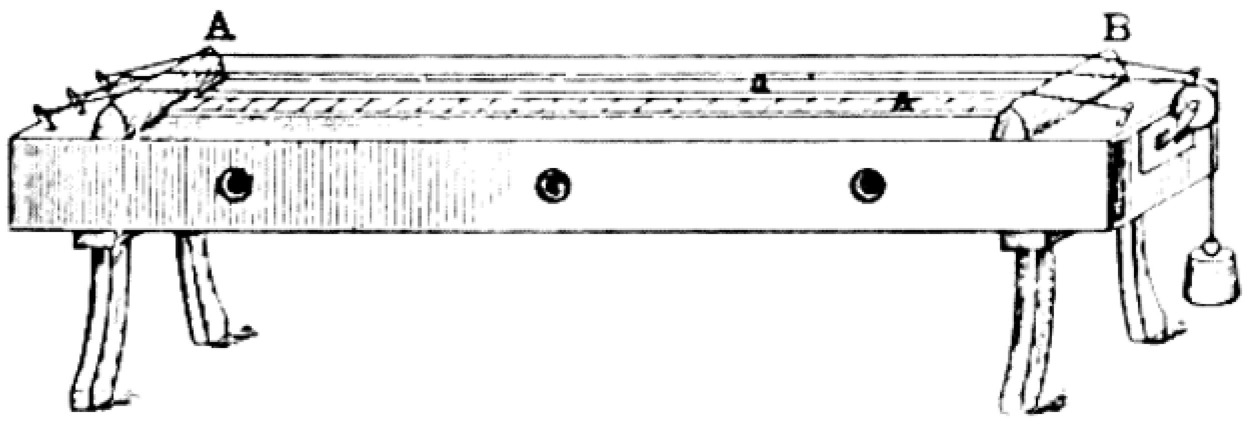


FIG. 171.—The Sonometer.

fixed bridges, A, B, at the ends confine the vibrations to a definite length of wire; there are also movable bridges under each wire if a still shorter length is required. A scale on the top gives the length of wire in use. The box acts as a sounding board and increases the loudness of the sound. When the wires are put in vibration, either by bowing with a violin bow or by plucking, they cause the box also to oscillate, this communicates its motion to the surrounding air and a much larger mass is put in vibration than would be brought about by the wires alone, hence the sound is louder. The wooden body of a violin performs the same function, as does also the sounding board of a piano. When a grand piano is played its top is raised so that the energy may escape outwards.

EXPERIMENT.—If the handle of a vibrating tuning fork is pressed on the table the sound is much louder for a similar reason, but it dies away more

[1] Barton and Black, "Practical Physics," p. 108.

rapidly as the energy is more quickly communicated to the surrounding medium.

EXPERIMENT.—*To prove that $n \propto 1/l$.* Keep the tension of a wire constant but alter its length by the movable bridge until it is in tune with a fork of known frequency. The final tuning is best done by means of beats. A disc of wood about 4 in. in diameter is fixed to one end of a short wooden rod with its plane perpendicular to the latter, this forms a kind of stethoscope to magnify the sound. The free end of the rod is placed on the sonometer and the disc pressed to the ear, the wire is plucked and the handle of the sounding fork is held on the sonometer box. If the two are nearly in tune the beats can easily be heard. One bridge is moved until the beats are very slow when the length between the bridges is taken. Repeat the observations with forks of different known frequencies. It will be found very approximately that

$$n_1 : n_2 : n_3 = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$$

EXPERIMENT.—*To show that $n \propto \sqrt{1/m}$.* Find by weighing the mass of 1 cm. of each of a number of wires. Take the heaviest, fix one end of it to the sonometer, pass it over the pulley and hang a weight of several kgms. to its free end. Fasten any other wire on the sonometer by pegs at each end and adjust it until nearly its whole length is in tune with the length AB (Fig. 171) of the first. The second wire is to act as a standard, by noting what length of it is in tune with the fixed length AB of the other wires, the same weight being used in each case, we can find how the frequency varies when m only is altered. Let l_1 be the length of the standard in tune with the first wire, n_1 its frequency. Replace the first weighed wire by a second and let l_2 and n_2 be the corresponding quantities for the standard. Then for the latter wire $\frac{n_1}{n_2} = \frac{l_2}{l_1}$ from the last experiment.

For the weighed wires l and T are constant, hence if equation (1) is true

$$\frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\therefore \frac{l_2}{l_1} = \sqrt{\frac{m_2}{m_1}}$$

This relation should be verified for several wires. In a similar way if the wires are made of the same material, so that the density is constant, it may be proved that $\frac{n_1}{n_2} = \frac{r_2}{r_1}$, or if the radius is constant but the wires are of different materials

$$\frac{n_1}{n_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

EXPERIMENT.—*To find how n varies with the tension.* One wire is stretched by hanging weights at its end and for each load the length l of the standard in tune with the part AB (Fig. 171) is found. The frequency of the loaded wire is proportional to $1/l$. Plot your results on a curve taking the square roots

of the tensions as abscissæ and the corresponding values of $1/l$ as ordinates. This will be very approximately a straight line, thus proving that $n \propto \sqrt{T}$.

The equation (1) may be written $n = c\sqrt{T}$, where c is a constant for a given wire. Taking logarithms we have

$$\log n = \log c + \tfrac{1}{2} \log T$$

hence if the observations of the last experiment are plotted again using log T for abscissæ and log $(1/l)$ for ordinates, the tangent of the angle which the line makes with the horizontal axis should be 1/2. The equation $V = \sqrt{\frac{T}{m}}$ is obtained on the assumption that the wire is perfectly flexible, whereas actually it opposes a resistance to change of shape, in other words it possesses rigidity (p. 2). This extra force brings it back more quickly to its mean position when displaced ; thus its frequency is increased. The effect is greater for short thick wires than it is for long thin ones.

Use of a Wire in determining Frequency.[1]—Having verified equation (1) it may now be used to find the frequency of a fork or other source of sound. The necessary tension is produced by weights, a *thin* wire is used and the length in tune with the fork is noted. The mass of 1 cm. is found by weighing a known length, after which the frequency can be calculated from the equation. In particular it can be proved that if one fork is the octave of the other the ratio of their frequencies is 2 : 1.

Harmonics.—In the vibrations we have been considering the wire has two nodes, one at each end ; it is then giving its lowest or *fundamental* note, but it is possible to make it vibrate with 3, 4, or more nodes, as in Fig. 172. All that is necessary is to press it lightly at the point *a* where a node should appear and bow it with a violin bow at the point *b*, where an antinode should come. The part between two adjacent nodes is called a ventral segment. As the distance between successive nodes is $\lambda/2$ it is clear that the wave-length in (1) (Fig. 172) is l, where l is the length of the string. In (2) $\lambda = 2/3 \, . \, l$, in (3) $\lambda = l/2$, and so on. The velocity of a wave along the wire is constant, being equal to $\sqrt{T/m}$, hence $n\lambda = V$ is constant and the frequency is inversely proportional to the wave-length. Calling λ_1

[1] Barton and Black, "Practical Physics," p. 111.

the wave-length of the lowest note, λ_2 that corresponding to (1) in the figure and so on, we have

$$\lambda_1 : \lambda_2 : \lambda_3 = 2l : 2l/2 : 2l/3$$

or for the corresponding frequencies,

$$n_1 : n_2 : n_3 = 1 : 2 : 3$$

The lowest note that the wire can produce is called its **fundamental**, those whose frequencies are twice, thrice, etc., that of the fundamental are called its 1st, 2nd, etc., **harmonics**. It would be more logical to

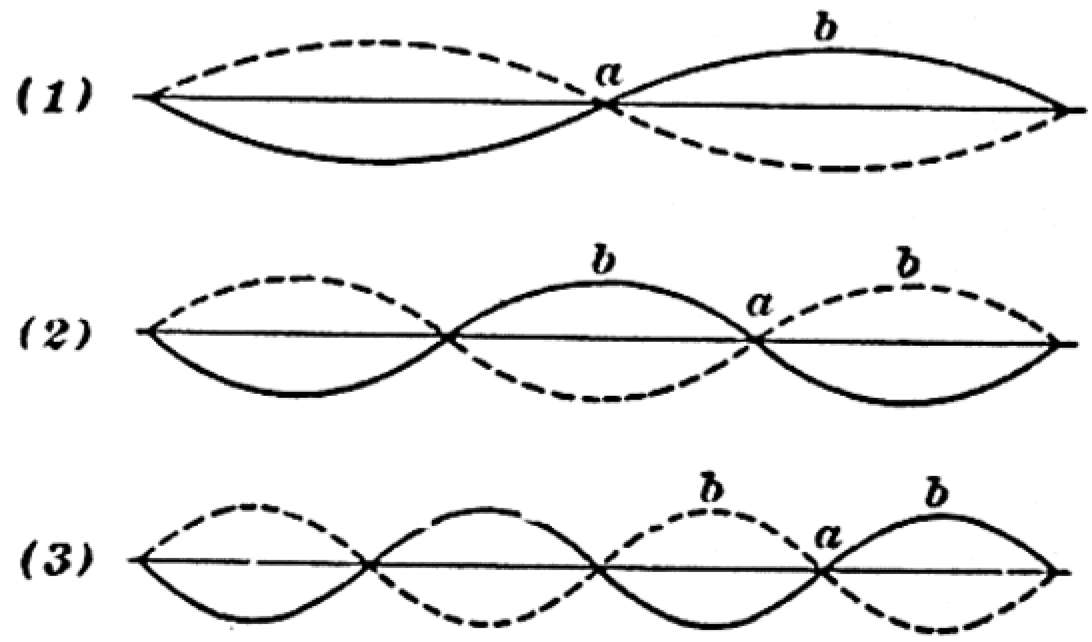


Fig. 172.—Showing the Different Modes in which a Wire can vibrate.

call the fundamental the 1st harmonic and the others of the series the 2nd, 3rd, etc.

Melde's Experiment.—The vibrations of strings can be shown in a very elegant manner by an experiment due to Melde. A piece of linen thread about 1 m. long is attached at one end to the prong of a tuning fork, the other end passes over a glass rod and carries a light pan. By properly adjusting the tension the string may be made to vibrate in its fundamental mode (Fig. 173 *A*). In order to have a definite length the string should be pressed at the point where it crosses the rod. If now, keeping the length the same, the fork is turned into the position shown in Fig. 173 *B*, it is found that the string vibrates with a node at its middle point, or its frequency is doubled. The reason for this is easy to see. Suppose (figure *A*) the string occupies the lower dotted position when the prong is at its maximum displacement to the left; half a vibration later the prong has moved its greatest distance to the right and the string is stretched taut, while at the end of one vibration it is slack enough to take up

the upper dotted position. Hence the frequency of the fork is double that of the string. In figure B string and fork have the same period.

EXPERIMENT.—Set up the experiment as in figure A, keep the length constant but add weights until the string vibrates in succession in the modes shown in Fig. 171. Prove that the frequency varies as $\sqrt{\text{Tension}}$. Repeat with the fork in the second position.

In making the experiment it will be found somewhat difficult to judge when the proper weight has been chosen, a small change in the tension merely causes the vibrations to take place in a different

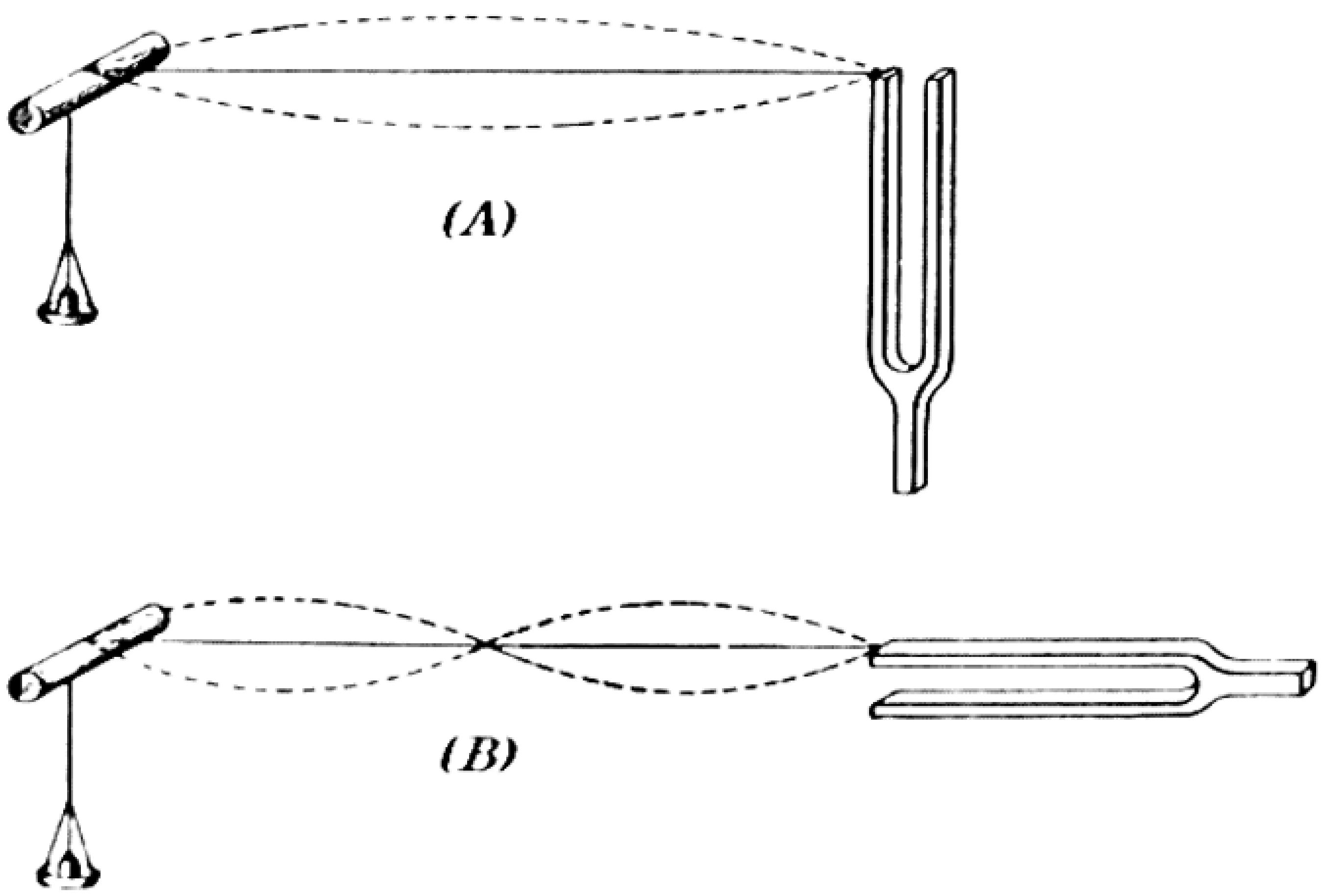


FIG. 173.—Melde's Experiment.

plane without altering their amplitude greatly. Recent experiments show that equation (1) is obeyed only when the displacements are vertical, the adjustments must be continued until this condition is obtained. The transverse vibrations of strings have numerous applications in the construction of musical instruments such as the piano, violin, harp, etc. The vibrations of rods are too complicated for us to deal with (for tuning fork, see p. 300).

VIBRATIONS OF COLUMNS OF GAS. RESONANCE

Tube closed at one End.—The vibrations of columns of gas depend on the principles explained on pp. 276–278. For a tube closed at one end the apparatus shown in Fig. 174 may conveniently be used.

AC is a glass tube about 5 cms. in diameter and 1 m. long. It is connected at its lower end by rubber tubing to a reservoir containing water. The reservoir is raised until the water is near the top of the tube, when the clip E is closed and the reservoir hung at a lower level. By cautiously opening the clip the water surface may be adjusted to any point. If a sounding tuning fork is held near A it is found when the water reaches certain positions B and C that the tube gives out a loud note. A convenient frequency for the fork is 512. To understand the reason for this behaviour let us see what happens to a wave of compression which starts down the tube with an amplitude a. If the water stands at B it is reflected from this point as a compression (p. 275), but when it reaches the open end of the tube it finds the layers of air more easy to move, for they can relieve themselves from their strained condition by expanding sideways as they are no longer confined by the tube. Hence the end layer moves over a distance larger than a, and a reflected wave of rarefaction is sent downwards exactly as if the medium above A were less dense than the air in the tube. Similarly when the wave of rarefaction arrives again at A after reflexion at B the external air moves easily towards the rarefied portions and produces a reflected wave of compression. Suppose now the time taken by the waves to travel from A to B and back again is T/2, where T is the period of the fork. A compression starting down the tube is reflected at A as a rarefaction at a time T/2 later, but at the same instant the fork itself is sending a rarefaction down the tube, thus the reflected and direct waves are in the same phase, their effects are added, and the column of gas is thrown into violent stationary vibrations with a node at B. If $AB = L$ the waves travel a distance 2L in the time T/2, and therefore 4L in the time T. But the distance passed over in one period is the wave-length, hence $\lambda = 4L$. Thus $AB = \lambda/4$ and A, the point where the displacements are largest, is an antinode. Suppose next the water stands at C where $BC = \lambda/2$, then the

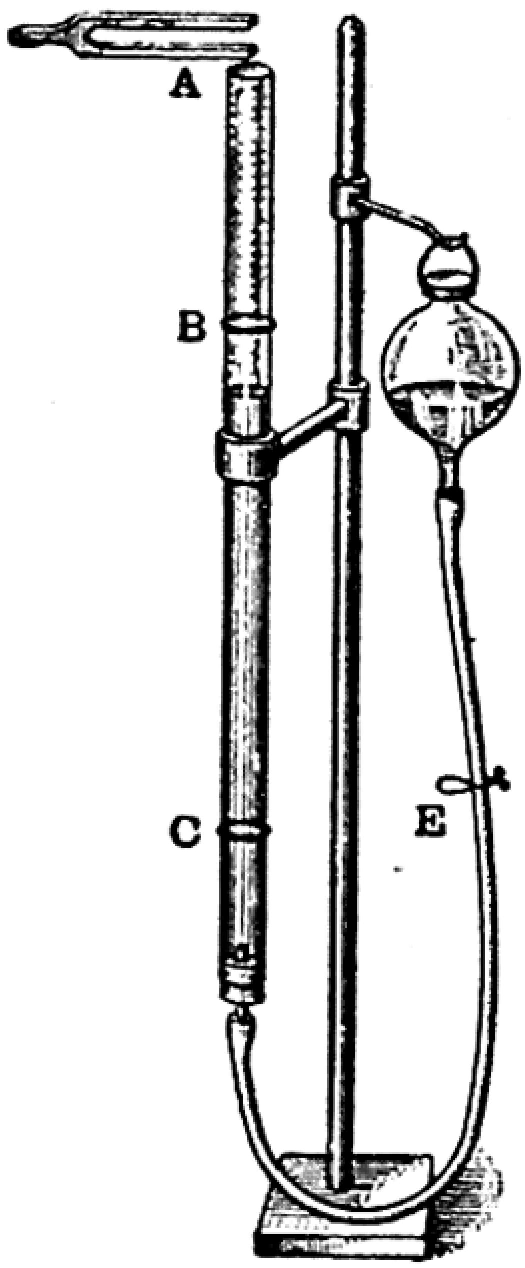


FIG. 174.—Resonance Tube.

reflected waves arrive at A exactly one period later than they did in the previous case, since they have to travel over the additional distance $2BC = \lambda$, and they are again in phase with the waves produced by the fork. Hence a loud note is again heard, but there are now nodes at B and C. If the tube is long enough other nodes may be found. The time a body takes to perform one complete vibration if started and then left to itself is called its **free period**, the vibrations it undergoes in these conditions are called **free vibrations.** The cause of the strong vibrations of the air in the tube is now clear; it receives a succession of impulses from the fork which are so timed as to assist its free vibrations, the effects are added and produce a much larger amplitude than a single impulse. A body oscillating under such conditions is said to be in **resonance** with the exciting force. In Fig. 173 *A* the string receives an impulse each half period, in *B* the free period of the string is equal to that of the fork. Fig. 175 shows possible positions of the nodes for a tube of length L when excited with different forks. Denoting by λ_1, λ_2, etc., the wave-lengths of the corresponding notes, n_1, n_2, etc., the frequencies, we have

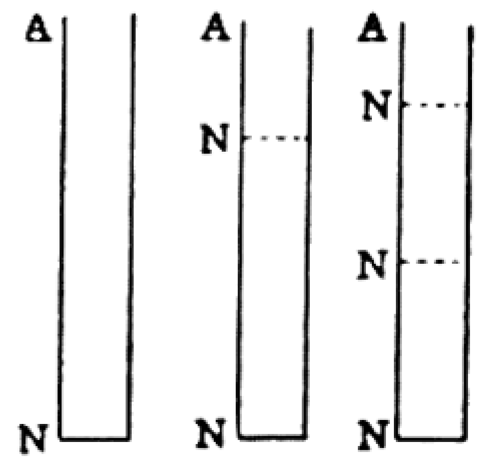


FIG. 175. — Showing possible Modes of Vibration in a Tube closed at one End.

$$L = \frac{\lambda_1}{4}, \quad L = \frac{3\lambda_2}{4}, \quad L = \frac{5\lambda_3}{4}, \text{ etc.}$$

or

$$\lambda_1 = 4L, \quad \lambda_2 = \frac{4L}{3}, \quad \lambda_3 = \frac{4L}{5}, \text{ etc.}$$

or, as $n \propto 1/\lambda_1$

$$n_1 : n_2 : n_3 = 1 : 3 : 5.$$

This shows that a tube closed at one end gives only the alternate harmonics.

Further Examples of Resonance.—The following experiments illustrate further the principle of resonance.

EXPERIMENT.—Open a piano front and sing or whistle a loud note, the piano returns the same note. Those strings whose frequencies bear a simple relation to that of the note have been thrown into resonant vibration.

Tuning forks are frequently mounted on resonance boxes, these

are simply boxes of such a size that the air in them resonates to the fork.

Experiment.—Place two forks of the same pitch on their resonance boxes and let the open ends of the latter face each other. Bow one fork and then immediately stop it, the second fork is found to emit a loud sound.

Experiment.—Place the two resonance boxes of the last experiment one horizontally the other vertically with the mouths near each other. Hold a sounding fork near them so that its prongs vibrate in a vertical plane; no sound is heard except when one box is closed. It is seen from p. 279 that one box is receiving a compression when the other is receiving a rarefaction and the two effects interfere. This experiment is, I believe, due to Prof. A. M. Mayer.[1]

Experiment.—Make a simple pendulum with a heavy bob and having a small hook below. Suspend from this a lighter pendulum of different length. When the heavy bob is made to oscillate the lighter follows suit, but its amplitude is never large. It is said to perform *forced* vibrations. If, however, the lengths are adjusted until the free periods are equal, then a slight displacement of the heavier bob rapidly produces in the lighter pendulum a vibration of considerable amplitude. The forces applied at its point of suspension are then timed in such a way that their effects are added.

Experiment.—Support a metre stick at its ends and hang two helical springs, with weights attached, from two points near its centre (Fig. 176). Pull A downwards, then let it go. Its vibrations are transmitted to B through the metre stick; if the periods of the springs are equal B gets up a large amplitude until it has absorbed all the energy from A, which is then reduced to rest. The interchange of energy then proceeds in the reverse direction. A third spring of different period should be attached to the stick at the same time to show the smallness of the amplitude in forced vibrations.

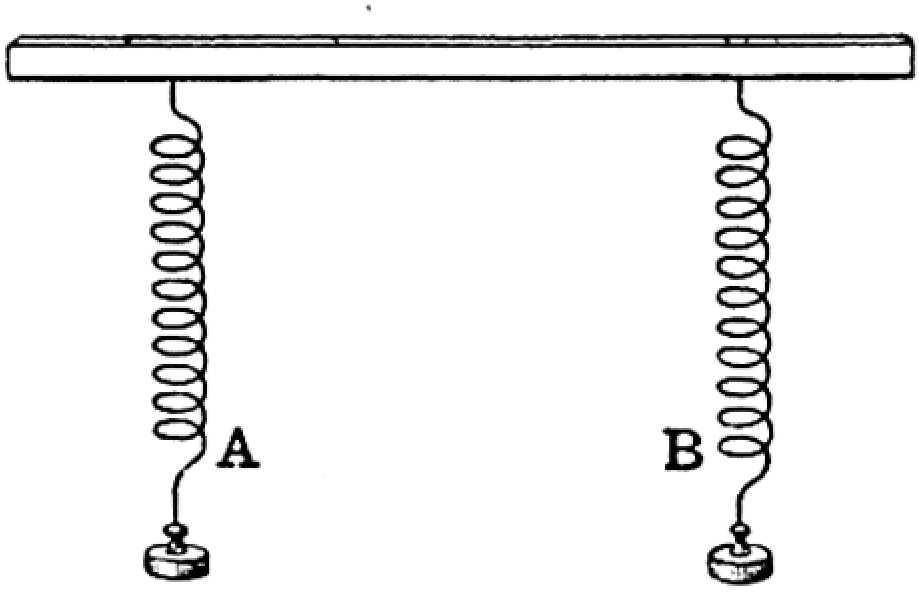


Fig. 176.—Resonating Springs.

Experiment.—The last experiment can be repeated on the sonometer with two stretched wires. If their frequencies are equal and one wire is bowed the other is thrown into vibration. The alternations of rest and oscillation take place as in the last case.

Tube open at Both Ends.—Resonance effects similar to those on p. 294 are produced if the tube is open at each end when a sounding fork is held near one extremity. In this instance a compression is

[1] For a large number of experiments on sound capable of performance with simple apparatus the student should refer to Prof. Mayer's book on Sound. (Macmillan.)

reflected as a rarefaction at the further end, and when this arrives at the starting point it is returned as a compression. The reflected wave must therefore reach the fork at the instant the next compression is starting down the tube, *i.e.* the disturbance must travel down the tube and back again in one period of the fork. Hence the length L of the tube must be $\lambda/2$ or $\lambda = 2L$. The wave-length necessary for resonance is therefore half of what it was in the previous case with the same length of tube, or the frequency is doubled. The note is thus the octave of that given by a tube of the same length closed at one end. Each open end is an antinode, and when the tube is giving its fundamental there is a node at the middle. Other possible modes of vibration are indicated in Fig. 177. With the same notation as before

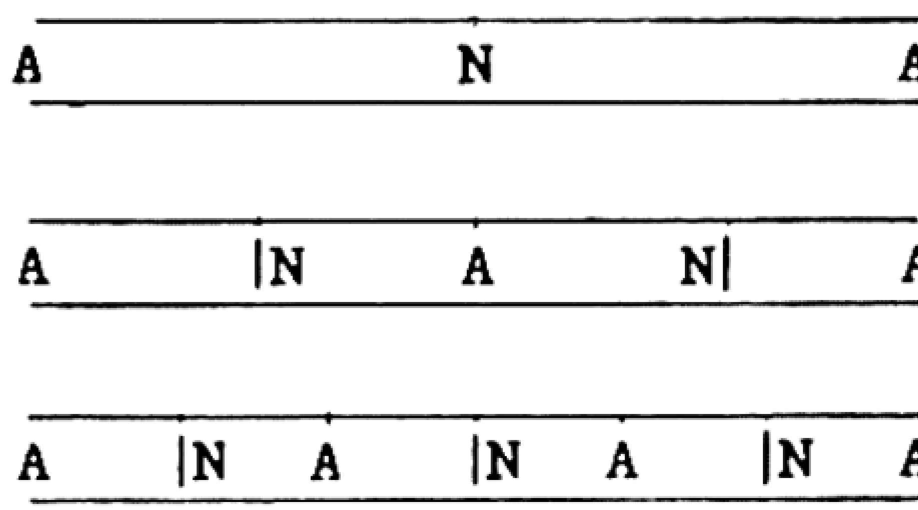


FIG. 177.—Showing possible Modes of Vibration in a Tube open at each End.

$$L = \frac{\lambda_1}{2}, \quad L = \lambda_2, \quad L = \frac{3\lambda_3}{2}, \quad L = \frac{4\lambda_4}{2}$$

or

$$\lambda_1 = 2L, \quad \lambda_2 = \frac{2L}{2}, \quad \lambda_3 = \frac{2L}{3}, \quad \lambda_4 = \frac{2L}{4}$$

or

$$n_1 : n_2 : n_3 : n_4 = 1 : 2 : 3 : 4.$$

Thus the whole series of harmonics can be produced. The vibrations in an open tube are most conveniently studied by having two tubes, one of which slides into the other, so as to provide an adjustable length. It has been mentioned that the air expands sideways at the open end of a tube, the effect of this can be shown with a tuning fork.

EXPERIMENT.—While a fork is sounding hold the edge of a steel scale close to one prong and parallel with it, the sound is much louder; as the air cannot escape round the edges of the fork the energy is hindered from spreading in all directions. This is a lecture experiment of Sir G. Stokes'.

Measurement of λ by Resonance Tubes. End Correction.—The resonance phenomena described evidently provide a direct method of measuring wave-lengths; if, further, the frequency of the fork is

known the velocity of sound can be calculated from the equation $V = n\lambda$. To give an idea of the magnitude of λ if $V = 332$m./sec. and $n = 512$, $\lambda = 64{\cdot}8$ cms.

Experiment.—Use the apparatus of Fig. 174 to find the position of the nodes B and C. It will be found that BC is larger than 2AB, although, according to the theory, they should be equal.

Experiment.—Measure the wave-length with a tube open at each end using the same fork as in the previous experiment. The result is approximately 4AB of the preceding case, but is less than 2BC.

The reason for these discrepancies lies in the theory; it has been assumed that the open end is an antinode or place where the density does not change, in actual fact the antinode is situated a short distance outside the tube. The magnitude of this end correction, according to Lord Rayleigh, is 0·6R, where R is the tube radius. Hence in Fig. 174, $\lambda = 4(AB + 0{\cdot}6R)$. As the correction is rather uncertain when a tuning fork is held near the open end, it is best when measuring the wave-length to find the nodes at B, C, the distance between them is $\lambda/2$. For an open tube the correction must be added for each end, the wave-length to which such a tube resonates in its fundamental mode is therefore $2(L + 1{\cdot}2R)$. For a closed tube of the same size the fundamental note has a wave-length $4(L + 0{\cdot}6R)$, as this is not quite double the other the frequencies are not in the ratio 2 : 1; the pitch of the higher note is slightly too low. Vibrating air columns play an important rôle in wind instruments such as the organ, clarinet, flute, etc.

Organ Pipes.—An organ pipe is a wooden or metal tube, square or circular in section, provided with an opening and lip at its lower end (Fig. 178). When placed in communication with a wind chest air under pressure is blown against the lip and sets the air column in vibration. Experiments with injections of smoke show that the issuing jet passes alternately to the left and right of the lip thus maintaining a series of condensations and rarefactions. A (Fig. 178) is an antinode since the movement of the air is there greatest, the other end is a node or antinode according as it is closed or open. The pitch is not given exactly by the simple theory above as the end correction at the lip is uncertain. The tuning is done by experiment; for a closed tube a movable piston is pushed to and fro until the correct note is given. When the end is open the size of the opening is varied by a flap B (see figure) which alters the end correction

and therefore the pitch. The harmonics are produced by strong air blasts. If a hole is bored through the wall of the tube at a point corresponding to a node the density of the air is made equal to that outside, the point in question can no longer be a node and the pitch is altered. On the other hand a boring at an antinode is without effect. For demonstrating the pressure changes at different points in the tube a manometric flame is convenient. A hole is bored through the wall and is closed by a rubber membrane A (Fig. 179)

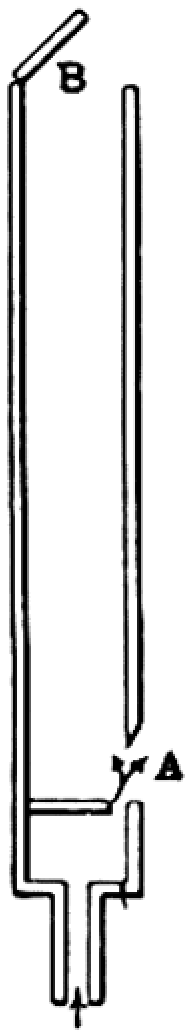


FIG. 178.—Organ Pipe.

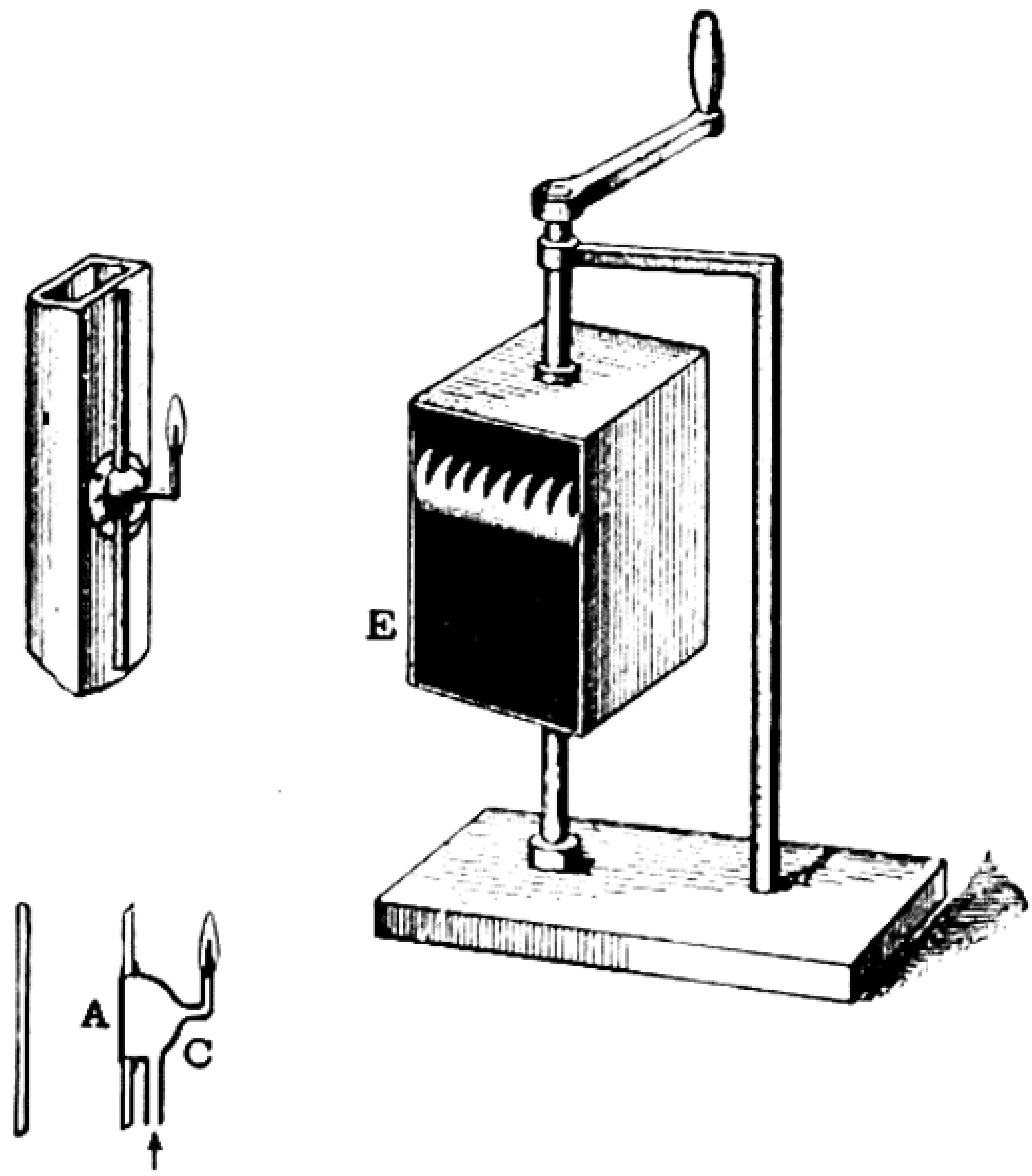


FIG. 179.—Manometric Flame.

which forms one side of a small chamber C. Gas is led into the chamber from the mains and escapes at a small jet where it is lighted. If the pressure in the tube varies the membrane is pushed to the right or left and the length of the flame momentarily increases or decreases. When the tube is "speaking" the variations are too rapid to be followed by the eye. This difficulty is overcome by observing the flame in a rotating mirror E ; owing to the persistence of retinal impressions a number of images are seen simultaneously in the mirror, and any oscillations can easily be followed. The flame flickers most violently at a node but scarcely moves at an antinode. The manometric flame can also be used with a Quincke's

tube (Fig. 165); if placed at D it is stationary when the two sets of waves destroy each other.

Koenig has found the position of the nodes and antinodes by direct ear observations. A slit was cut along the whole length of one side of the tube and it was fixed horizontally with the slit just immersed in a large trough of water. One end of a bent glass tube was passed through the slit while the other end was held near the ear. At an antinode there was silence, but at a node a loud sound was heard.

EXPERIMENT.—*Rubens' tube.* As a lecture experiment the position of the nodes can be shown in the following manner. BC (Fig. 180) is a brass tube about 8 cms. in diameter and 4 m. long. One end B is closed with a brass plate. Along one side a number of holes are bored about 2 mms. in diameter and 3 cms. apart. A slightly smaller tube about 50 cms. long slides in the open end, the outer extremity of this is closed with a thin rubber membrane A.

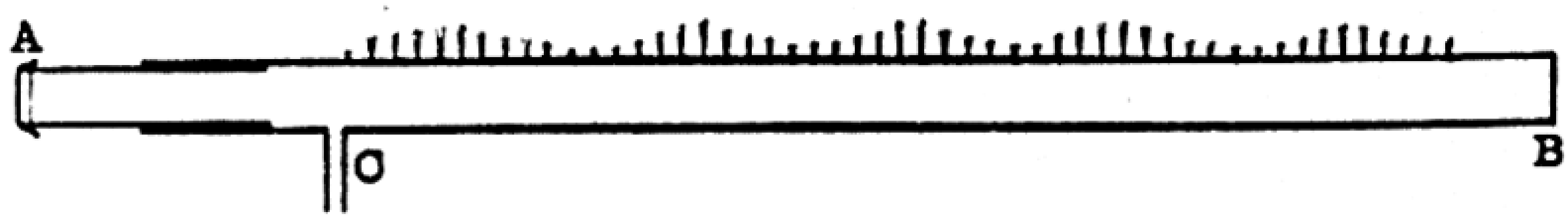


FIG. 180.—Rubens' Tube.

Gas from the mains is allowed to enter at the side tube C, after a few minutes it may be lighted with safety at the small holes. If a loud note is sounded near A and the position of the sliding tube is adjusted the enclosed gas is thrown into resonance and the jets vary in length. A whistle or tuning fork on resonance box serves as a convenient source of sound.

In wind instruments like the flute and clarinet the tube is pierced by a number of holes which can be opened or closed by stops. The position of the nodes and hence the pitch can therefore be varied at will.

Singing Flames.—A column of gas can be thrown into vibration by the following method.

EXPERIMENT.—Draw off a glass tube about 40 cms. long to a fine jet about 2 mms. in diameter. Connect it to the mains and light the gas at the orifice. If the jet is gradually pushed up a wide glass tube about 1 m. long it is found that in a certain position the tube speaks with a loud and unpleasant note. When the flame is viewed in a rotating mirror it is seen to be in violent oscillation. The vibrations apparently depend on periodic supplies of heat to the air column which cause it to expand and contract, if these are properly timed resonance occurs, but the phenomena are too complicated for us to deal with.

Tuning Fork.—A tuning fork can be regarded as a bar bent at the

middle point where the handle is attached, when vibrating this point is a node while the ends of the prongs are antinodes. Its importance lies in the purity of its note ; a string or column of gas, as we have seen, can give a whole series of harmonics, the first overtone of a fork is about six octaves above the fundamental and is moreover very weak. When the temperature rises the length of the prongs increases and the elasticity of the steel is diminished. Each of these changes lowers the frequency but the latter is of the most importance.

Kundt's Tube.—Resonating columns of gas can be used to study the *longitudinal* vibrations of rods. The apparatus, called a Kundt's tube, is shown in Fig. 181. A rod AC about 1 m. long is clamped at its middle point E, at one end it carries a circular cardboard disc A which projects into a long glass tube about 5 cms. in diameter. The further end of the tube is closed by a movable piston B, and between the two discs a small amount of lycopodium powder is scattered.

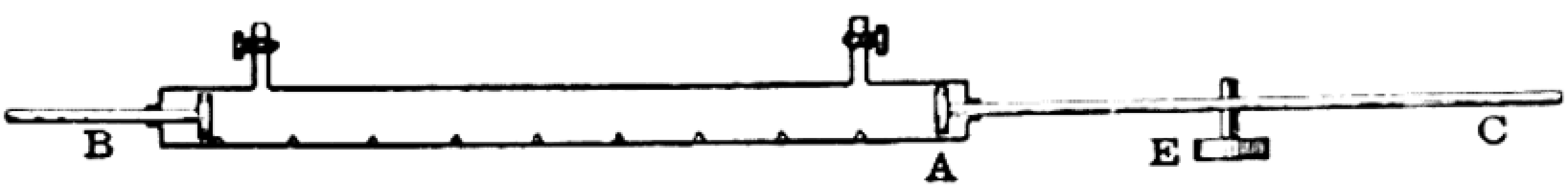


Fig. 181.—Kundt's Tube.

If the free half of the rod is grasped firmly with a resined cloth which is pulled towards C it is thrown into longitudinal vibration with a node at the middle and an antinode at each end. By altering the position of the piston B the length of the air column can be adjusted until it is in resonance with the rod. When this occurs the powder is violently agitated and finally settles down in small heaps at the nodes. The average distance between the heaps gives the half wave-length in air, while AC is the corresponding quantity for the rod. Let λ_1 and λ_2 be the wave-lengths in air and rod respectively,

then $$\frac{\text{velocity of sound in the rod}}{\text{velocity of sound in air}} = \frac{n\lambda_2}{n\lambda_1} = \frac{\lambda_2}{\lambda_1}$$

For the rod $V = \sqrt{\frac{Y}{\rho}}$, hence the Young's modulus may be deduced if the velocity of sound in the gas is known. For the experiment to succeed the tube must be thoroughly dry and clean and *the powder also must be dry*. It is a good plan to keep the lycopodium in a desiccator for some hours before use. The tube may be filled with different gases and the velocities, which are proportional to the

wave-lengths, can be compared. In this way it may be shown that $V \propto \sqrt{1/\rho}$. By the use of heating jackets the variation of the velocity with temperature can also be investigated. Apparatus of this type has been used to measure the velocity of sound in mercury, sodium, and potassium vapours, and in helium and argon. Since $V = \sqrt{\frac{\gamma . P}{\rho}}$ the ratio of the specific heats can be deduced. It was found in all these cases that $\gamma = 1{\cdot}66$ approximately. This is the ratio that theory predicts in the case of monatomic gases; hence it is concluded that the molecule of these substances contains only one atom. An application such as this shows in a striking manner the help which one branch of science can give to another.

EXAMPLES ON CHAPTER XXVI

1. Four strings are all of the same length and material, but of diameters in the ratios of 1 : 2 : 3 : 4, and are all stretched to half their breaking stress. Compare the pitches of their fundamental notes. (L. '80.)

2. A string stretched with a weight of 25 lbs., when made to vibrate transversely, gives a certain note. What tension must be applied to a string of the same material, but of twice the length and thickness, to make it give the octave above that note? (L. '87.)

3. Four exactly similar and equal strings stretched with the same tension are vibrating side by side; how will the note emitted be affected if they be fastened together so as to form one string by winding round them an extremely thin piece of silk? (L. '88.)

4. A string is stretched by such a weight that a hump runs along it at the rate of 64 ft./sec. Two points on this string, 4 ft. apart, are firmly clamped to a board without altering the tension of the string; if this part of the string be tweaked what is the pitch of the note it will emit? (L. '90.)

5. Compare the frequencies of vibration of two strings stretched with weights of 10 kgms. and 1 kgm. respectively. They are each 1 metre long, and they are of the same diameter, but their densities are 7·8 to 1 respectively. (L. '02.)

6. A sounding organ pipe is warmed from 16° to 127° C. What is the effect on the note it emits? (L. '91.)

7. Taking the fundamental mode, indicate, by carefully drawn figures, the motion of the air at various points of a sounding pipe at some one instant, and the distribution of pressure in the tube at some one instant. (L. '94.)

8. If two organ pipes give four beats per sec. when sounded together in air at 15°, how many will they give in air at 0°? (L. '10.)

9. A tuning fork making 1028 vibs./sec. is sounded opposite one end of an open tube 3 ft. 2¼ ins. long, which reinforces the note of the fork. Calculate the positions of the various points where maximum movement and maximum pressure change occur in the air column in the tube. (L. 1900.)

10. A column of air and a tuning fork produce four beats per second when sounded together, the fork giving the lower note and the temperature of the air being 15°. When the temperature has fallen to 10° the two produce three beats per second. Find the frequency of the fork. (L. '08.)

11. If the velocity of sound in air at 0° is 332 metres/sec., find the shortest length of a tube, open at both ends, that will be thrown into resonant vibration by a fork whose frequency is 256 when the temperature of the air is 51° C. (L. '09.)

CHAPTER XXVII

AUDITION. QUALITY OF SOUNDS

Organs of Speech and Hearing. Limiting Frequencies for Audible Sounds.—As we are concerned with the physical rather than the physiological effects of sound waves it will be unnecessary to describe the structure of the ear. It will suffice if we state that the external ear leads to an air passage which is closed at its further end by a thin membrane called the ear drum. When the air is subjected to density changes, owing to sound waves, the drum is set in vibration like the rubber membrane in a manometric capsule, and this produces the sensation of sound. If the frequency lies below a certain limit a continuous note is not heard, but instead one recognises a succession of separate impulses. From experiments on long organ pipes and large tuning forks Helmholtz came to the conclusion that the lowest frequency which produces a continuous note is about 30 per second. The upper limit with average persons is about 15,000 per second. Waves having a greater frequency than this do not produce the sensation of sound but they can still be detected with sensitive flames. The method of determining the upper limit of audibility has already been described on p. 278. The factors which enable us to judge from which direction a sound is coming are not altogether known. There is no doubt that one factor is the different intensity at the two ears which we interpret by the help of previous experience. A source on our right will affect the corresponding ear more strongly than the other, which is partly screened by the head. Some animals, like the horse, can turn the external ears in various directions, and the alterations in intensity no doubt assist in locating the source. Experiments in recent years show that the phase difference of the waves at the two ears has also an effect. This is illustrated by the following experiment. Let a rubber tube be placed in each ear and let these tubes join a longer one

which passes into an adjacent room so that direct waves cannot reach the observer. If a sounding fork is held at the further end the apparent direction of the source alters as the length of one of the branches is varied in length so as to change the relative phases of the waves at the ears. The human voice is produced by two stretched membranes, called the vocal chords, which are situated in the larynx and form the edges of a narrow slit. Their tension and distance apart can be varied at will. When air from the lungs is forced past them they are made to vibrate and a sound is produced. The pitch of the note is determined by the tension of the chords, its quality largely by resonance of the air in the throat, mouth, and nose cavities. In a baby's cry there is little resonance, the effect is produced simply by the vocal chords. How large a part experience plays in the matter of voice production can hardly be appreciated until we have listened to the attempts of a young child to sing. The effect of a head cold on the voice shows what happens when resonance in the nasal cavity is suppressed.

Experiment.—Sound a tuning fork and sing the same note, then cease singing but keep the mouth in the same position. If the sounding fork is held near the open lips the mouth cavity resonates to the fork.

Experiment.—Sing a low chest note; the upper part of the chest can be felt to be in vibration if the fingers are placed on it.

Fourier's Theorem.—We have shown (Figs. 153 and 154) how to find the resultant of two or more S.H.M.'s in the same straight line. The converse operation is of frequent importance in many branches of physics, viz. given a resultant curve to find the simple harmonic components from which it is built up. Any curve which repeats itself time after time is called a periodic curve. Fig. 154 is an example of a periodic curve which is not simple harmonic yet is built up of simple harmonic components. This suggests the question, Can *every* periodic curve be regarded as the resultant of a number of S.H. components, and if so, can these components be chosen in more than one way? A component is completely specified when its period, phase, and amplitude are known. This problem has been solved by Fourier. According to **Fourier's theorem, every periodic curve can be resolved into S.H. components which can be chosen in only one way.** He showed further that if T is the time the resultant takes to repeat itself, in other words if its period is T, then the periods of the components are T, T/2, T/3, etc. Some of these may

be missing in certain cases, *e.g.* in Fig. 154 if T is the period of the resultant those of the components are T/4 and T/5. The methods of getting the proper amplitude and phase for each component are too complicated for us to consider. The importance of Fourier's theorem in sound will be seen from the following considerations. Suppose a sonometer wire is struck, and, by means which have been devised, let its displacement curve be taken. It will not in general be a sine curve, but it may be resolved into a number of such having periods T, T/2, T/3, etc. The question is, Are vibrations corresponding to these periodic times actually present, or is Fourier's theorem merely a mathematical device which bears no relation to physical facts ? If the former alternative is the true one, then, in addition to a note whose frequency n is that of the fundamental, there should be notes of frequency $2n$, $3n$, etc. Helmholtz has found that this is actually the case ; a brief account of his experiments will now be given.

Helmholtz's Experiments. Quality of Sounds.—Let n be the frequency of the wire when giving its fundamental. Helmholtz constructed a series of brass resonators of the shape shown in Fig. 182 ; the frequencies to which they responded were n, $2n$, $3n$, etc. They were placed with their open ends A near the wire, the narrow stem B could be held near the ear. When the wire was struck if a harmonic of frequency $6n$ was present the corresponding resonator would be thrown into vibration. By this means he showed that as many as 15 harmonics could be detected. Their relative strengths could be estimated roughly from the intensity of the resonance. When the same note was produced by another source such as an organ, violin, or the human voice, some of the harmonics were missing and their relative strengths were altogether different. This gives a clue as to the origin of the quality of sounds. Quality is determined by the superposition on the fundamental of a series of harmonics whose number and strength vary with the source. By damping a piano wire at different points, corresponding to the position of an antinode, certain harmonics can be excluded ; this is found to alter the character of the sound. Similarly the quality of the voice depends on what

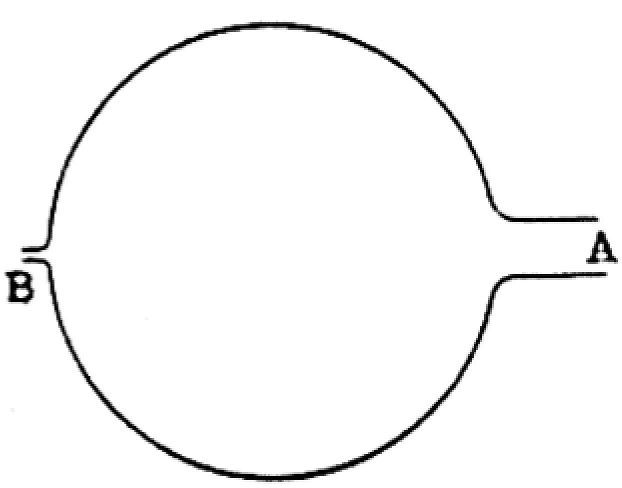


Fig. 182.—Helmholtz Resonator.

harmonics the vocal chords produce and how their relative intensities are altered by resonance in the mouth, etc. A trained singer learns by practice how to exclude those which detract from the pleasing quality of the voice. A note which cannot be resolved into more than one simple harmonic component is called **a tone.** Having analysed a note into its tones Helmholtz performed the converse experiment, viz. produced a note of given quality from its simple harmonic components. Each of the resonators used to analyse a piano note was placed with its mouth in front of a tuning fork of corresponding frequency, when all the forks were excited and the amplitude of the resonance properly adjusted it was found that a note was produced of practically the same quality as that given by the piano wire.

EXAMPLES ON CHAPTER XXVII

1. What conditions determine the velocity of transmission of a transverse wave in a stretched string? What constitutes the difference between sounds of the same pitch emitted by different instruments? (L. '81.)

2. What is a musical sound? Describe exactly what it is that determines the intensity, pitch, and quality of a musical note. (L. '92.

CHAPTER XXVIII

MAGNETIC POLES. LINES OF FORCE. THE INVERSE SQUARE LAW

Preliminary Facts.—It has been known for some hundreds of years that certain iron ores, consisting chiefly of Fe_3O_4, possess the property of attracting iron filings. These ores are called natural magnets, and the property in virtue of which this attraction takes place is called **magnetism.** If a natural magnet is rubbed along a steel bar it is found that the steel becomes endowed with magnetic properties, or, as is usually said, it becomes magnetized without in any way affecting the ore. The bar is then called an artificial magnet, or more briefly, a magnet. Steel magnets possess the great advantage over the natural product that they can be made in a variety of convenient shapes; the forms usually chosen are cylindrical or rectangular bars, either straight or bent into horseshoe shape. We shall see that the strongest magnets are made by passing an electric current through a number of turns of wire wrapped round a soft iron core; these are called electro-magnets.

Experiment.—Dip a bar magnet into iron filings; it is found that they adhere most strongly near the ends of the bar. Those points where the magnetism is most strongly exhibited are called the poles of the magnet. (A more exact definition is given later.)

Experiment.—Magnetize a steel knitting needle by rubbing it, always in the same direction, with the pole of a magnet; mark the end last touched by the pole. Suspend the needle by a cotton thread from a wooden stand (Fig. 183). If the thread is first damped and drawn past the edge of the thumb-nail its tendency to untwist will be greatly reduced. Bring the pole already used in magnetizing it near the marked end of the needle, it is attracted; bring the same pole near the unmarked end, repulsion takes place.

Evidently the poles of the needle possess dissimilar properties. If the magnetized needle is replaced by an unmagnetized one each

end is attracted by the pole of a magnet. Repulsion, therefore, is the only sure test of the presence of magnetism.

EXPERIMENT.—Again suspend the magnetized needle and notice the direction in which it points ; this will be found to be roughly north and south. Displace it from this position, or suspend it with its poles reversed, it will be found to come to rest in the same direction as before, with the same end pointing north.

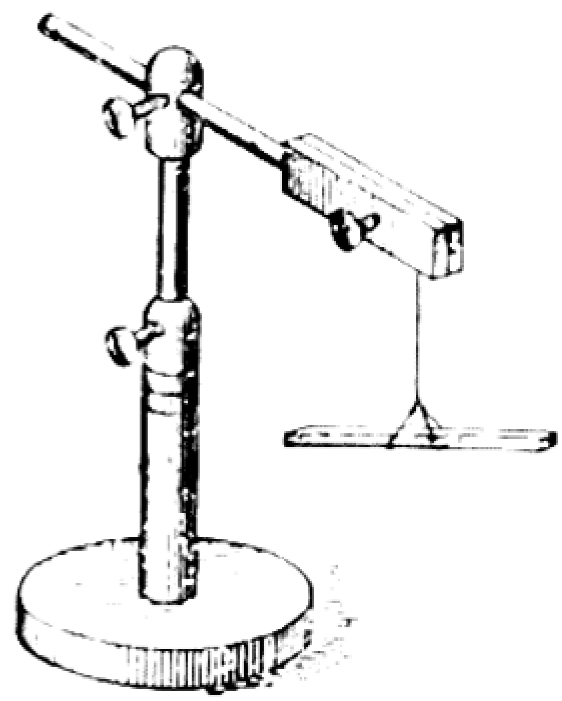


FIG. 183.

That end which points north is called the north-seeking, North, or positive pole, the other is called the south-seeking, South, or negative pole.

The straight line joining the poles is called the **magnetic axis of the magnet**; its positive direction is taken to be from the S. to the N. pole. The vertical plane passing through the magnetic axis of a freely suspended magnet at rest is called the **magnetic meridian,** this plane is said to run magnetic N. and S.

EXPERIMENT.—Suspend several magnetized knitting needles in succession, as in the last experiment, and from their direction when at rest determine their positive poles. Mark these and suspend one of the needles. Now show that its positive pole is repelled by the positive pole of a second needle, but is attracted by a negative pole ; and similarly that negative poles repel each other.

It is seen that **like poles repel and unlike poles attract each other.**

The second experiment above shows that when a rod is magnetized by rubbing with a positive pole the end of the rod that is touched last becomes a negative pole.

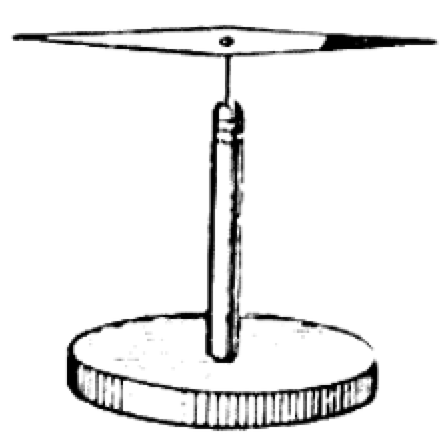


FIG. 184.—Compass Needle.

For experiments such as these it is very convenient to have a test needle, with its positive pole marked, supported on a fine point as shown in Fig. 184. Such a magnet is called a compass needle.

It is found impossible to make a positive pole in a bar without creating a negative pole at some other point of it; these unlike poles are not necessarily at the ends.

EXPERIMENT.—Stroke a knitting needle with the positive pole of a magnet starting from the middle and going to each end in turn. Tests with the compass

needle will show that the ends are negative poles, and that there is a positive pole near the centre.

Such an arrangement may be regarded as consisting of two magnets with their positive poles in contact, forming what is called a **consequent pole** at the middle of the needle.

Magnetic Substances. Experiment.—Hang a bit of magnetized watch-spring by a silk fibre over the poles of a horseshoe magnet. It sets along the line joining the poles. Now replace the watch-spring by a short piece of bismuth. (This may readily be cast in a suitable form.) It is found to set at right angles to the line joining the poles. An electromagnet is usually required for this experiment. A piece of copper is apparently uninfluenced.

Accurate experiment shows that all substances may be divided into two classes; members of the one resemble steel in setting along the line joining the poles, the others set in a perpendicular direction like bismuth. The former are called **para-magnetic,** the latter **diamagnetic,** substances; we shall confine our attention to members of the first class, Only a few of these, viz. iron, steel, nickel, cobalt, and certain alloys, exhibit magnetic properties to a very marked degree; they are called magnetic or **ferro-magnetic** substances. Very careful experiments are necessary to detect magnetic properties in other materials, we shall therefore regard all other substances as being non-magnetic.

Permanent and Temporary Magnetism.—The following experiments show that the magnetic properties of iron and steel have important differences.

Experiment.—Place one end of a soft iron bar in contact with the pole of a strong magnet. The further end of the bar can now attract iron filings and repel one of the poles of a compass needle, when, however, the magnet is removed the magnetism almost entirely disappears. If the iron bar is suspended over the poles of a horseshoe magnet it can be attracted and repelled by another magnet, it has thus become temporarily a magnet. Repeat the experiments with a steel rod and it will be found that the magnetism developed is very small.

Experiment.—Rub an iron bar with the pole of a magnet; it is only feebly magnetized and the greater part of this magnetism disappears when it is struck on the bench. Steel, as has been seen, can be permanently magnetized.

These facts may be collected in the statement that iron readily becomes magnetized while under the influence of a magnet, but its magnetism is only temporary; steel, on the other hand, is more difficult to magnetize but its magnetism is permanent. When a

substance acquires magnetic properties by being placed in the neighbourhood of a magnet it is said to be magnetized by **induction or influence.** The latter term is preferable, as the word "induction" is used later in a quantitative sense. It is evidently important that we should be able to make magnets capable of retaining their magnetism in undiminished quantity. For example, the accuracy of certain electrical instruments depends on the constancy of the magnets used in their construction. The best magnets for the purpose are made from tungsten steel (steel containing about 5 per cent. of tungsten). The form given to the magnet has also great influence on its permanence; the most suitable shape is that of a nearly closed ring which brings the poles together. In the case of bar magnets constancy is partially attained by laying two of them side by side and connecting unlike poles by a piece of soft iron, called a keeper, as in Fig. 185. The reason for this will appear later.

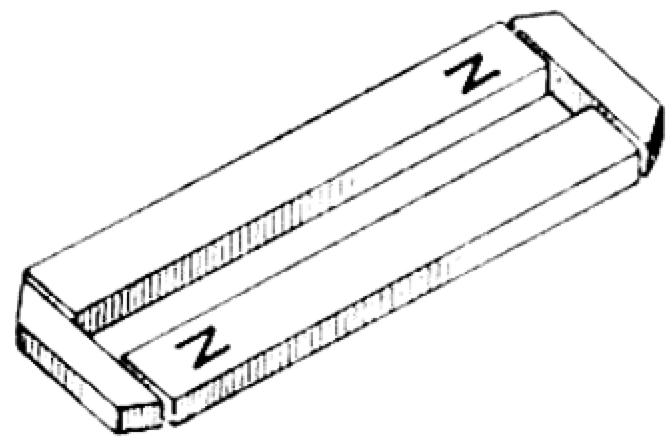


FIG. 185.—Bar Magnets and Keepers.

The Inverse Square Law.—The preceding experiments show that between the poles of two magnets there exists a force of attraction or repulsion; it is important to determine how this varies with their distance apart. A convenient apparatus for the purpose, due to Grimsehl, is shown in Fig. 186.[1] AB represents a magnetized knitting needle about 20 cms. long attached to a similar rod of brass AC. The whole is balanced round a knife edge at A. Another magnetized needle DE can be moved up and down the brass rod P, and the distance apart of the poles at D and B can be read off a vertical scale.

EXPERIMENT.—Determine the approximate position of the poles by dipping each magnet in iron filings, and place like poles at D and B. On account of the force of repulsion B is now pushed downwards, but the needle is brought back to its horizontal position by moving a sliding weight Q. Let F be the vertical force on the pole B, R the distance BD, w the weight of Q. Then by the principle of moments $F \cdot AB = w \cdot AQ$, or $F = w \cdot AQ/AB$. Find the length AQ for different values of R, as w and AB are constant F varies as AQ. It

[1] See also Barton and Black, "Practical Physics," p. 121, where a similar apparatus due to Hibbert is described.

will be found that $AQ \times BD^2$ is very approximately constant, or FR^2 is constant, hence

$$F \propto \frac{1}{R^2}$$

Thus the force exerted by one pole on another varies inversely as the square of their distance apart. For example, if BD is halved it will be found necessary to make AQ four times larger, halving the distance between the poles has increased the repulsion in the ratio 4 : 1. This is the inverse square law first discovered by Coulomb. In the above the attraction between the unlike poles at B and E has been neglected ; this will be justifiable if the distance BD is not too

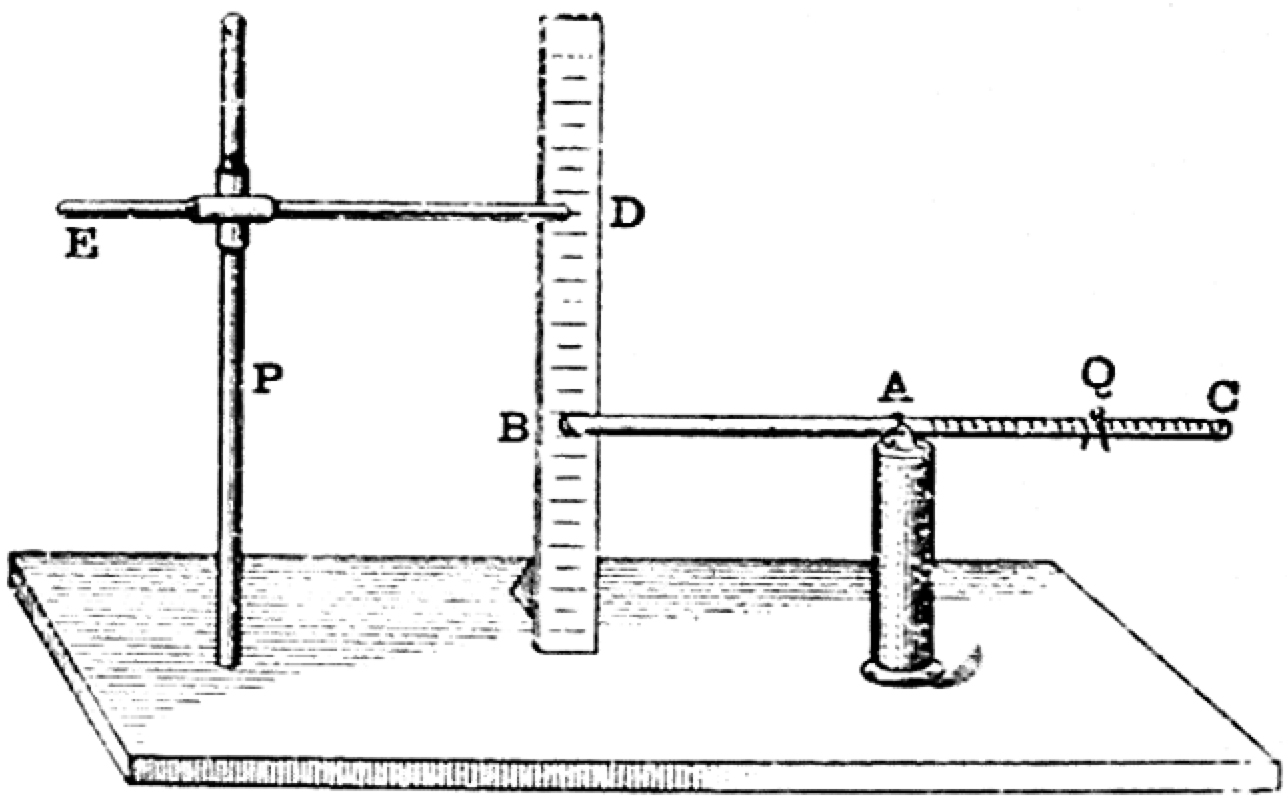


Fig. 186.—Balance Method of proving the Inverse Square Law.

large. The attraction between the poles at D and A is also negligible, since the latter pole is near the fulcrum and the moment of the force tending to turn the rod is very small. A more exact proof of the inverse law is given later.

Evidently if there could be added to the pole B another exactly like it the force of repulsion would be doubled ; if at the same time the pole at D could be increased threefold the force would be increased sixfold. Thus the force between two poles varies as the product of the pole-strengths. Combining the two results, if m and m' are the strengths of two poles separated by a distance R,

$$F \propto \frac{mm'}{R^2}$$

or

$$F = \frac{1}{\mu} \cdot \frac{mm'}{R^2}$$

where μ is a constant depending on the nature of the medium in which the poles are immersed. Experiment shows that the force is the same in all non-magnetic media. In such media μ is arbitrarily put equal to unity; thus in air

$$F = \frac{mm'}{R^2}$$

This equation is used to define the unit pole. Let two equal and similar poles, concentrated at points, be placed 1 cm. apart in air, and let us suppose the strength of each can be altered by the same amount until the force of repulsion is 1 dyne. The equation then becomes, since F and R are each unity,

$$1 = m^2$$

i.e. $m = 1$, or the poles are of unit strength. We thus get the definition: **If two equal and similar poles concentrated at points 1 cm. apart in air repel each other with a force of 1 dyne, then each is a unit pole.**

Magnetic Field.—The space around a magnet in which the magnetic force can be detected is called the field of the magnet. The **intensity of the magnetic field** at a point is measured by the force in dynes which would act on a unit positive pole if placed at that point, the presence of the pole being supposed to produce no disturbance. If the force is 1 dyne the field is called unit field; to this the name **Gauss** has been given. When a pole of strength m is placed in a field of H Gausses the force acting upon it is mH dynes. If the field is everywhere the same in direction and intensity it is called a uniform field. Let us find the intensity of the field due to a pole of strength m at a point R cms. away from it. Imagine a unit N. pole placed at the point in question, the intensity of the field is the force which acts on it. The repulsion between the two is $F = \frac{m \times 1}{R^2}$ or $F = \frac{m}{R^2}$. Since a compass needle removed from other magnets is acted on by forces which cause it to set in a definite direction, we must suppose there is a magnetic field due to the earth. This is uniform over a small space if all magnetic materials are removed to a distance.

EXPERIMENT.—Lay a short magnet on a cork and float it on water. It sets in a definite direction but does not move as a whole to the side of the

vessel. If H is the intensity of the earth's field, m and $-m'$ the strengths of the poles, the resultant force acting on the magnet is $(m-m')$H. As the magnet is at rest this is zero, hence $m=m'$, or *the poles of a magnet are of equal strength.*

Lines of Force.—As the strength of a field is measured by the force which acts on a unit pole, it may be represented by a line drawn in a definite direction and of proper magnitude. Dealing only for the moment with the direction, a curve may be drawn in such a manner that the tangent to it at any point is parallel to the direction of the field at that point. Such a curve is called **a line of force.** The whole space round a magnet may be mapped out by a series of these lines showing everywhere the direction of the field.

When a very short compass needle is placed near a magnet, throughout the small space it occupies we may suppose that the field is uniform, and as the poles are equal they are acted upon by equal forces in opposite directions. The needle therefore comes to rest with its axis parallel to the field, or coinciding with the line of force. This provides a means of plotting the lines.

EXPERIMENT.—Place a magnet and a short compass needle on a sheet of paper and mark the position of the ends of the needle. Shift it so that its negative pole is at the point previously occupied by the positive and again mark the position of the ends. Proceeding in this way we get a series of points which if joined by a continuous curve give a line of force.

As the lines are drawn to show the direction in which a positive pole moves they must start at a positive and end at a negative pole. It is extremely useful to picture all space in a magnetic field as filled with these lines. Fig. 187 shows them for a few typical cases; they are supposed to be running from N to S. They may be more easily shown by sprinkling iron filings over a sheet of paper on which the necessary magnets are placed. Each scrap of iron then becomes a magnet by influence and sets along a line of force when the paper is gently tapped. Fig. 188 (a) shows the result for a horseshoe magnet.

From Fig. 187 we see that the lines crowd together near the poles where the field is strong; they give not only the direction but some idea of the relative intensities of the field at different points. They can be made to represent the intensity accurately if, instead of an indefinite number, they are drawn according to the following rule: Suppose a small area held with its plane perpendicular to the

lines, then the number passing per cm.^{2} is made numerically equal to the intensity of the field at the centre of the area. This number is frequently spoken of as the density of the lines; thus in a field of 10 Gausses the density of the lines is 10. In a uniform field the lines are equidistant and parallel. Let a unit pole be placed at the centre of a sphere 1 cm. in radius. The field on the surface is m/R^2 and is therefore unity; hence we must draw one line through each cm.^{2} and 4π lines through the whole surface. Each unit pole therefore gives rise to 4π lines if the rule just given is adhered to.

A number of properties of the magnetic field may be explained in terms of the lines of force if we suppose, as Faraday did, that (1) they are in a state of tension, (2) they repel each other sideways. Thus the attraction of unlike poles may be ascribed to the tension in the lines running from one to the other; this tends to draw the poles together. If the lines experience a sideways repulsion, Fig. 187 *C* shows that unlike poles will repel each other. The curvature of the lines in Fig. 187 *B* is due to the same cause; they spread out until the repulsion is small enough to be balanced

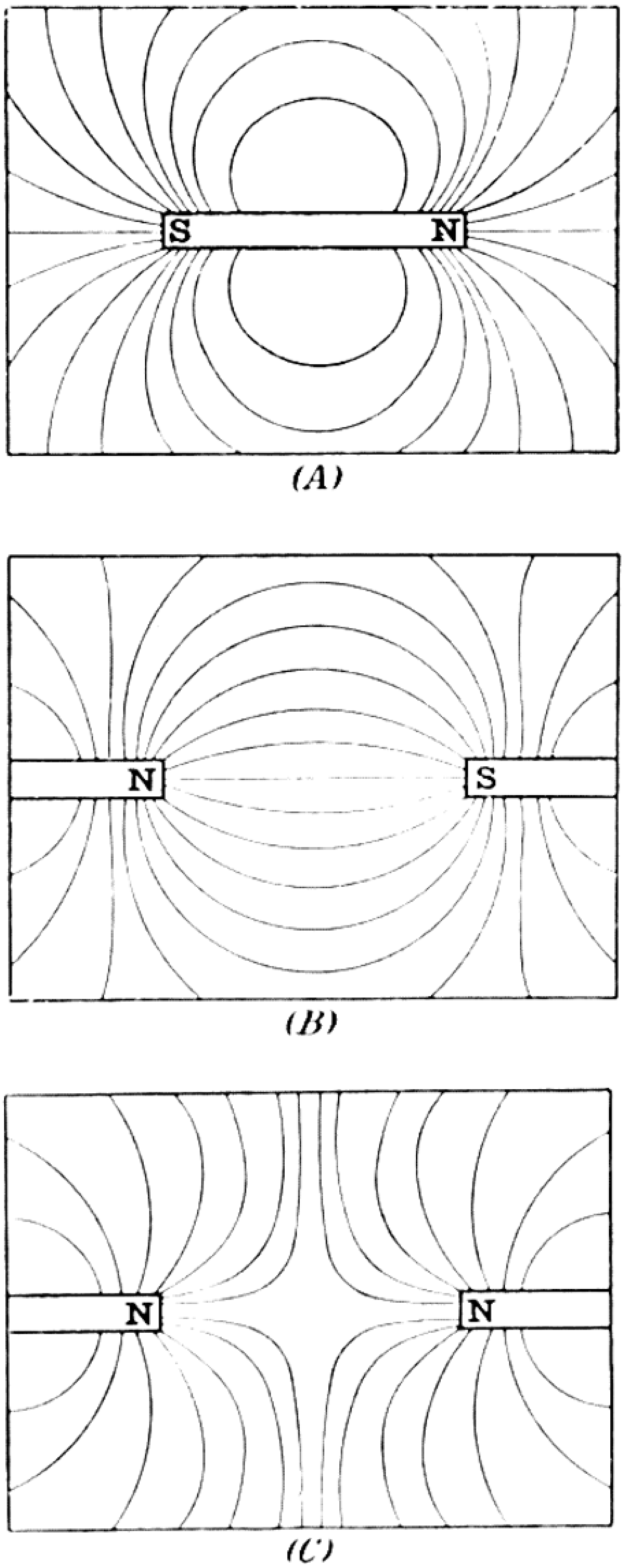
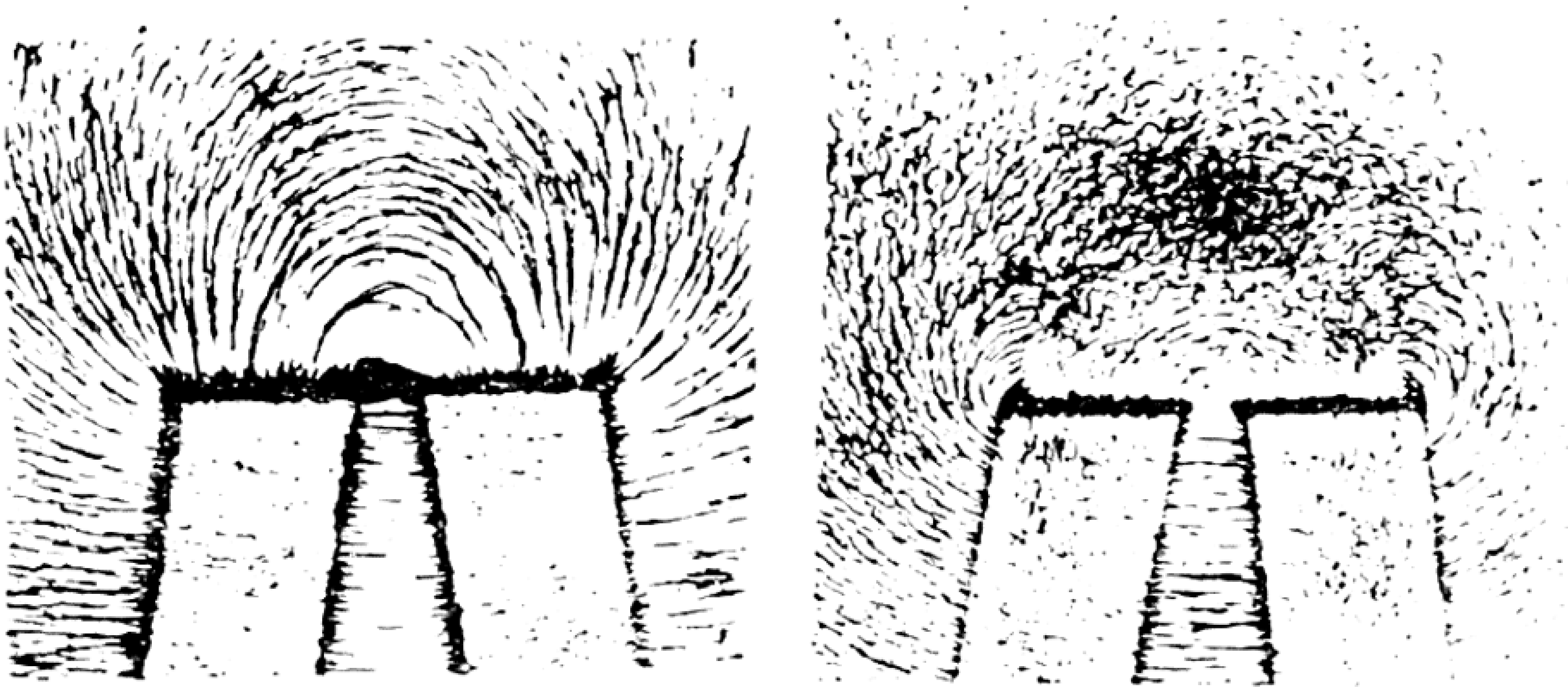


Fig. 187.—Typical Cases showing the Distribution of Lines of Force.

by the tension. It can be shown that the lines pass more readily through iron than air.

EXPERIMENT.—Cover a horseshoe magnet with a sheet of paper, sprinkle over it iron filings and so obtain the usual figure. Place a keeper near the poles and repeat the experiment; scarcely any lines can now be seen, they



(a) (b)

FIG. 188.—Distribution of Lines of Force by means of Iron Filings.

pass from pole to pole through the iron rather than through the air. In Fig. 188 (a) was taken with the keeper off, (b) with the keeper near the poles; note the absence of lines above the keeper.

Fig. 189 shows how a piece of soft iron distorts a uniform field on account of this crowding of the lines into the metal. Where they enter and leave they create a negative and positive pole respectively.

Molecular Theory of Magnetism.—One cannot work long with iron filings and magnets before the question presents itself, What is the change produced in an iron bar which causes it to show magnetic properties when held near a magnet?

The following experiment suggests one step in the solution of the problem.

EXPERIMENT.—Half fill a test-tube with iron filings, pass in to the centre a small brass disc attached to a wire handle, then add more filings until the whole tube is loosely filled. The tube attracts each pole of a compass needle, showing that it is magnetically neutral. Draw the pole of a magnet along it several times; some of the filings are seen to arrange themselves with their lengths parallel to the axis of the tube, and if it is now tested it is found to

exhibit polarity at the ends. Shake the tube and the magnetism disappears. Remagnetize it; there is no pole at the centre, but if the brass plug is pulled out so as to remove one-half of the filings a pole appears at this point which is of the same sign as that which has been removed at the end.

In explanation of this it is supposed that the molecules of iron are small magnets—how they acquired their magnetism we cannot inquire—initially their axes are turned in all directions thus accounting for the absence of poles in the tube. When it is magnetized all that is done is to turn a number of the axes into the same direction as each other, as in Fig. 190; this results in a N. pole at one end and a S. pole at the other. When the part to the right is removed we evidently have at C a pole of the same sign as that at B. If the tube is shaken this regular arrangement disappears and with it the polarity of the tube. This method of regarding the process of magnetization is called

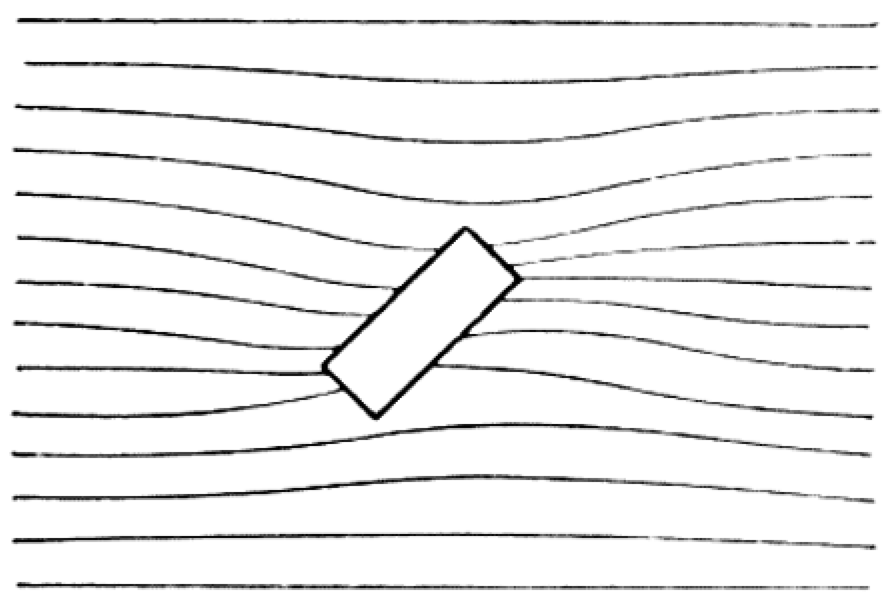


FIG. 189.—Distortion of Lines of Force by a piece of Soft Iron.

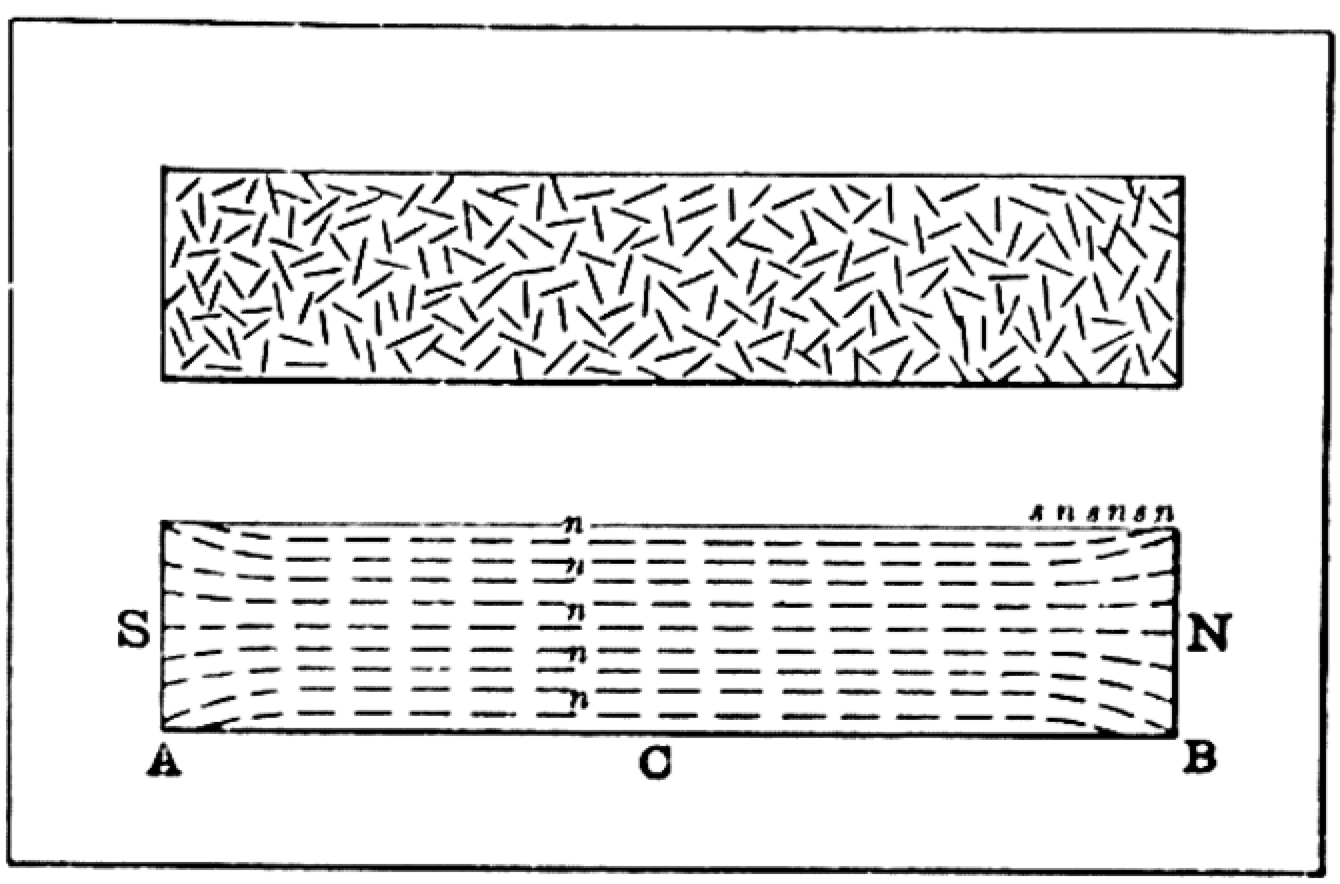


FIG. 190.—To illustrate the Molecular Theory of Magnetism.

the molecular theory of magnetism; it is supposed to apply equally to the case of actual magnets. According to this theory,

the more strongly a bar is magnetized the greater is the proportion of the molecules which have fallen into the regular arrangement; if all the small magnets were so directed the magnet could not be made stronger. Such a stage has been reached with soft iron. To explain the differences in the behaviour of iron and steel it is supposed that in the former the molecular magnets can easily be turned, but readily fall out of alignment when the magnetic field is removed, while in steel the movements are performed less easily, thus accounting for the greater difficulty of magnetizing it and for its permanent magnetism.

EXPERIMENT.—Break a magnetized knitting needle in two pieces; the broken surfaces exhibit opposite polarities and each part is a complete magnet, as in the analogous experiment with the tube of filings.

Any rough treatment such as hammering or jolting will tend to disturb the molecules, especially near the ends of the bar where there are no attracting poles to hold them in position. Similarly if a magnet is heated the molecules are thrown into more vigorous vibration (p. 8), and the regular arrangement may be destroyed. In each case the bar will lose some, or the whole, of its magnetism. Suppose now the poles of a horseshoe magnet are joined by a soft iron keeper, the iron becomes magnetized and there is a complete chain of molecules from pole to pole. It will thus be difficult for a molecular magnet to turn out of its position and the magnetism is rendered more stable. This is why keepers are used. A similar effect is produced if a magnet is made in the form of a nearly closed ring with a narrow gap separating the poles; the lines of force run across the gap and keep the molecules in position.

Poles and Magnetic Moment of a Magnet.—Iron filings adhere to the ends of a magnet not merely at points but over considerable areas, hence we must regard the pole as consisting of a collection of unit poles distributed throughout a certain space. When such a bar is placed in a uniform field each half is subjected to a number of equal and parallel forces arising from the action of the field on the unit poles; this multitude of forces may be replaced by single resultants acting on each half at definite points—the centres of the parallel forces. These points are the two poles of the magnet; in a uniform field the bar will behave as if its magnetism were concentrated at these points. Thus **the poles of a magnet are at the centres**

of the two systems of parallel forces which act on each half of the magnet when it is placed in a uniform field.

The poles may be found approximately by plotting the lines of force near each end of the magnet and finding where they meet when the bar is removed. If the strength of the poles is m and their distance apart is l, the product ml is called the **magnetic moment of the magnet.** We will denote it by M.

Couple acting on a Magnet.—Let a magnet whose pole strength is m and the distance between whose poles is $2l$ be placed in a uniform field of intensity H, and let it be deflected so that its axis makes an angle θ with the lines of the field (Fig. 191). Each pole is acted upon by a force mH tending to reduce θ. Let us calculate the moment of the restoring couple formed by these forces. Draw BC perpendicular to the field; the moment required is evidently $m\mathrm{H} \,.\, \mathrm{BC}$, but $\mathrm{BC} = \mathrm{AB} \,.\, \sin\theta = 2l \,.\, \sin\theta$.

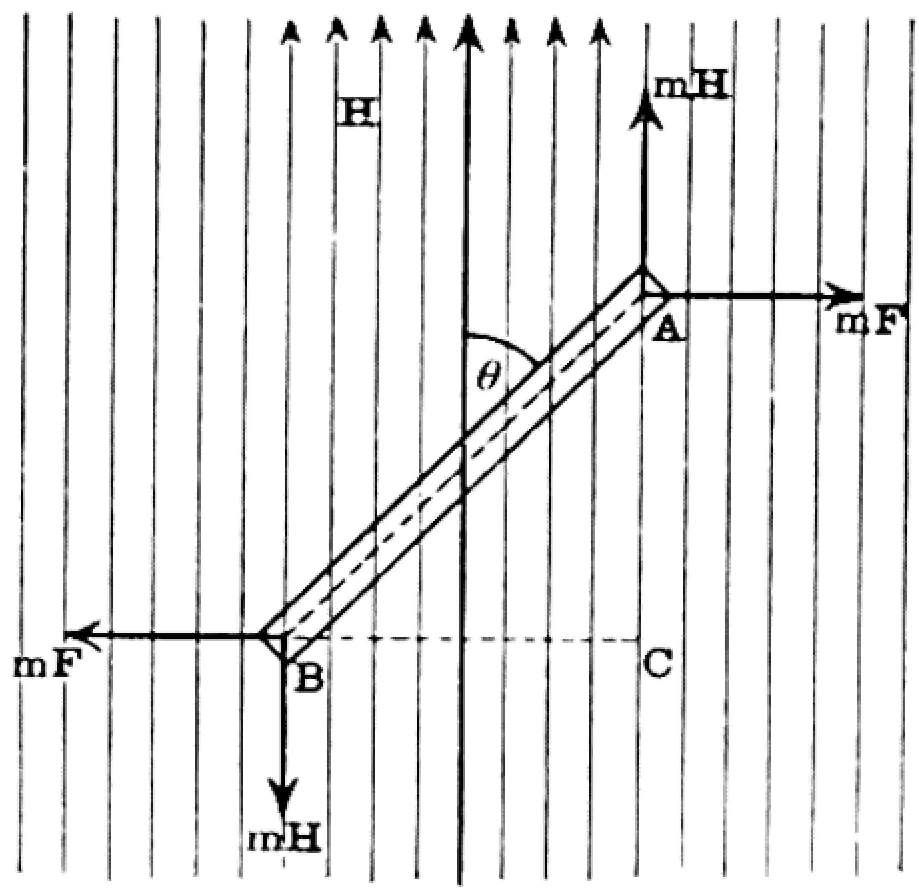


Fig. 191.—Couple acting on a Magnet in a Uniform Field.

$$\therefore \text{restoring couple} = m \,.\, 2l \,.\, \mathrm{H} \sin\theta = \mathrm{MH} \sin\theta$$

Suppose that the deflexion θ is produced by a uniform field F perpendicular to H. The force on each pole due to F is mF, and the moment of the couple they produce is $m\mathrm{F} \,.\, \mathrm{AC}$. If the magnet is in equilibrium the couples due to F and H are equal,

$$\therefore m\mathrm{F} \,.\, \mathrm{AC} = m\mathrm{H} \,.\, \mathrm{BC}$$

or
$$\mathrm{F} = \mathrm{H} \,.\, \frac{\mathrm{BC}}{\mathrm{AC}} = \mathrm{H} \,.\, \tan\theta$$

Hence if H is known F can be found. These two results should be remembered; in using them it should be noted that we suppose the fields are (1) uniform and (2) perpendicular to each other.

Experiment.—The formulæ will hold whether the forces arise from magnetism or other causes, hence the tangent formula may be verified by the

apparatus shown in Fig. 192.[1] The small needle at the centre, which represents the bar magnet of the formula, carries a long pointer at right angles to it which moves over a graduated circle. Two strings are fastened to each end of the needle, these pass over pulleys and carry weights at their free ends. The pulleys can slide on their supports so that the forces constituting one couple can be kept parallel and at right angles to those forming the other. One pair of weights is kept constant and the other is varied. It should be proved that the ratio of the tensions in the strings is equal to the tangent of the angle of deflexion of the needle.

A better definition of the moment of a magnet can now be given;

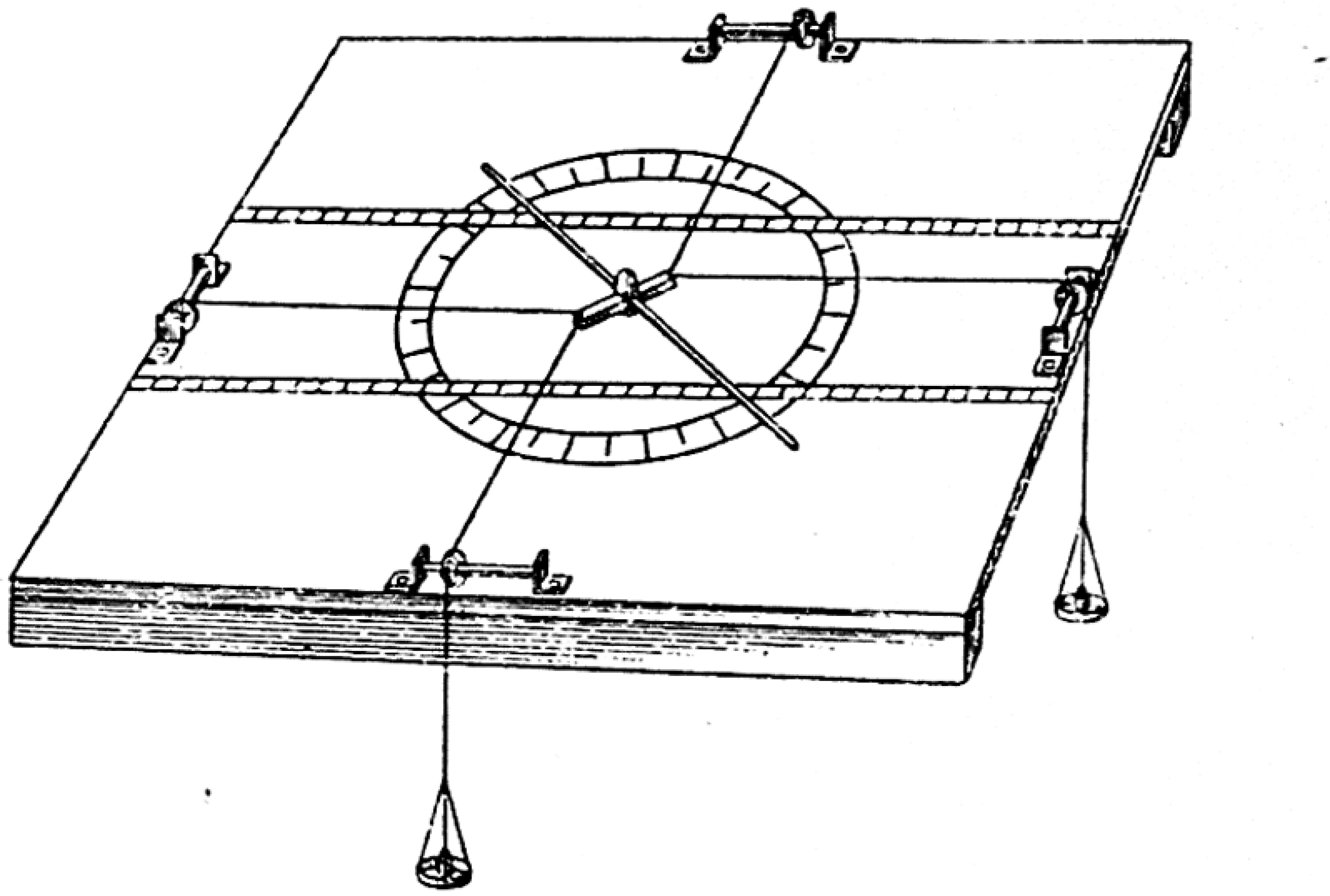


FIG. 192.—Burton's Apparatus for proving the Tangent Law.

that on p. 319 is unsatisfactory as it is not known exactly where the poles are situated. Let a magnet be held perpendicular to the lines of force in a field of unit intensity, then the restoring couple is $MH \sin \theta = M \sin \frac{\pi}{2} = M$. Hence its **magnetic moment is the moment of the couple required to hold it perpendicular to the lines of force in a field of unit intensity.**

EXAMPLES ON CHAPTER XXVIII

1. A magnet suspended by a fine vertical wire hangs in the magnetic meridian when the wire is untwisted. If on turning the upper end of the wire half round the magnet is deflected through 30° from the meridian, show how

[1] To be obtained from Messrs. Pye, Cambridge.

much the upper end of the wire must be turned in order to deflect the magnet 45° and 60° respectively. (L. '84.)

2. A bar magnet is laid on a sheet of paper on a drawing-board. Supposing it to have its poles concentrated at two given points, how would you determine by measurement and calculation the direction of the force at a given point on the paper? and, given a small compass needle, how would you test the result? (L. '95.)

3. By what experiments would you show that the two poles of a magnet are of equal strength and of opposite polarity? How is the equality accounted for on the molecular theory of the constitution of a magnet? (L. '01.)

4. Calculate in C.G.S. units the couple on a bar magnet 4 cms. in length, with poles each of strength 150 units, when placed with its axis at right angles to a magnetic field of intensity 0·18. (L. '06.)

5. Two magnets each of effective length 8 cms. and moment 80 units, lie in the same straight line with their N. poles 6 cms. apart. Calculate the repulsive force between them. (L. '09.)

CHAPTER XXIX

MAGNETIC MEASUREMENTS

Time of Oscillation of a Magnet.—If a magnet is allowed to oscillate freely in a horizontal plane in a uniform field it can be shown that the motion is simple harmonic, provided the amplitude is small. The period in seconds is given by

$$T = 2\pi\sqrt{\frac{K}{MH}}$$

where M is the magnetic moment of the magnet, H is the horizontal intensity of the field in Gausses, and K is a constant for a given magnet, called its moment of inertia. The value of K depends on the size, shape, and mass of the bar, and upon the axis about which the oscillations take place. For a cylindrical bar oscillating about an axis perpendicular to its length and passing through its centre of gravity

$$K = m\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$$

where l is the length, r the radius of the cylinder, and m its mass. If the bar is rectangular, of length a and breadth b, and it oscillates about an axis through the centre of gravity perpendicular to the plane containing a and b its moment of inertia is

$$K = m \cdot \left(\frac{a^2 + b^2}{12}\right)$$

EXPERIMENT.—The equation may be used to compare the moments of two magnets if the moments of inertia can be calculated. Suspend a magnet in a horizontal position by means of a thin thread and let it oscillate under the action of the earth's field H. Repeat with the second. If T_1 and T_2 are the periods

$$\frac{T_1^2}{T_2^2} = \frac{K_1/M_1H}{K_2/M_2H}$$

whence

$$\frac{M_1}{M_2} = \frac{K_1}{K_2} \cdot \frac{T_2^2}{T_1^2}$$

Comparison of Fields by the Oscillation Method.[1]—If a short magnet is caused to oscillate at different points the same equation may be used to compare the fields at these points; in this case K and M are constant. If T_1 and T_2 are the periods where the fields are H_1 and H_2 respectively, then

$$\frac{T_1^2}{T_2^2}=\frac{1/H_1}{1/H_2}$$

or

$$\frac{H_2}{H_1}=\frac{1/T_2^2}{1/T_1^2}$$

When the fields differ greatly in intensity it is possible that the magnetic moment M of the oscillating needle may be altered by influence; in that case the method is inapplicable. The oscillation needle shown in Fig. 193 is due to Mr. Searle. The magnet passes through a brass block which tapers below to a point. Its position can easily be marked on the table and the weight of the block renders the oscillations slow enough to be counted readily.

Experiment.—*To compare the fields produced by two magnets at a given distance away from them.* Let the needle oscillate in the earth's field alone and note the time of a number of oscillations, hence deduce the period T. Place one of the magnets due S. of the needle, and at the given distance away, with its N. pole pointing north. The lines of force due to the earth and magnet are parallel at the needle and the field is $(F_1 + H)$, where F_1 is field due to the magnet alone and H that of the earth. Let the period be T_1,

the

$$\frac{F_1 + H}{H} = \frac{1/T_1^2}{1/T^2}$$

$$\therefore \frac{F_1}{H} + 1 = \frac{1/T_1^2}{1/T^2}$$

and

$$\frac{F_1}{H} = \frac{\frac{1}{T_1^2} - \frac{1}{T^2}}{\frac{1}{T^2}}$$

Repeat the observations with the second magnet, then

$$\frac{F_2}{H} = \frac{\frac{1}{T_2^2} - \frac{1}{T^2}}{\frac{1}{T^2}}$$

[1] See also Barton and Black, "Practical Physics," pp. 117-133.

Dividing one equation by the other

$$\frac{F_1}{F_2} = \frac{\frac{1}{T_1^2} - \frac{1}{T^2}}{\frac{1}{T_2^2} - \frac{1}{T^2}}$$

The reason the experiment is done in this way is to make it possible to allow for the earth's field, as this cannot be got rid of in the observations. A similar set of measurements enables us to prove the inverse square law by a method originally used by Coulomb. It is best for this experiment to use a Robison magnet (Fig. 193).

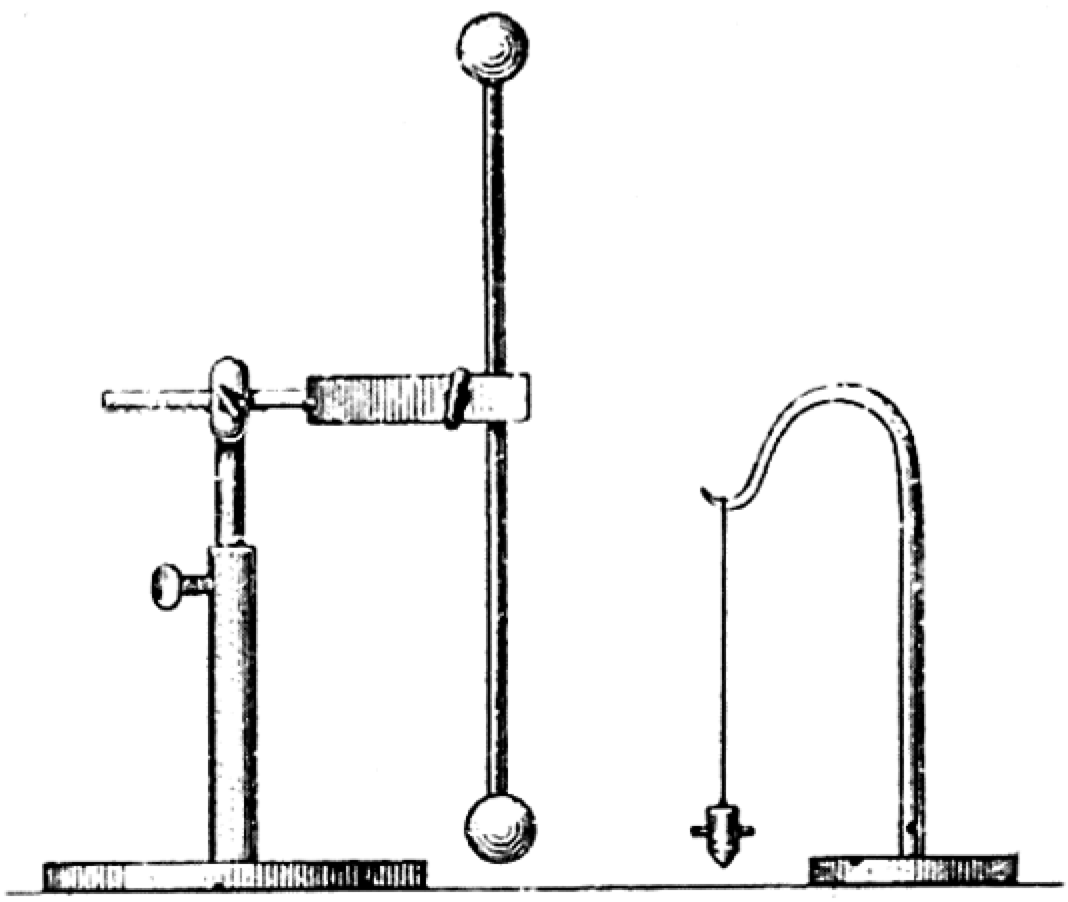


FIG. 193.—Oscillation Method of proving the Inverse Square Law.

This is a long steel rod with spherical balls of the same metal at each end; by plotting lines of force it can be shown that the poles are situated at the centres of the spheres, hence we get a definite point from which to measure.

EXPERIMENT.—Note the period T of the compass needle when vibrating in the earth's field, then place the Robison magnet in a vertical position due S. of the needle with its N. pole at the same level (Fig. 193). The field at the needle is now $(F_1 + H)$; let the period be T_1 and the distance from the pole be d_1. Alter the distance to d_2 and observe the new time of vibration T_2. Then

$$\frac{F_1}{F_2} = \frac{\frac{1}{T_1^2} - \frac{1}{T^2}}{\frac{1}{T_2^2} - \frac{1}{T^2}}$$

Hence prove that $\frac{F_1}{F_2} = \frac{d_2^3}{d_1^3}$. The field due to the upper S. pole is negligible if the magnet is long and d_1 and d_2 are not too large.

Field due to a Bar Magnet in Two Standard Positions.—It is required to find the field due to a bar magnet at a point on its axis. Let P be the point which is at a distance d from the *centre* of the magnet, also let m be the pole strength and $2l$ the distance between the poles

FIG. 194.—End-on Position.

(Fig. 194). Then the field at P is the resultant of m at N and $-m$ at S and is evidently in the direction NP.

$$\text{Hence the field } F_1 = \frac{m}{NP^2} - \frac{m}{SP^2}$$

$$= m\left\{\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right\}$$

$$\therefore\ F_1 = \frac{4ml \cdot d}{(d^2 - l^2)^2}$$

But the moment of the magnet $M = 2l \cdot m$,

$$\therefore\ F_1 = \frac{2Md}{(d^2 - l^2)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If l is small compared with d, which means that a short magnet must be used, we may neglect l^2 in comparison with d^2 in the denominator and

$$F_1 = \frac{2M}{d^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Next let the point P be on the line bisecting the magnet at right angles and at a distance d from the centre (Fig. 195). The field at P is now made up of two equal components, (a) m/NP^2 along PQ and (b) m/SP^2 along PS. The $\triangle$ NPS has its sides NP and PS parallel to these forces, if therefore these lines are taken to represent the forces the resultant F_2 is represented in magnitude and direction by the line NS. But $ON/NP = \cos ONP$,

$$\therefore\ NS = 2ON = 2PN \cdot \cos ONP$$

Replacing the lines by the forces they represent we have

$$F_2 = \frac{2m}{NP^2} \cdot \cos ONP$$

But $$NP^2 = d^2 + l^2$$

and $$\cos ONP = \frac{l}{(d^2 + l^2)^{\frac{1}{2}}}$$

$$\therefore\ F_2 = \frac{2l \cdot m}{(d^2 + l^2)^{\frac{3}{2}}} = \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} \quad . \quad . \quad . \quad (3)$$

If l^2 can be neglected as before

$$F_2 = \frac{M}{d^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Taking the two simple formulæ it is seen that the field in the second case is half that in the first, and, as the figures show, is oppositely directed. These formulæ are direct consequences of the inverse square law; if it had been assumed that $F \propto 1/R^n$ we should have obtained, when l^2 is negligible in comparison with d^2,

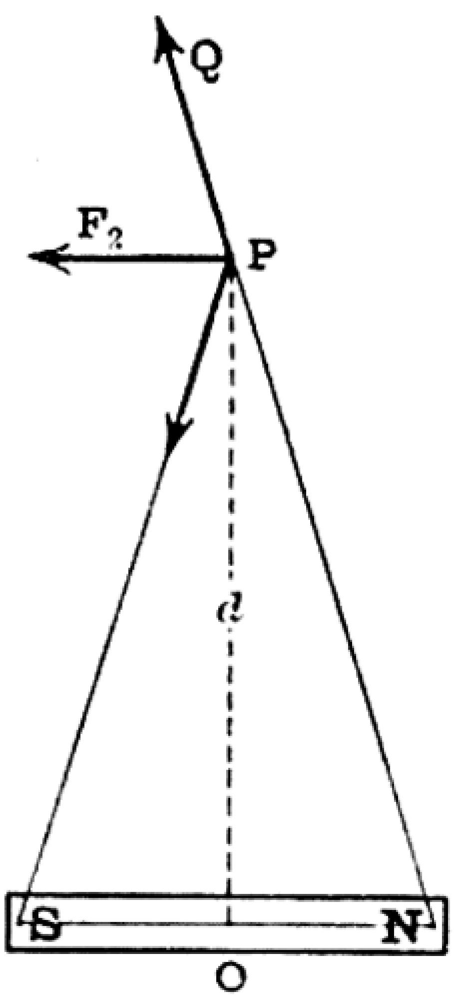


FIG. 195.—Broadside-on Position.

$$F_1 = nF_2 \text{ (see p. 334)}$$

Hence by finding the ratio of F_1 to F_2 the truth of the inverse square law can be tested. An oscillation method is here given; for a more accurate means, see p. 331.

EXPERIMENT.—Find the time of oscillation of a Searle compass needle in the earth's field H, then place due S. of it a short bar magnet with its axis in the meridian and its positive pole pointing N. The distance between the needle and the centre of the magnet should be about 20 cms. and a strong magnet should be used. Find the new period in the field $(F_1 + H)$. As the poles may not be symmetrical with respect to the centre, it is best to repeat the last observation with the magnet at the same distance due N. of the needle. Call the mean period with the magnet in position T_1. Turn the magnet so that its negative pole points N., and place it in the meridian with its centre due W. of the needle and the same distance away as before. The lines due to the earth and the magnet are again parallel and the field at the needle is $(F_2 + H)$. Find the period of oscillation T_2 and repeat with the magnet due E. of the

needle. The ratio of F_1 to F_2 can then be found from the formula on p. 324; it should be 2 very approximately if the inverse square law is true.

EXPERIMENT.—Compare the fields due to the magnet at different distances from it in either of the two standard positions; hence show that $F \propto \frac{1}{d^3}$, where d is the distance from the *centre.*

The A and B Tangent Positions of Gauss.—In case I. just considered let there be placed at P a short compass needle, and let the axis of the bar magnet be at right angles to the earth's lines of force (Fig. 196). Before the bar magnet is brought up the needle will lie in the meridian, afterwards it will be deflected through an angle θ_1; let us calculate this deflexion. Let H be the earth's horizontal field

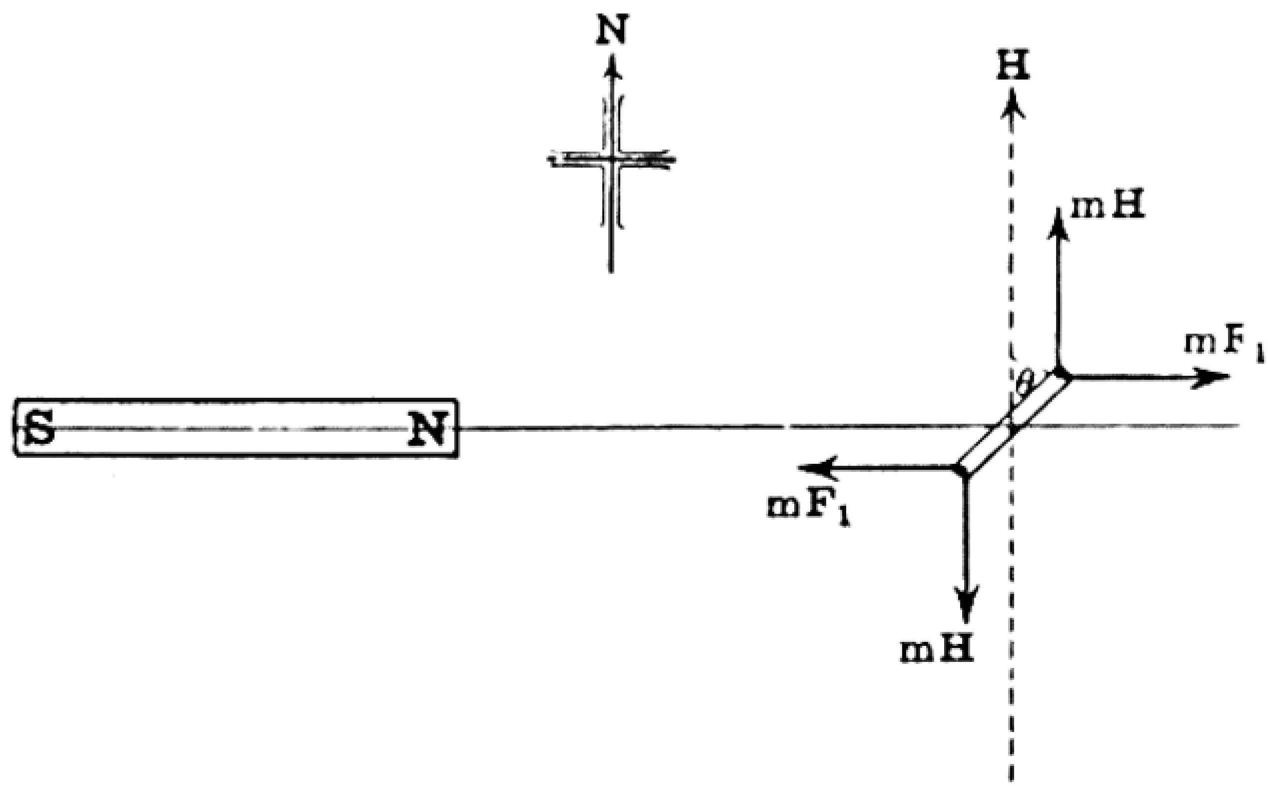


FIG. 196.—A Tangent Position of Gauss.

and the rest of the notation be as before. Since F_1 and H are at right angles, and each is uniform in the small space surrounding the needle,

$$F_1 = H \, . \tan \theta_1 \text{ (p. 319)}$$

But

$$F_1 = \frac{2Md}{(d^2 - l^2)^2}$$

$$\therefore \frac{2Md}{(d^2 - l^2)^2} = H \, . \tan \theta_1$$

or

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \, . \tan \theta_1 \quad . \quad . \quad . \quad . \quad (5)$$

If the approximate formula (2) is used this becomes

$$\frac{M}{H} = \frac{d^3}{2} \, . \tan \theta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Similarly, if needle and magnet are placed as in Fig. 197, F_2 and H are perpendicular; if the deflexion is θ_2

$$F_2 = H \cdot \tan \theta_2$$

$$\therefore \text{ from (3) } \frac{M}{(d^2 + l^2)^{\frac{3}{2}}} = H \cdot \tan \theta_2$$

and
$$\frac{M}{H} = (d^2 + l^2)^{\frac{3}{2}} \cdot \tan \theta_2 \quad . \quad . \quad (7)$$

or approximately from (4)
$$\frac{M}{H} = d^3 \cdot \tan \theta_2 \quad . \quad . \quad . \quad . \quad (8)$$

These two arrangements of magnet and needle are called respectively the A and B tangent positions of Gauss. It should be noted that in each case the deflecting magnet is at right angles to the earth's lines of force.

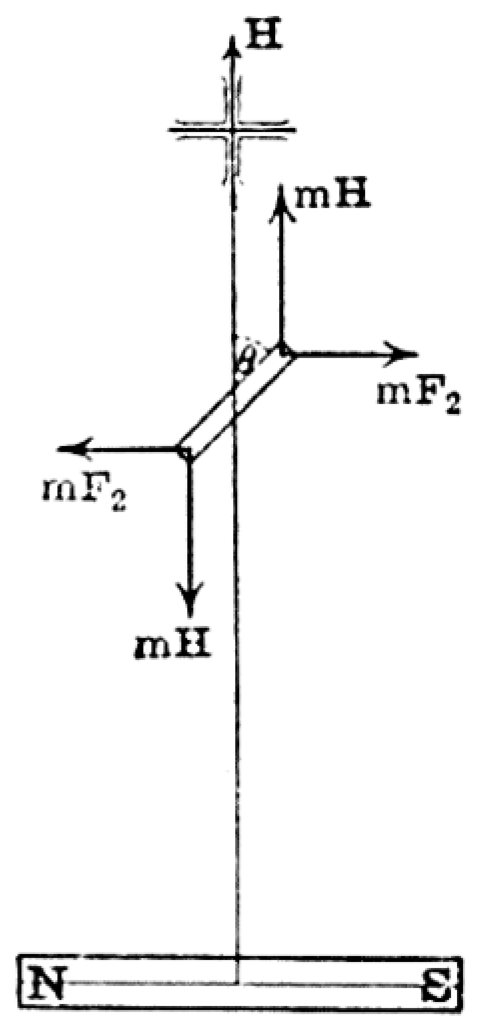


Fig. 197.—**B** Tangent Position of Gauss.

Magnetometer.—From the foregoing it is seen that there are two methods of comparing magnetic fields, (1) The oscillation method (p. 323), (2) The deflexion method of the last paragraph. In the latter it is arranged that one of the fields is perpendicular to that of the earth, then $F = H \cdot \tan \theta$, where θ is the deflexion of the compass needle. Thus the intensity of F varies as the tangent of the angle through which the needle is deflected. For many observations by this method a magnetometer is convenient. This instrument consists, in its simplest form, of a short compass needle supported at the centre of a graduated circle and carrying a long pointer at right angles. The compass box is itself fixed on a graduated bar (Fig. 198). In the more sensitive reflecting form the needle is about 5 mm. long, and it is stuck to the back of a concave or plane mirror which is suspended by a fine silk fibre; the deflexions are read by one of the methods described on pp. 145, 160.

Comparison of Magnetic Moments.—There are several ways in which magnetic moments can be compared; a method of determining a moment absolutely is given on p. 338.

(1) One method of comparison has already been given on p. 322. The following modification renders it unnecessary to know the ratio of the moments of inertia. Two pieces of glass tubing A, B, Fig. 199, are fastened together by wire and suspended by a single thread. The magnets are placed in the tubes and are allowed to oscillate torsionally, first with their axes in the same direction and secondly when they are opposed. Let T_1 and T_2 be the corresponding

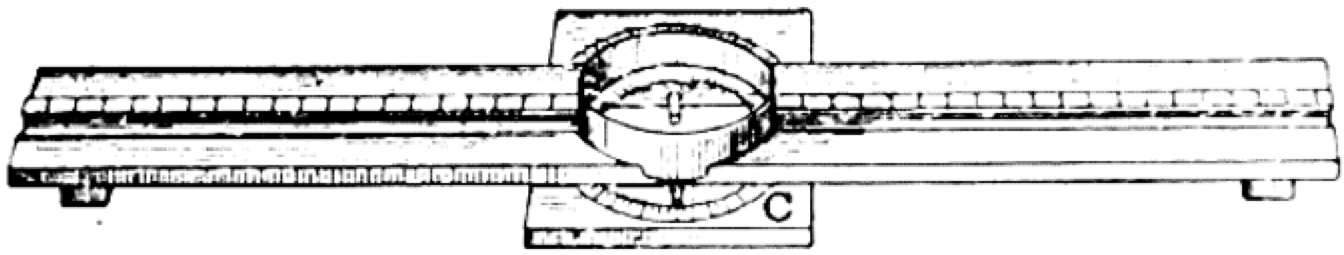


FIG. 198.—Simple Magnetometer.

periods, M_1 and M_2 the moments to be compared. In the first case the moment is $(M_1 + M_2)$ and in the second $(M_1 - M_2)$, while the moment of inertia remains constant, hence

$$T_1 = 2\pi\sqrt{\frac{K}{(M_1 + M_2)H}}$$

$$T_2 = 2\pi\sqrt{\frac{K}{(M_1 - M_2)H}}$$

Whence
$$\frac{T_1^2}{T_2^2} = \frac{M_1 - M_2}{M_1 + M_2}$$

and
$$\frac{M_1}{M_2} = \frac{T_1^2 + T_2^2}{T_1^2 - T_2^2} \quad \text{(see note, p. 189)}$$

That magnet which points N. in each case has the greater moment M_1.

(2) *Using the A tangent position of Gauss.*—The graduated arms of the simple magnetometer are arranged to point magnetic E. and W., when the pointer should stand at zero on the circular scale. One magnet is placed on the bar with its length perpendicular to the meridian and the reading at each end of the pointer is taken. The magnet is next reversed end for end, keeping its centre at the same distance from the needle, and the readings repeated. It is then moved to the corresponding position on the other side of the compass box and four more readings are made. The mean of the eight observations is taken to be the deflexion θ_1 in Equation (5) or (6). The deflexion θ_2 due to the second magnet, when placed with its centre the same distance from the needle, is found in a similar way. Then using (6) $M_1/M_2 = \tan\theta_1/\tan\theta_2$; for greater accuracy Equation (5) may be used.

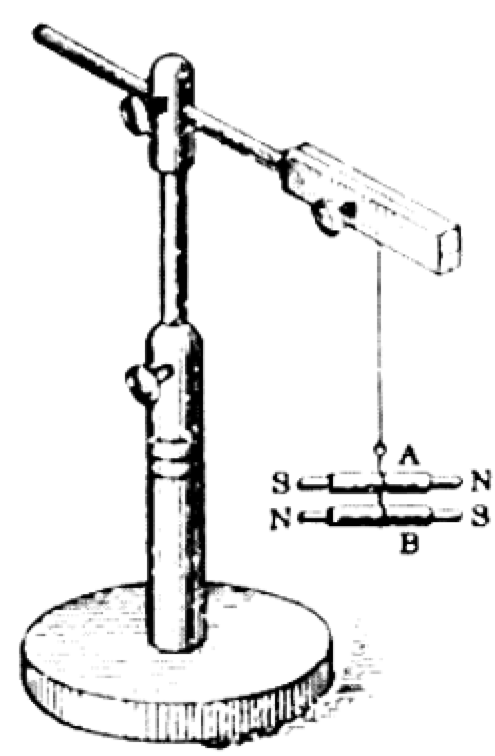


FIG. 199.

As the above is a typical experiment in magnetic measurements

we will give briefly the reasons for taking the observations in the manner described. Suppose that the centre of the circle is at O (Fig. 200), and that through a slight error in construction the point of support of the needle is at O′. It is clear that the reading at one end of the needle will be too small, while that at the other is too large by an equal amount, hence the mean is free from error. If the poles are not quite symmetrical about the centre of the deflecting magnet the deflexion is too large in one position and too small when the bar is reversed; the mean gives a more correct result.

(3) *The B tangent position.* The arms of the magnetometer are pointed N. and S., the magnet is placed across them in the B position (perpendicular to the meridian note), and four pairs of readings taken as before for each magnet. Then from (8) $M_1/M_2 = \tan\theta_1/\tan\theta_2$. Dissymmetry of the poles has less influence in this case.

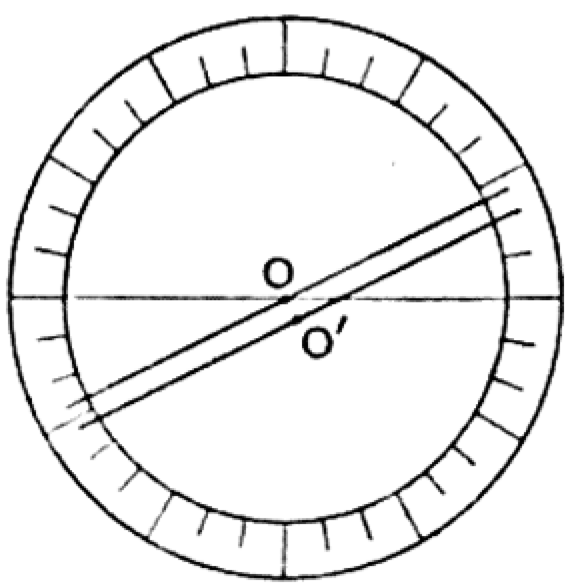


FIG. 200.

(4) *Null method.*—Instead of keeping d constant in the expression $M/H = \frac{d^3}{2}\tan\theta$, we may arrange to have θ the same for each magnet. Then $M_1/M_2 = d_1^3/d_2^3$. One magnet is placed E. of the needle, the other on the W., with their poles directed to produce deflexions in opposite directions. One is moved until the deflexion is zero, and d_1 and d_2, the distances from the centres of the magnets to the needle, are measured. The magnets are reversed end for end, keeping d_1 constant, and a new d_2 is found; the mean value of d_2 is taken as the correct one. The B tangent position can also be used.

The Sine Method.—Moments may also be compared by the sine method. Place a bar magnet on the magnetometer in the A position and turn the whole instrument round until the pointer is again at zero. Magnet and needle are again at right angles, and each is inclined at an angle θ_1 to its original direction (Fig. 201). If m' is the strength of the poles of the needle, l' the distance between them, the moment of the couple due to the bar magnet is $m'F_1l'$, and that arising from the earth's field is $m'H \,.\, l'\sin\theta_1$. These must be equal,

$$\therefore\ H\sin\theta_1 = F_1 = \frac{2M}{d^3}\ \text{(from (2))}$$

or

$$\frac{M}{H} = \frac{d^3}{2}\,.\,\sin\theta_1$$

The angle θ_1 may be found from an additional scale C, as in Fig. 198, or by merely removing the bar magnet the needle swings through θ_1 back to the meridian, the rotation may then be read on the ordinary graduated circle. If θ_2 is the rotation for another magnet at the same distance, $M_1/M_2 = \sin\theta_1/\sin\theta_2$. We may also start from the B position before swinging the magnetometer round; in either case the magnets should be reversed as before.

Gauss's Method of proving the Inverse Square Law.—For this experiment it is best to use a reflecting magnetometer, the deflecting

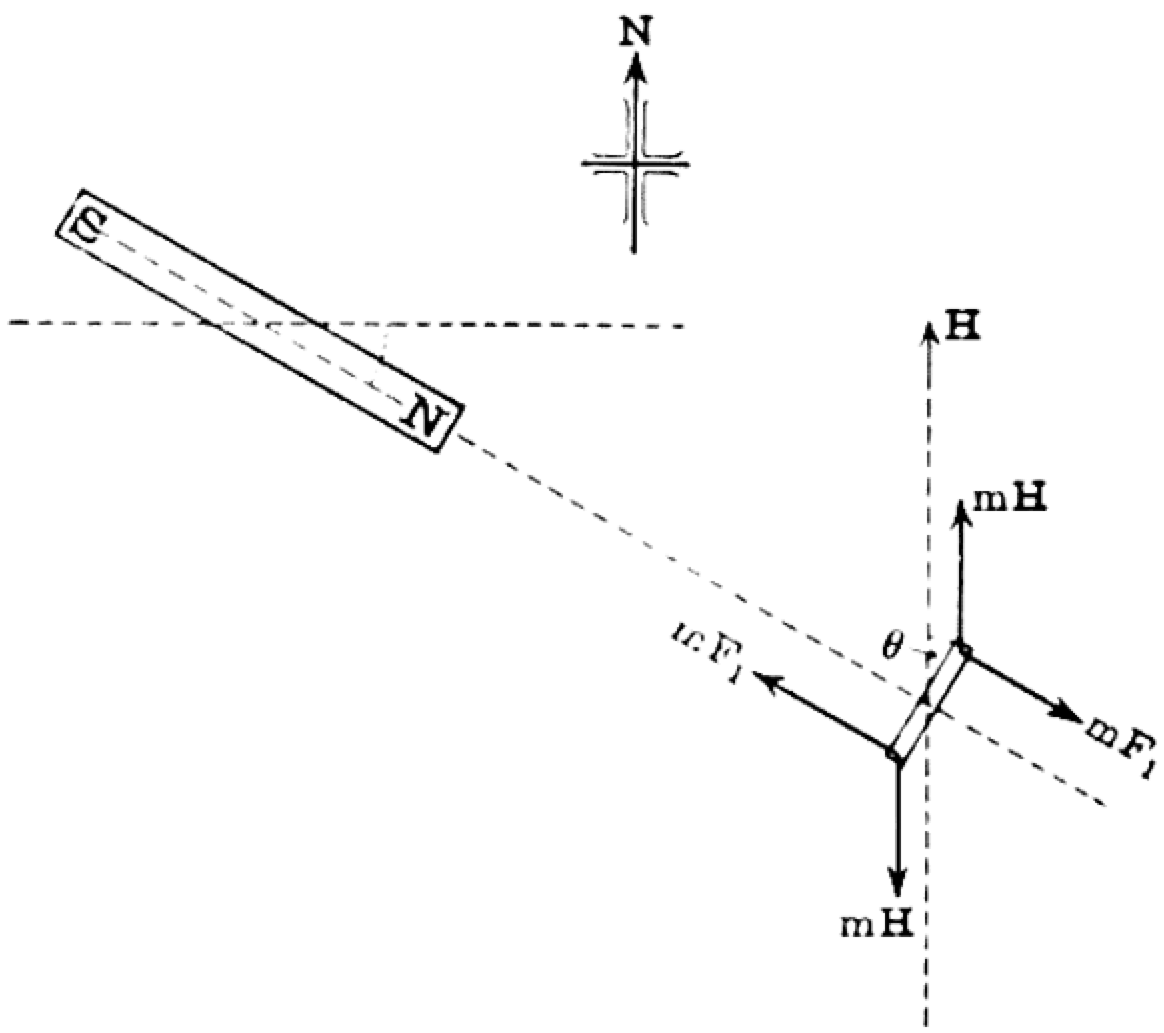


FIG. 201.—Sine Position of Magnet and Needle.

magnet may then be a thick knitting needle about 3 cms. long, and formulæ (2) and (4) can be safely used. It is required to prove that $F_1 = 2F_2$ (p. 326).

EXPERIMENT.—The magnet is placed in the A tangent position, with its centre at a distance d from the needle, and the deflexions are noted as in the second method of the last paragraph, except of course that each end of the needle cannot be read with the reflector. The corresponding deflexion is found with the bar in the B tangent position at the same distance d. Prove $\tan\theta_1/\tan\theta_2 = 2$.

EXPERIMENT.—In the A tangent position $F_1 = 2M/d^3$, from which we have deduced Equation (6). The latter equation and therefore (2) can be verified by noting the deflexions for different values of d, for since M/H is constant the product $d^3 . \tan\theta$ should also be constant. Use a reflecting magnetometer for this experiment, else the more complicated equation (5) must be employed.

Additional Experiments.—The following experiments are instructive as applications of the principles explained in this chapter. A note may be made here. We have seen that two methods can conveniently be used to compare magnetic fields, one making use of oscillations, the other of a magnetometer. In each case the earth's field has to be taken into account, but it should be observed that in the second method the fields F_1, F_2, which we seek to compare, are made perpendicular to that of the earth, while in the first method they are made parallel.

EXPERIMENT.—*Distribution of magnetism along a bar.* Observe the time of oscillation of a Searle needle in the earth's field H, let this be T. Now place due magnetic S. of it, at a distance of 3 cms., a long vertical bar magnet, with its positive end at the same height as the needle. The horizontal field at the needle is now $F_1 + H$, where F_1 arises chiefly from the magnetism on the bar at the point adjacent to the needle, and may be taken as proportional to the amount of this magnetism. Let the new period be T_1. Raise the bar magnet 2 cms. and observe the period T_2 due to the field $F_2 + H$. Then as on p. 324

$$\frac{F_1}{F_2} = \frac{\frac{1}{T_1^2} - \frac{1}{T^2}}{\frac{1}{T_2^2} - \frac{1}{T^2}}$$

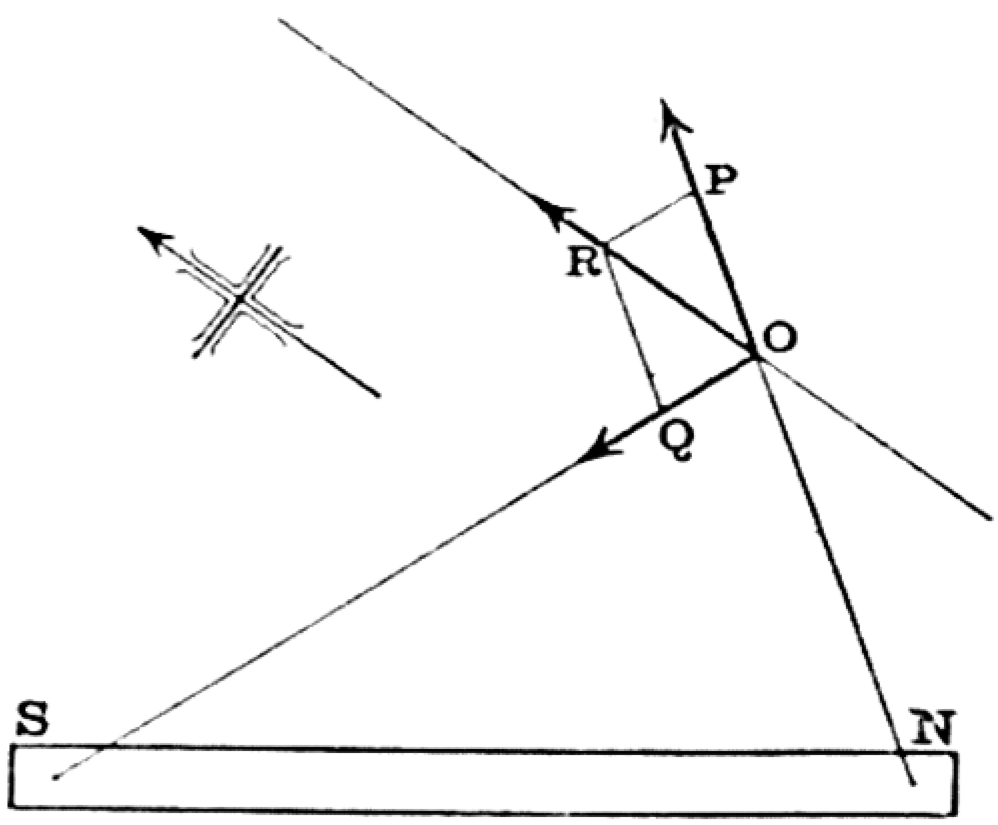


FIG. 202.—Graphical Method of proving the Inverse Square Law.

In this way the amounts of magnetism at different points of the bar may be compared. Show the results in the form of a curve. It has been assumed that the moment of the needle is constant, this may not be true since the fields may differ greatly and the magnetisation may be altered by influence; the method is not very exact on this account.

EXPERIMENT.—*Graphical method of proving inverse square law.* Find the position of the poles N. and S. of a bar magnet and draw the lines ON, OS, where O is any point on the paper (Fig. 202). The fields at O due to the poles are m/ON^2 and m/OS^2. On any suitable scale mark off on NO produced and on OS lengths OP, OQ, proportional to these fields; complete the parallelogram OPRQ. The resultant field is represented by OR, hence a short needle if placed at O should set along OR if the inverse law is true. To eliminate the effect of the earth's field the paper must be turned round until OR is in the magnetic meridian.

EXPERIMENT.—*Neutral points of a field and pole strength of a magnet.* Plot the lines of force due to a bar magnet which is placed on a sheet of paper with its axis pointing N. The result on one side of the magnet is shown in Fig. 203. At the point A the compass sets indifferently in any direction; it is a neutral point, where the earth's field is exactly equal and opposite to that of the magnet. Find the poles and draw the parallelogram of forces as in the last experiment. Since the field at A is zero AED is the triangle of forces, and DA represents the earth's field H. Taking this as 0·18 Gauss (p. 338), the forces represented by AE and AC can be found by measurement; but these forces are m/AS^2

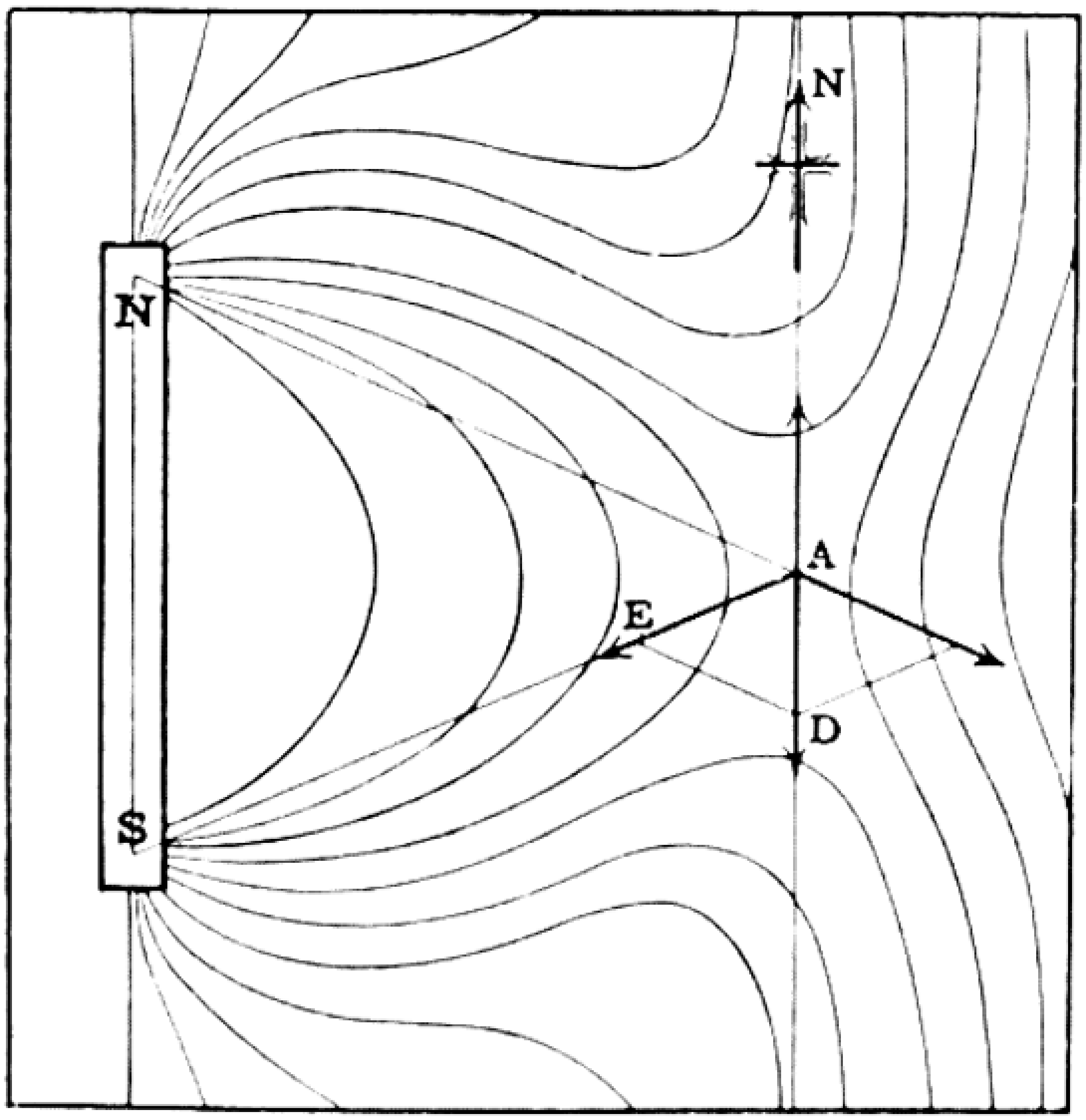


FIG. 203.—Showing a Neutral Point.

and m/AN^2 respectively. Hence if AS and AN are measured the strength of each pole can be found. Other positions of the bar magnet should be used.

Astatic Needle.—An astatic needle consists of two magnets of equal moment, with their axes parallel but pointing in opposite directions (see Fig. 199). A system of this type will experience no couple when it is placed in a uniform field. In practice it is found impossible to fulfil the above conditions perfectly, but the directive action of the field can be greatly reduced with care. If the axes are parallel but the moments M and M′ are unequal, the needle will act like a magnet of moment (M — M′). Let us find how such a system will set in the earth's field if the moments are unequal and the axes not quite parallel. Let M and M′ be the moments, θ and θ' the

corresponding angles between H and the positive direction of the axes when the needle is at rest. The restoring couples must be equal,

$$\therefore \; M'H \sin\theta' = MH \sin\theta$$
$$\therefore \; \sin\theta'/\sin\theta = M/M'$$

If the moments are equal $\theta = \theta'$, and the magnets will point approximately E. and W.

EXAMPLES ON CHAPTER XXIX

To prove the formulæ on p. 326, we have

$$F_1 = m\left\{\frac{1}{(d-l)^n} - \frac{1}{(d+l)^n}\right\}$$

Dividing and multiplying the denominators by d^n

$$F_1 = \frac{m}{d^n}\left\{\frac{1}{\left(1-\frac{l}{d}\right)^n} - \frac{1}{\left(1+\frac{l}{d}\right)^n}\right\}$$

The expression in brackets can be developed by the binomial theorem, neglecting squares and higher power of l/d we obtain

$$F_1 = \frac{m}{d^n}\left(1 + n\,.\,\frac{l}{d} - 1 + n\frac{l}{d}\right)$$
$$= \frac{2lmn}{d^{n+1}}$$
$$= \frac{nM}{d^{n+1}}$$

Similarly for F_2 we have

$$F_2 = \frac{2m}{NP^n}\,.\cos ONP$$

$$\text{also } NP = (d^2 + l^2)^{\frac{1}{2}}, \quad \cos ONP = \frac{l}{(d^2 + l^2)^{\frac{1}{2}}}$$

$$\therefore \; F_2 = \frac{2ml}{(d^2 + l^2)^{\frac{1}{2}}} \cdot \frac{1}{(d^2 + l^2)^{\frac{n}{2}}}$$
$$= \frac{M}{(d^2 + l^2)^{\frac{n+1}{2}}}$$

or, if l^2 can be neglected,

$$F_2 = \frac{M}{d^{n+1}}$$

1. A horizontally suspended magnet vibrates 12 times per minute at a place where the horizontal intensity of the earth's magnetic field is 0·18. How many times per minute will it vibrate at a place where the horizontal intensity is 0·245 ? (L. '93.)

2. A small suspended magnet makes 10 oscillations per minute under the influence of the earth's field alone. A bar magnet is brought near it so as not to disturb the direction of the pointing of the suspended magnet, but so that the latter now makes 14 oscillations per minute. What would be the frequency if the bar magnet were now reversed pole for pole ? (L. 1900.)

3. Explain how the law of the mutual attraction of magnetic poles at different distances may be investigated by means of the turning moment one produces on the other. (L. '04.)

4. A short magnet 50 cms. to the west of a compass needle deflects it through 45°. Find approximately the magnetic moment of the magnet, the value of the earth's horizontal field being 0·18 C.G.S. units. (L. '05.)

CHAPTER XXX

THE EARTH'S MAGNETIC FIELD

The Earth's Magnetic Elements.—It has already been noticed that a bar magnet if suspended by a fibre, after oscillating for some time, comes to rest in a definite direction, viz. North and South. We might conclude from this that the earth's field is horizontal; a simple experiment will, however, show that this is not so.

EXPERIMENT.—Support an unmagnetised steel bar in the manner indicated in Fig. 204, so that it can turn round both horizontal and vertical axes. Arrange that it sets horizontally, then magnetise it. It will now be found to set in a N. and S. direction but it will, in general, be inclined to the horizontal. In the northern hemisphere it is the N. pole which dips, in the southern hemisphere the S. pole.

It is evident from this that the resultant force is inclined to the horizontal, and is moreover parallel to a given vertical plane directed nearly N. and S. in England. The vertical plane containing the magnetic axis of the magnet when it is at rest is called the **magnetic meridian.** The angle between the geographical[1] and magnetic meridians is called the **angle of declination**; it is about 16° W. in London, *i.e.* a compass points about 16° west of north. The angle which the magnetic axis of the freely suspended needle makes with the horizontal is called the **angle of dip, or the inclination**; it is about 67° in London. The complete determination of the magnetic intensity at any point thus resolves itself into finding the three magnetic elements:—(1) Total intensity; (2) Declination; (3) Inclination. If I is the total intensity, which is parallel to the magnetic axis of the needle in Fig. 204, we can resolve it into its horizontal and vertical components H and V and $I^2 = V^2 + H^2$.

[1] The geographical meridian at any point is the vertical plane which is directed to the geographical north and south poles. Its direction must be determined by astronomical means.

Let OI (Fig. 204) represent the total intensity in magnitude and direction at the point O; the components are represented by OH and OV. If the angle of dip IOH be represented by ϕ, then

$$\frac{V}{H}=\frac{OV}{OH}=\frac{HI}{OH}=\tan\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also
$$\frac{H}{I}=\frac{OH}{OI}=\cos\phi$$

or
$$H = I\cos\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and similarly
$$V = I\sin\phi$$

Evidently OH and OI are in the same vertical plane, viz. the magnetic meridian. We see that if at any point the direction and magnitude of the horizontal component are known, and also the angle of dip, the field is completely determined; for V can be calculated from (1) and I from (2). An account of the determination of these quantities will now be given.

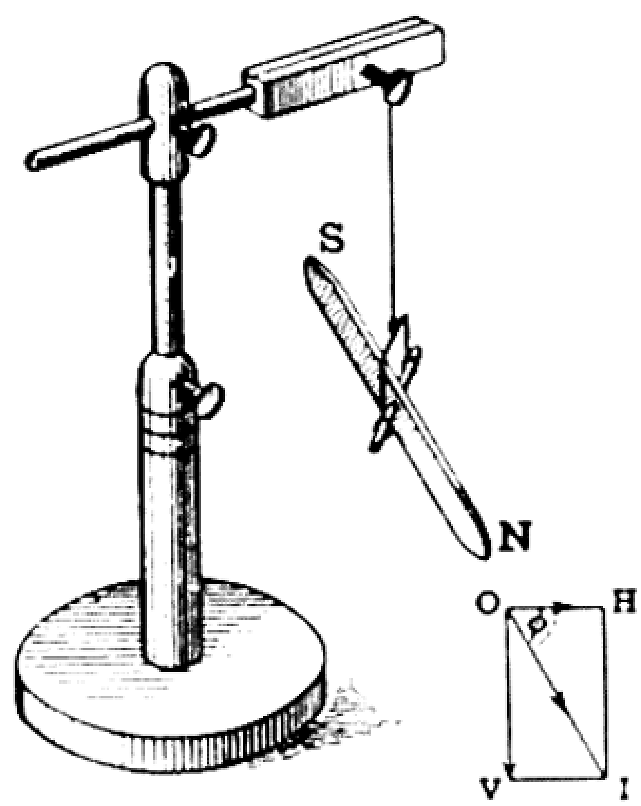


FIG. 204.—Freely-suspended Magnet.

Measurement of the Horizontal Component.—This is important, as the value of H is frequently required in calculations and measurements in magnetism and electricity. A direct measurement of V or I would be difficult, as it is awkward to work with vertical or inclined needles. Two experiments are necessary: (*a*) The magnetometer experiment; (*b*) The oscillation experiment. If M is the moment of a bar magnet, it has already been shown that the value of M/H can be found from observations with a magnetometer by either the tangent or sine methods. Taking the A tangent position we have

$$\frac{M}{H}=\frac{d^3}{2}.\tan\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where θ is the deflexion of the magnetometer needle. Both d and θ can be observed, but as M is unknown another equation connecting this quantity and H must be found, then from the two either M or H can be calculated The bar magnet used in the magnetometer

experiment is next suspended in a horizontal position by a fibre, and caused to oscillate under the influence of the earth's horizontal field. The time of vibration T is observed, then

$$T = 2\pi\sqrt{\frac{K}{MH}} \quad (p. 322)$$

or

$$MH = \frac{4\pi^2 K}{T^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Multiplying the Equations (1) and (2) together we get M^2, and dividing the second by the first we find H^2. The same observations thus give us either the moment of the magnet or the earth's horizontal field. The deflexion experiment is made in the manner described on p. 329, with the simple form of Kew magnetometer shown in Fig. 198. The oscillations are observed in the box shown in Fig. 205. It consists of a wooden box provided with movable windows back and front and surmounted by a vertical tube T. The magnet is suspended from the rod A and hangs in the box, which shields it from draughts. In order to free the suspension fibre from twist a bar of brass is first suspended in place of the magnet, and the rod A is turned until the bar comes to rest parallel with the sides of the box. The magnet is now replaced, care being taken that the fibre does not twist while the interchange is being made. The box is then turned until the magnet is parallel to the sides, when the fibre will be without twist and the magnet will lie in the magnetic meridian. Observations of the time of oscillation can now be made. The moment of inertia of the bar, K in Equation (2), is calculated from the appropriate formula on p. 322. The value of the earth's horizontal component in London is 0·18 Gauss approximately.

Measurement of the Angle of Dip.—A simplified form of the Kew dip circle is shown in Fig. 206. The axle of the needle rotates upon two horizontal knife-edges and is brought to rest upon them at the centre of the scale by two sliding pieces with V-shaped ends. The whole is contained in a case with glass windows back and front to shield the needle from draughts. The case can be turned round a vertical axis, its azimuth being given by a vernier moving over a horizontal circular scale on the base. After the instrument has been levelled the case is rotated until the needle sets vertically; in this position the horizontal component has no effect, or the needle would be inclined somewhere between the horizontal and the vertical.

Hence the axle of the needle must be in the meridian, for when this is so the horizontal field has no turning moment in the plane of rotation. If the case is now turned through 90° the needle is in the magnetic meridian, and the angle of dip may be read off the vertical circular scale. The result obtained may be affected by certain errors arising from the following circumstances: (*a*) The axis about which the needle rotates may not pass through the centre of the

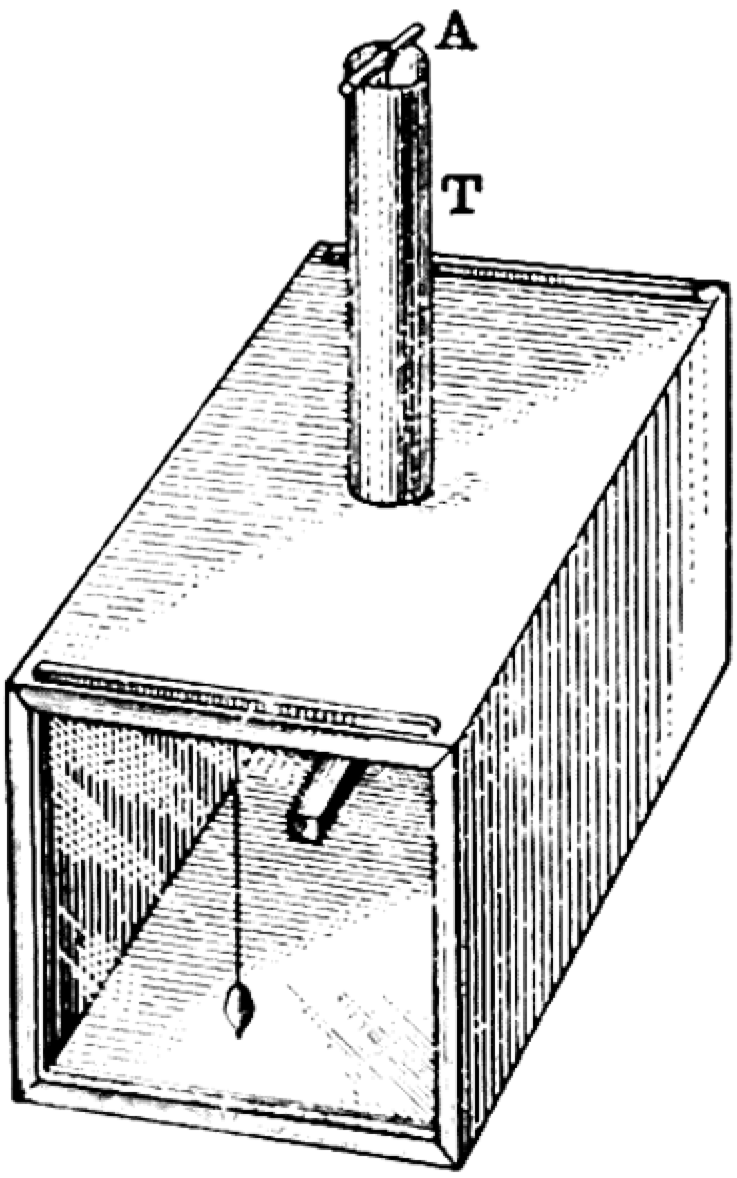


Fig. 205.—Vibration Box.

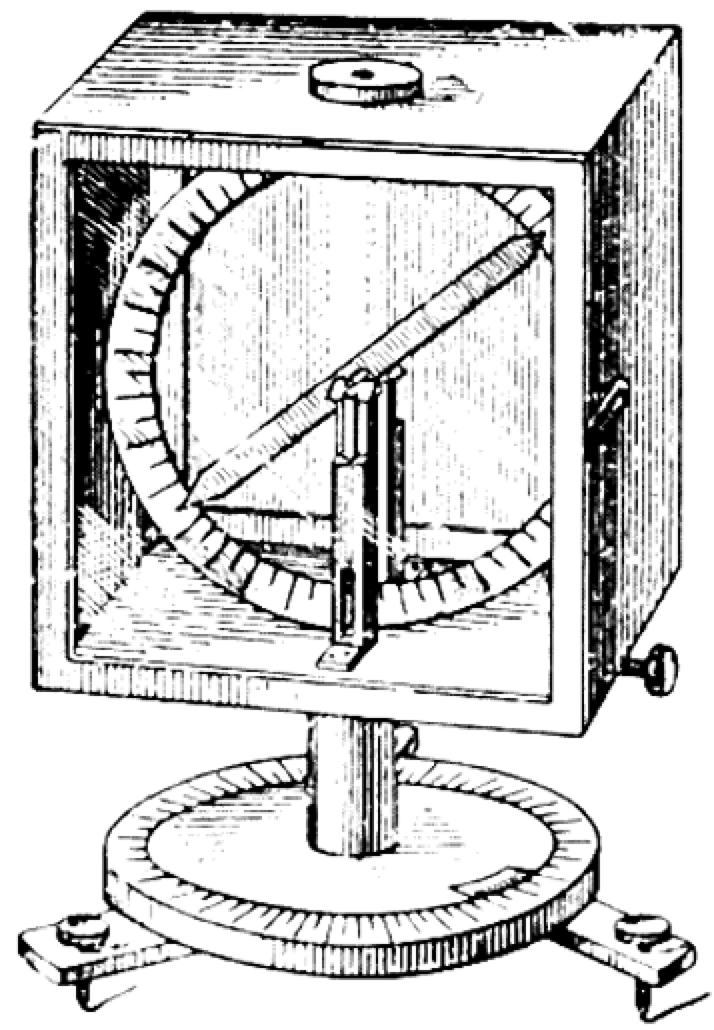


Fig. 206.—Dip Circle.

vertical scale; (*b*) The magnetic axis of the needle may not coincide with its geometric axis; (*c*) The zero line of the vertical circle may not be exactly horizontal; (*d*) The centre of gravity of the needle may not be on the axis about which it revolves. We proceed to show how, by taking the observations in the proper manner, these errors may be eliminated.

From p. 330 it is clear that (*a*) is got rid of by reading both ends of the needle and taking the mean. If the mean be taken at the end of the adjustments we have at present two observations. To eliminate (*b*) the needle is removed from its bearings and reversed back for front, as the magnetic axis always sets in the same direction, Fig. 207 shows for a similar instance that the mean result will be

free from this error. We have now four readings. The case is next turned round the vertical axis through 180°; if in the first instance the zero line of the vertical circle was too high on the right it will now be too high on the left; the four previous observations are repeated and error (*c*) is eliminated. If in all these experiments the centre of gravity of the needle is below the point of support, the weight will pull the lower end of the needle down, and the dip obtained will be too large. To eliminate the resulting error the needle is remagnetised in the opposite direction, when its other end dips, and the eight readings above are repeated. The mean of the sixteen observations gives a result free from instrumental defects.

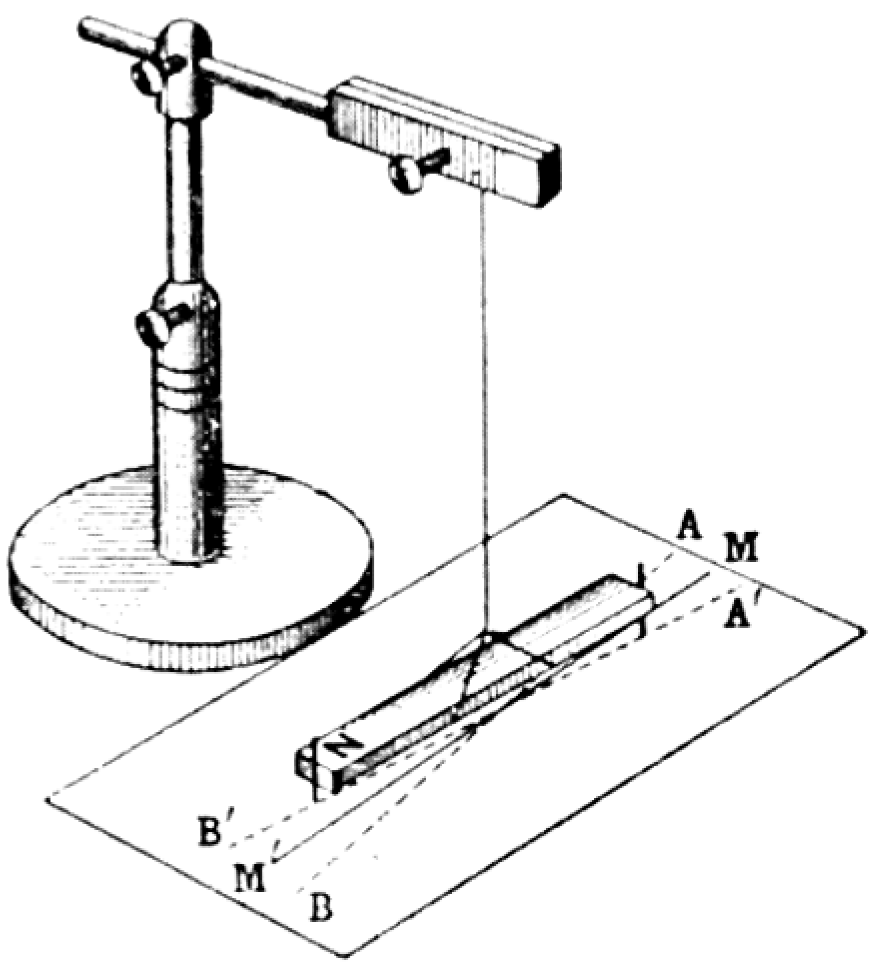


FIG. 207.—Method of finding the Magnetic Meridian.

The Angle of Declination.—Let a bar magnet be suspended horizontally by a torsionless fibre above a circular scale. It comes to rest with its magnetic axis in the magnetic meridian. If now the direction of the geographical meridian is ascertained with reference to the same scale, the angle between the two, *i.e.* the declination, can be read off at once. As astronomical observations are necessary to find the geographical meridian we will describe merely a method of finding the magnetic meridian.

EXPERIMENT.—*To determine the magnetic axis of a magnet and also the direction of the magnetic meridian.*—Fasten to the ends of a bar magnet two straight pieces of wire (non-magnetic) as shown in Fig. 207. Place it in a copper wire stirrup and suspend it by a long piece of silk fibre so that the ends of the wires just clear the bench. When it has come to rest mark the positions A, B, of the wires on the bench and draw the line AB. If the pins were on the magnetic axis this line would represent the direction of the magnetic meridian. If this condition is not fulfilled, and so far we have given no method of testing, we must proceed as in the elimination of error (*b*) above. The magnet is removed from the stirrup, its lower face turned uppermost, and the observation repeated. This time the wires are at A′, B′. The line MM′ which bisects the angle between AB and A′B′ is in the plane of the magnetic meridian; if the magnet is lowered on to the bench after removing the wires this line also gives

the direction of the magnetic axis. For the second position is obtained from the first by rotating the bar about its magnetic axis, which points in a fixed direction, hence the line joining the pins must be equally inclined to this line in the two positions.

Magnetic Maps.—The variation of the magnetic elements over the earth's surface can be shown most conveniently by plotting lines of equal horizontal intensity, of equal dip, or of equal declination, upon geographical maps. Lines of equal dip are called **isoclinic lines,** those which join places of equal declination are **isogonic lines.** Observations show that the distribution of field is roughly such as would be obtained if the earth contained a strong magnet whose length is shorter than the diameter. At certain points the total field is perpendicular to the earth's surface, a dipping needle sets vertically while a compass points indifferently in any azimuth. These are called the magnetic poles, the N. pole lies in lat. 70° N. and long. 97° W. As the direction of dip is reversed in going from the northern to the southern hemisphere there is a line of zero dip, called the **magnetic equator.** It lies near the geographical equator. The sets of lines are not distributed evenly over the earth's surface—this is no doubt partly due to the presence of magnetic ores—but generally speaking the isoclinic lines and those of equal horizontal intensity run parallel to the magnetic equator like lines of latitude. There are two lines along which the declination is zero; these are called **agonic lines.** If the field were due entirely to an internal magnet there would be one agonic line, viz. the great circle joining the magnetic and geographical poles. One of the agonic lines runs roughly in this direction, the other is situated in Siberia and is oval in shape.

Variations in the Magnetic Elements.—The magnetic elements have now been observed for a large number of years, and the records show that they are undergoing slow variations extending over centuries. These are called **secular changes.** For instance, in the year 1600 the declination in London was about 10° E., from that time until 1800 it changed gradually to about 25° W., since then it has been moving back to zero and is, as has been stated, about 16° W. at the present time. Corresponding changes have taken place in the positions of the magnetic poles. If these changes are periodic they will take nearly 1000 years to perform one cycle. Observations with continuously self-recording instruments show that there are, in addition, small annual and daily variations which

are periodic in character. Thus the declination is a maximum in the afternoon, it then decreases and reaches a minimum value about eight o'clock the following morning, rising again as the day advances. Occasionally these more or less regular changes are disturbed by very irregular variations which are called magnetic storms. Their occurrence seems to be closely connected with other well-known phenomena, such as the appearance of the Aurora Borealis or of sun spots. The Aurora is undoubtedly of electro-magnetic origin. This would give us the idea that part, at any rate, of the earth's field is due to external causes.

EXAMPLES ON CHAPTER XXX

1. The moment of a magnet is 1000 C.G.S. units. How much work is done in turning it through 90° from the magnetic meridian in a horizontal plane at a place where the horizontal intensity is 0·16 C.G.S. unit ? (L. '02.)

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2. A bar magnet whose moment is 9860 C.G.S. units is turned in a horizontal plane through 60° from the meridian. Find the work done and the couple required to maintain it in that position. $H = 0{\cdot}2$ dyne per unit pole. (L. '10.)

3. A dip needle oscillates in the meridian at the rate of 35 oscillations/min. in a locality where the dip is 60°. In another locality, where the dip is 45°, it is found that the needle makes 40 oscillations/min. Find (*a*) the ratio of the earth's total intensities, and (*b*) the ratio of the horizontal components of the earth's magnetic field at the two places. (L. '08.)

CHAPTER XXXI

ELECTROSTATICS

Preliminary Ideas.—It has been known for 2000 years that amber, when rubbed with a suitable rubber, acquires the power of attracting light bodies. Gilbert, in the sixteenth century, discovered other substances which behave in a similar manner. A body which has become possessed of this property is said to be electrified or charged with electricity. An account is given in the following pages of the methods of electrifying bodies, the results produced by electricity, and the methods of measurement. That branch of the subject which deals with electricity at rest is called electrostatics.

EXPERIMENT.—Rub an ebonite rod with a woollen or fur rubber, it can then alternately attract and repel bits of paper or cork brought near it; it is electrified or charged with electricity.

EXPERIMENT.—Suspend horizontally, by a single thread, a light ebonite rod that has been electrified by friction with wool, and bring near it another ebonite rod that has been charged in a similar manner; repulsion takes place. Replace the ebonite rods by rods of glass rubbed with silk, repulsion again occurs; but if a glass rod, charged by rubbing with silk, is brought near a suspended ebonite rod that has been electrified with flannel the two are seen to attract each other.

We infer, exactly as in the parallel case of magnets, that there are two kinds of electricity, and that like charges repel and unlike charges attract each other. The electricity produced on glass when it is rubbed with silk is called positive electricity, that produced on ebonite when rubbed with flannel is called negative electricity. The sign of the charge produced on any body depends on the state of its surface, the temperature, the nature of the rubber, etc. Thus a ground glass rod is negatively charged by rubbing with flannel; a smooth glass rod, if made very hot, is negatively electrified by friction with silk.

It was thought for some years that the power to become electrified

was possessed only by certain non-metallic bodies; it is now known that all solids can be electrified. If a brass tube is held in the hand while it is rubbed it is impossible to electrify it, but if it is held by an ebonite handle thrust into the tube it becomes negatively charged when rubbed with flannel. This charge at once disappears when the brass is touched with the finger, showing clearly why it was not electrified in the first instance; as fast as the electricity was produced it escaped through the hand of the experimenter. Substances like brass, which allow electricity to travel along them, are called **conductors**; those like ebonite, which do not allow this movement, are called **insulators or dielectrics.** There is, however, no definite boundary between the two classes, all substances conduct in a greater or less degree, and no substance is known which does not offer some resistance to the movement of electricity through it. The best insulators are quartz, amber, sulphur, paraffin, ebonite, dry gases, and certain organic liquids; the metals, particularly silver and copper, are the best conductors. If a conductor is required to retain its charge it must be supported by an insulator, it is then said to be insulated.

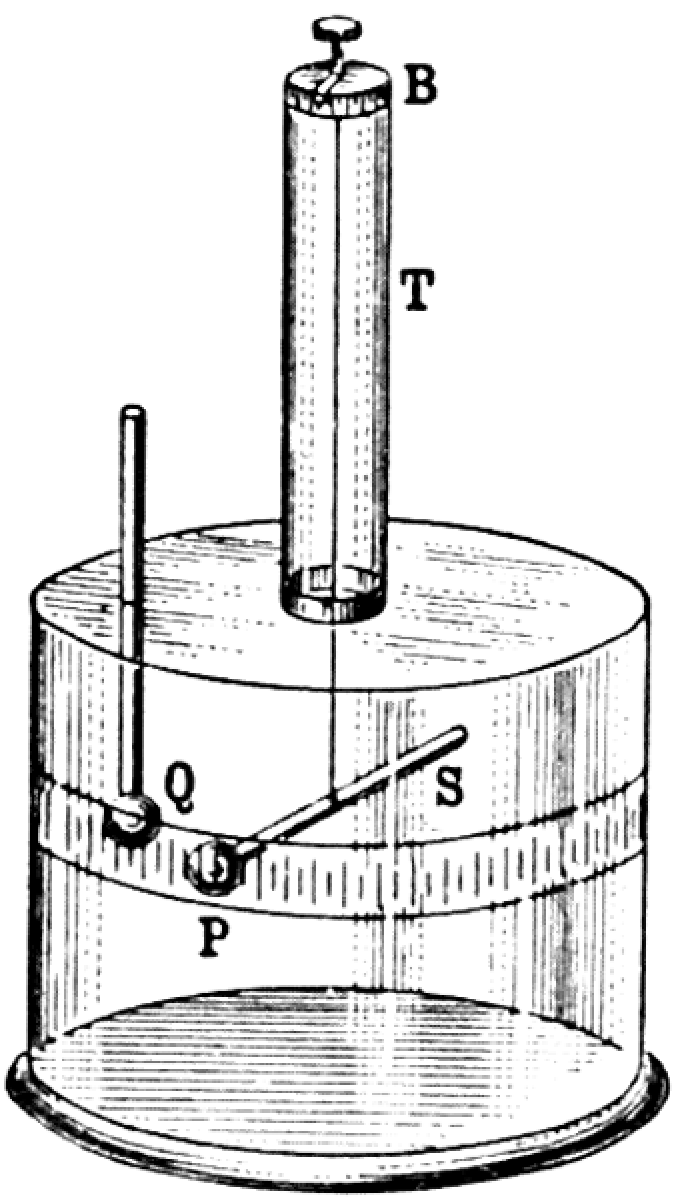


FIG. 208.—Torsion Balance.

Inverse Square Law.—It is found, as in magnetism, that two charges Q and Q′, if placed at a distance R apart, repel each other with a force which varies as $1/R^2$. Priestley first established the law in 1767, but the first direct proof, nearly twenty years later, is due to Coulomb, who used a torsion balance (Fig. 208). A thin wire T, fixed in a torsion head B, carries at its lower end a light insulating rod S. At the extremity of this rod there is a small gilt pith-ball P; another small conductor Q is supported on an insulating rod which passes through the top of the case. It is arranged that there is no twist on the wire when P and Q are in contact and one end of S points to the zero of a circular scale on the glass case. Q is charged with electricity and placed in position; it shares its charge

with P and the two conductors repel each other. The rod S is now acted upon by two couples, one arising from the force of repulsion F between the two charges, the other due to the twist on the wire; the latter couple is proportional to the torsion. An example will best show how the law is proved. In a certain case, Coulomb found that P was repelled through 36°, and in order to reduce this to 18° it was found necessary to turn the torsion head B clockwise through 126°. The distance between the charges was now approximately halved, and, since the wire was turned through 126° at the top and 18° in the opposite direction at bottom, the total torsion was 144°, *i.e.* $36° \times 4$. To reduce the distance by half the torsion had therefore to be increased fourfold, or the ratio of the forces at distances R and 2R are 4 : 1, hence $F \propto 1/R^2$. If Q is touched by another conducting sphere of the same size half the charge will be transferred to it; this may be removed, when it is found that the force exerted by the charge on Q is halved, showing that the force is proportional to the charge. Hence the force between two charges Q and Q′ separated by a distance R varies as QQ'/R^2, or

$$F = \frac{1}{K} \cdot \frac{QQ'}{R^2}$$

Experiment shows that the force between two charges varies with the medium by which they are surrounded; the constant K depends on the medium. Owing to the gradual leaking away of the charges along the insulating supports, and to other reasons, the method is not capable of giving more than a rough indication of the law. Another proof is given on p. 355.

Unit Charge and Intensity of an Electric Field.—The equation just given is used exactly as the corresponding one in magnetism to define the unit quantity. Since, as we shall see shortly, the presence of a charge on one sphere influences the distribution of the electricity on a neighbouring sphere, we must suppose in our definition that the charges are concentrated at points. The constant K is arbitrarily made unity for air. If now there are two equal charges Q, separated in air by 1 cm., and they are found to repel each other with a force of 1 dyne, then each charge is unit charge; for F and R are each unity, hence $Q^2 = 1$. We have, therefore, the following definition: **The electrostatic unit of electricity is that quantity which, if concentrated at a point, repels an equal and similar quantity concentrated at**

another point 1 cm. away with a force of 1 dyne, the medium between them being air.

When one charge is placed in the neighbourhood of another it is usually acted upon by a force of attraction or repulsion; the space throughout which this force can be detected is called the electric field of the second charge. The intensity of the field at a given point is measured by the force in dynes which acts on a unit positive charge if placed at that point, the presence of this test charge being supposed to leave the original field undisturbed. The direction of the field is that along which the unit positive charge tends to move. The phrase " intensity of the electric field " is frequently abbreviated to " the electric field." To find the field due to a charge Q at a distance R away, it is supposed that a unit positive charge is placed at the point in question, then the repulsion is $\frac{Q \times 1}{R^2}$, or the field $F = Q/R^2$.

Electrical Lines of Force.—Just as in magnetism a curve may be drawn so that the tangent at any point is parallel to the electric field at that point; this constitutes an electrical line of force. As the lines show the direction in which a positive charge tends to move, they must start at a positive and end at a negative charge; an arrow is placed on them to show this direction. The convention adopted as to the number to be drawn to represent the intensity of the field differs from that followed in magnetism. The rule for drawing the magnetic lines of force was equivalent to supposing that each unit pole gave rise to 4π lines (p. 315), in the present case it is supposed that each positive unit of electricity gives rise to one line. Imagine a charge Q at the centre of a sphere of radius R; the intensity of the field at the surface of the sphere is $F = Q/R^2$, and the total number of lines is Q, distributed over an area $4\pi R^2$. The number, N, crossing each cm.^{2} of the surface is $\frac{1}{4\pi} \cdot \frac{Q}{R^2} = F/4\pi$. Hence $F = 4\pi N$, *i.e.* the intensity of the electric field is 4π times the density of the lines (cp. this with p. 315).

It is supposed, as in the magnetic case, that the lines are in a state of tension, and that they exert a sideways repulsion on each other, thus accounting for the attraction of dissimilar charges and the repulsion of those having the same sign (see p. 315).

Gold-leaf Electroscope.—The gold-leaf electroscope (Fig. 209)

affords a simple means of detecting the presence and sign of an electrical charge. It consists of a metal case A, closed by glass at the front and back, and carrying an ebonite stopper B; through the latter a thin metal rod C is passed which carries at its lower end two thin strips of gold-leaf. When the top of the rod, the knob of the electroscope as it is called, is put in metallic communication with an electrified conductor a portion of the charge runs into the gold-leaves, and, as similarly charged bodies repel each other, the leaves diverge, the greater the charge they receive the greater will be their divergence. The leaves may be torn if they receive too great a charge; to overcome this danger a small amount of electricity may be carried from the conductor to the electroscope by a proof-plane. This consists of a small disc of metal attached to an ebonite handle; it is caused to touch the charged conductor, thereby receiving part of the charge, this is then transferred to the electroscope by putting the plane in contact with the knob.

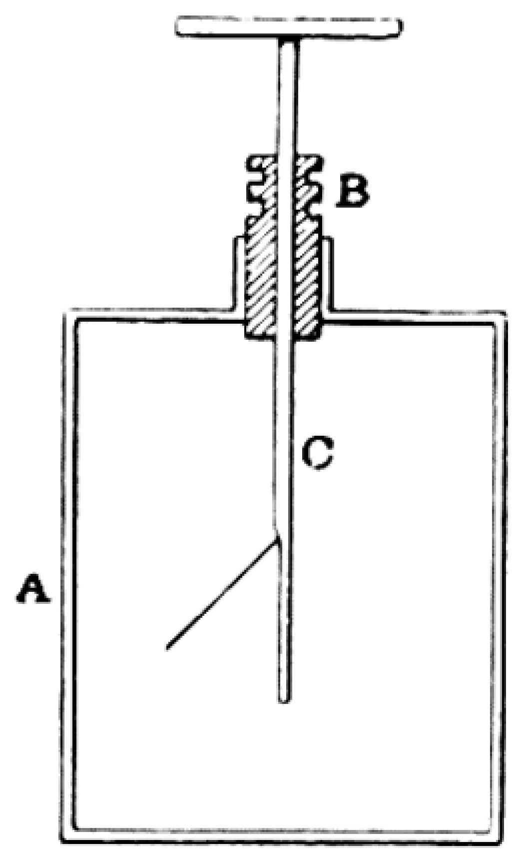


FIG. 209.—The Gold-leaf Electroscope.

EXPERIMENT.—Electrify a glass rod with silk, gently bring it in contact with the knob and then remove it. The electroscope has received a positive charge and the leaves accordingly diverge. Bring a positive charge near it; the leaves diverge still further. If a negative charge is gradually approached the leaves at first collapse and may then diverge again.

This gives us a means of identifying the sign of the charge on a body; if a charged electroscope is brought near it, and the leaves diverge more widely, the electricity on the body is of the same kind as that on the electroscope. A partial collapse of the leaves does not necessarily indicate a charge of the opposite sign, for if a neutral conductor is brought near the knob the leaves collapse slightly. The reason for this will appear shortly. When it is uncertain whether a body is neutral or charged with electricity of the opposite kind to that on the electroscope, we must test it with the leaves charged in succession positively and negatively, if they collapse in each case the body is neutral. A scale on the glass to show the amount of the divergence is useful for rough measurements.

Electrification by Influence (Induction).—The leaves of an electroscope may diverge without electricity being communicated to it.

EXPERIMENT.—Hold an electrified rod near an uncharged electroscope, the leaves diverge; remove the rod, they at once collapse.

It is supposed that an uncharged, or neutral, body contains equal quantities of positive and negative electricity, when a charged body is brought near it there is a separation of the two charges; these run together again when the external charge is removed.

EXPERIMENT.—Hold an electrified glass rod near an insulated conductor (Fig. 210), touch the end A with a proof-plane and transfer the charge, if any, to an electroscope. It is found to be negatively charged. Show similarly

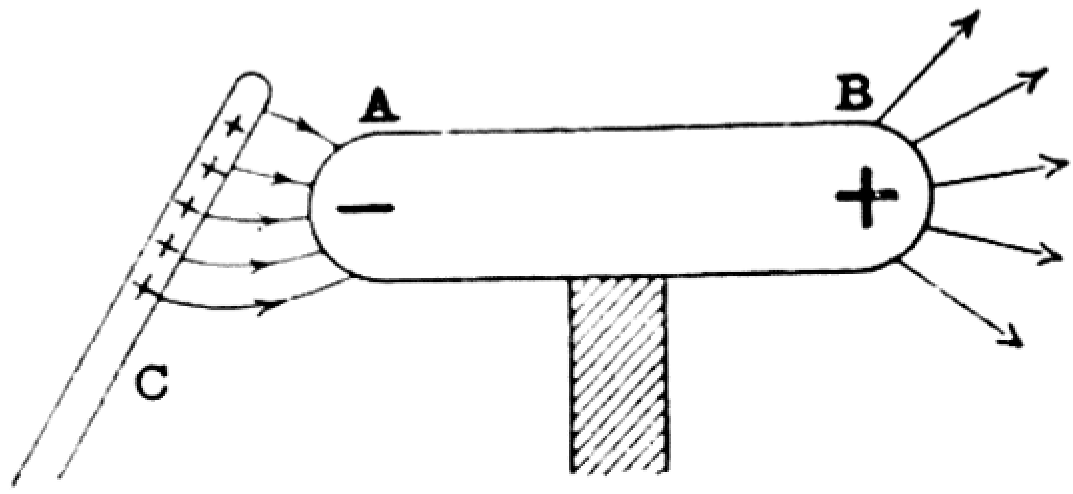


FIG. 210.—Electrification by Influence.

that the end B is positively charged. These charges disappear when the glass rod is removed.

The conductor is said to be electrified by influence (or induction) while it is in the neighbourhood of a charged body. There is evidently an analogy to magnetisation by influence (p. 311).

EXPERIMENT.—Modify the last experiment by first connecting an electroscope to the conductor by means of a wire. When the positively charged rod is held near the conductor the leaves diverge, and it can readily be shown that the charge on them is positive. Touch the conductor, the leaves collapse, showing that a charge has escaped. Remove the finger and then the rod, the leaves diverge with negative electricity. This procedure enables us to charge any conductor with electricity of the opposite kind to that on a charged rod or other conductor.

The theory of these effects is considered in the next chapter.

Electricity is confined to the Surface of a Conductor.—This can be shown as follows:—

EXPERIMENT.—Insulate a hollow metal can on a block of paraffin and give it a charge as in the last experiment. Tests with the proof-plane will now show that there is no electricity at points well inside the conductor, all the charge

is on the outside surface. If a metal rod is placed in the can, so that it projects at the top, a charge may be collected from the part external to the can but none from the portion within.

Similarly if a metal ball is charged and then allowed to touch the interior of an insulated, hollow vessel, all the electricity leaves the ball and flows to the outside surface of the vessel. The electroscope will show that the ball is completely discharged.

The Surface Density is greatest on the Sharply Curved Parts of a Conductor.—The quantity of electricity per cm.^{2} surface of an electrified body is called the **surface density** of the charge. It may be shown that it is greatest on those parts of a conductor that are most sharply curved; for if a pear-shaped conductor is charged, experiments with the proof-plane show that more electricity can be collected from the pointed end than from any other part. In a similar manner, when a flat disc is electrified, very little electricity can be collected from the flat portions compared with the amount that can be obtained from the edges.

The Induced and Inducing Charges are Equal.—When a charge induces electricity on other conductors the total positive and negative charges it induces on these bodies are each equal to the original charge. In the case of electricity placed on a conductor in the middle of a room, the induced charges are found partly on neighbouring conductors and partly on the walls and ceiling of the room; if it is wholly surrounded by another conductor all the induced charge will be found on the latter body.

EXPERIMENT.—Connect an insulated metal can to an uncharged electroscope. Lower into it an insulated metal ball charged positively; the leaves diverge with positive electricity, and they diverge more widely as the ball is gradually lowered, until it is completely inside. Beyond this position it may be moved about freely without affecting the leaves to any further extent. All the lines from the charge now end on the can. Touch the vessel with the finger; the leaves collapse, showing that the induced positive charge has escaped. Tests with the proof-plane show that the corresponding negative charge is on the interior of the can, where the lines from the inducing charge meet it. Touch the interior of the can with the ball; the positive and negative charges at the ends of the lines of force run together, no charge can then be detected on either the ball or the can. The positive charge on the ball is therefore equal to the negative charge it induces on the vessel, and, as the vessel was originally uncharged, the positive electricity that has escaped must contain an equal number of units.

We have supposed that a line of force starts from a unit positive

charge; the above results show that it ends on an equal negative one, since for each positive unit there exists a negative unit distributed on surrounding conductors. This experiment is usually referred to as Faraday's ice-pail experiment.

Positive and Negative Charges are always produced in Equal Amount.—This may be shown by the following experiment.

Experiment.—Fit over one end of an ebonite rod a woollen cap which can be removed by insulating silk threads. Revolve the rod in the cap and hold the two together over an electroscope; no charge can be detected, but, when the cap is removed by the threads, negative electricity is found on the rod and positive electricity on the wool. The two charges exactly neutralized each other and must therefore have been equal. Usually the charge on the rubber escapes through the hand, when the rubber and rod are separated, and so escapes detection.

EXAMPLE ON CHAPTER XXXI

1. A bar magnet is divided in the middle and the parts separated. An insulated conductor electrically charged by induction is similarly treated. Contrast and explain the state of affairs in the two cases. (L. '03.)

CHAPTER XXXII

POTENTIAL

Potential.—When two charged conductors are joined by a conducting wire electricity generally flows from one to the other; the charges are redistributed, but the total quantity of electricity remains the same, if by this we understand the algebraical sum of the charges. Thus the total quantity of electricity on three conductors whose respective charges are $+10$, -20 and $+35$ units, is $+25$ units; when the conductors are joined together this quantity will still be found spread over the whole extent of their surfaces.

Experiment.—Electrify two insulated metal spheres and compare their charges, as in the ice-pail experiment, by holding them one after the other inside a hollow conductor which is joined to an electroscope. The amounts by which the leaves diverge will give us some idea of the relative values of their charges. Hold them together in the hollow vessel, without allowing them to touch, and note the divergence of the leaves; now bring them in contact, the leaves do not alter, showing that there is still the same quantity of electricity on the two. If they are now tested again separately one will usually show a gain and the other a loss of electricity.

We must now inquire what are the conditions that determine which conductor loses electricity; in other words, what are the factors which fix the direction in which electricity shall flow. Experiments such as that above will soon convince us that the transfer is not necessarily from the large to the small conductor, nor even from the conductor containing more electricity to the one containing less. To prove the last statement a means is required of electrifying two conductors with charges whose relative magnitudes are known. This may be done with sufficient accuracy as follows: Charge an insulated spherical conductor negatively and bring near it another small insulated sphere. If the latter is touched by the finger it may be positively charged by influence as often as we please without affecting the charge on the larger conductor, and if it is always placed

in the same position the successive induced charges will be equal. These can be transferred to any hollow conductor by touching its interior with the small charged sphere. Let two hollow spherical conductors whose diameters are 10 and 20 cms. respectively be charged in this manner. It can then be shown that if five charges are given to the smaller and ten to the larger no transfer of electricity takes place when they are joined by a long thin wire, but if more than five are put on the smaller while the charge on the larger is the same as before, then electricity flows from the smaller to the larger sphere. If the proportionate charge on the larger sphere is increased beyond that given in the first instance the flow is in the opposite direction.

Parallel instances may be taken from other branches of physics. In the case of hot bodies heat flows from one possessing a higher temperature to another at a lower temperature, irrespective of which is the larger body or which contains the most heat. Or in hydrostatics, when two tanks containing water are allowed to communicate liquid flows from the one in which the level is the higher independently of their size. If we ask in the latter case why the flows take place, the answer is that by running from a higher to a lower level the water loses potential energy. A mass M at a height h has potential energy Mgh units, if it is allowed to move freely the motion is in the direction which diminishes this energy, it therefore falls to earth. The same principle applies in electricity. Suppose we charge a conductor by bringing to it successive units of positive electricity ; after it has received the first unit it is surrounded by an electrical field of force, and to bring up the remaining units they must be forced along against the field, in other words, work must be expended. This work finds its equivalent in the potential energy of the charge on the conductor ; a charged conductor therefore possesses energy. If now there are two charged conductors which are put in metallic communication, electricity will flow in such a direction as to diminish the joint potential energy of the charges. We can predict the direction of flow from the following considerations. In the case of two water tanks, A and B, if work has to be done against gravity to carry a gram of water from the surface in A to that in B then B is at the higher level, and water will flow from B to A when they are put in communication. Similarly if we have two conductors, A and B, and it requires the expenditure of work to carry unit positive charge from A to B, then positive electricity will flow

from B to A when they are connected by a wire. If, however, the forces in the field will move the test charge from A to B, then this will be the direction in which electricity will flow when they are connected. In the hydrostatic example we say that the level of the water is higher in B than A, in the electrical case we say that the **electrical potential** of B is greater than that of A. Electrical potential, therefore, is analogous to difference of level in hydrostatics. Positive electricity tends to flow from places of higher to those of lower potential, negative electricity goes in the opposite direction. **The difference of potential between two points is measured by the work, in ergs, that must be expended to carry a unit positive charge from one point to the other.**

In comparing the heights of two mountains we measure from sea-level, the sea being looked upon as such a large reservoir that any variation in the quantity of water it contains leaves its mean level unaltered. When we are dealing with electricity the earth is such a large conductor that any charge that can be communicated to it does not alter its potential, it is therefore taken as the conductor of zero potential. When it requires V ergs of work to carry a positive unit from the earth to a conductor the potential of the latter is V; if positive electricity tends to flow from the earth to the conductor the potential of the conductor is negative. If the potential at a point is V it will be necessary to expend VQ ergs of work to bring a charge of Q units from the earth to that point.

The Surface of a Conductor is an Equipotential Surface.—When a charge is moved in a direction perpendicular to the lines of force no work is done, since the field has no component in the direction of the motion. A surface which everywhere cuts the lines of force at right angles is called an equipotential surface, for a charge may be moved about on it without doing work. The surface of a conductor must fulfil this condition, otherwise two points on it will differ in potential and a flow of electricity from one to the other will ensue. When an electroscope is connected to a charged conductor electricity flows into the leaves until their potential is that of the conductor. Similarly when two conductors are connected a redistribution of electricity takes place which reduces them to the same common potential.

EXPERIMENT.—Join a proof-plane to an electroscope by a long wire, then move it over the surface of a charged, pear-shaped conductor. The leaves

show the same divergence for every position of the plane, proving that the conductor is an equipotential surface, although, as we have seen, the density of the charge varies from point to point.

Since lines of force show the direction in which a positive charge tends to move they must run from places of higher to those of lower potential. A line cannot start from and end on the same conductor, for this is an equipotential surface. Lines may, however, start from one part of a conductor where there is a positive charge, and other lines may end on a different part where there is a negative charge. An example of this occurs in electrification by influence as shown in Fig. 210. If there is no charge on a conductor an equal number of lines must start from and end on its surface.

Theory of Electrification by Influence (Induction).—We can now consider more fully the question of electrification by influence. When the charged rod C of Fig. 210 is brought near the conductor the latter becomes surrounded by an electric field; it would therefore require work to bring a positive unit from the earth up to the conductor against this field, *i.e.* the potential of the conductor is raised. This rise is greatest on those parts of the conductor which are nearest to the inducing charge, a redistribution of electricity accordingly takes place; a positive charge runs from A to B, thereby raising the potential of B and lowering that of A, until the conductor has the same potential at every point. If the conductor is now earth connected for a moment a positive charge runs out of it until its potential is reduced to zero; if then the rod C is removed the potential falls still further, becoming negative, and the negative electricity from A spreads over the surface until it again becomes an equipotential. The conductor is now charged and possesses energy; from whence does this energy come? It is seen that when the positive charge on C is removed to a distance we have to overcome the attraction of the negative electricity on the conductor; the energy is the equivalent of the work that is done against this attraction.

Exactly the same reasoning, of course, applies to the charging of an electroscope by influence, but it is instructive to view the process from the point of view of lines of force; the application of this method to the case just considered is left to the student. When a positively charged rod is brought near the knob lines of force end on the latter, and, as the electroscope is uncharged, an equal number must start from some other part of it, the leaves, and run to the metal case or

earth. The tension in these lines pulls the leaves apart. When the electroscope is momentarily earthed, the lines from the leaves disappear on account of the unlike charges on their ends running together, the leaves therefore collapse. If now the inducing positive charge is removed the lines which end on the knob repel each other sideways until they are spread all over the surface, and the leaves are again pulled apart, but this time by lines which start from the case and end on the leaves.

No Field inside a Conductor. Inverse Square Law.—Faraday showed that there was no field inside a hollow conductor by building a small chamber which he covered with tin-foil; this he insulated and charged strongly from the outside, he then took inside a delicate electroscope, but could not detect the slightest divergence of the leaves. As the leaves diverge when the electroscope is placed in a field, there must be zero field inside the conductor. We conclude that lines of force cannot penetrate the substance of a conductor, as magnetic lines of force pass through iron, they merely end on the surface.

EXPERIMENT.—Enclose an electroscope in an insulated wire cage and strongly charge the latter, the leaves do not diverge, verifying Faraday's conclusions.

It can be shown mathematically that if electric charges repelled each other according to any law other than the inverse square law there would be a field in the interior of a hollow conductor, the absence of such a field is the most accurate proof that the law is true.

Capacity.—It has been proved that the intensity of an electric field is $4\pi N$, where N is the density of the Faraday lines of force. If we double the density of the charge at every point of a conductor we shall double the density of the lines at every point in the surrounding field, hence the work required to bring a positive unit from the earth to the conductor will be doubled. The potential of a conductor is therefore proportional to the charge on it. **The ratio of the charge on a conductor to the potential to which this charge raises it is called the capacity of the conductor.** If Q is the charge, V the potential, and C the capacity, $C = Q/V$ or $Q = VC$. These expressions show that the capacity is numerically equal to the number of units of electricity required to raise the potential by unity (put $V = 1$ in the equation). Simple experiments show that the capacity of a conductor depends on its size, shape, and its nearness to other conductors.

EXPERIMENT.—Support a sheet of tinfoil on an ebonite rod, like a blind on its roller, connect it to an electroscope and give it a charge. Revolve the rod

so as to roll up some of the tinfoil, the leaves diverge more widely. Electricity resides only on the outer surface, as the sheet is rolled up its charge-carrying area is diminished, the capacity therefore decreases and the potential rises.

EXPERIMENT.—Support two plates, A, B, about a foot square (Fig. 211) on two ebonite strips and connect each to an electroscope. Charge A positively; each electroscope diverges with positive electricity. Push plate B closer to A; the leaves of electroscope C collapse slightly, and D shows a greater divergence on account of influence. Connect B to earth; the leaves of D collapse altogether and those of C to a very great extent. Positive electricity runs from B to earth, or, if we prefer to say so, negative electricity runs from the earth to B; this

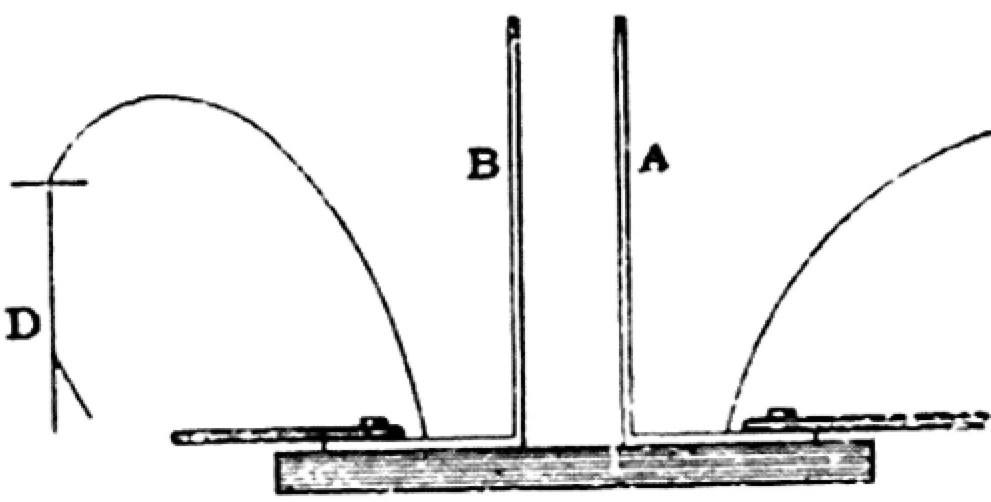


FIG. 211.—Principle of Condenser

greatly reduces the potential of A, and as its charge remains constant its capacity must be increased. Originally only a fraction of the lines from A end on B, but when the latter is earth connected they practically all pass from one plate to the other, because in that position their length is shortest between the unlike charges at their extremities.

This experiment shows that the capacity of a conductor may be artificially increased by bringing near it another conductor with a charge on it of opposite sign; such an arrangement is called a condenser. The capacity of one plate when a charge of the opposite kind can flow freely into the other (as, for example, when it is earthed), is called the capacity of the condenser. It is equal to the charge required to be put on one plate to raise the potential difference between the two by unity. In practice condensers are made of a number of sheets of tinfoil separated from each other by insulating layers of some dielectric, such as mica or paraffin, the alternate sheets are connected to one of two conducting rods forming the terminals of the apparatus (Fig. 212).

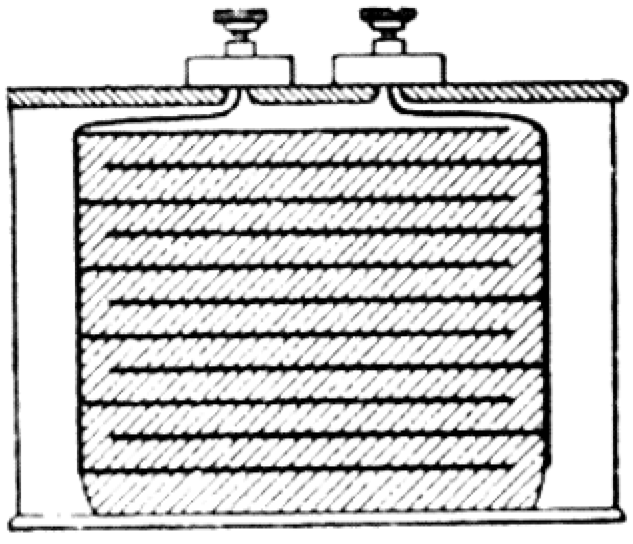


FIG. 212.—A Condenser.

Quadrant Electrometer.—The quadrant electrometer is an instrument for comparing potential differences more accurately than can be done by a gold-leaf electroscope. One form due to Dolezalek is shown in Fig. 213. A flat, cylindrical, box A is divided into four quadrants, each of which is insulated on a short amber pillar B. Diagonally opposite quadrants are permanently connected by thin wires, and communication is made with them from the exterior by two wires running through the amber. A very light conductor C called the needle, made of silvered paper, hangs inside the quadrants from the lower end of a very thin wire which passes at its upper end through an ebonite plug to a screw on the outside. The dotted portion of Fig. 213 (*a*) shows the shape of the needle and the position it ordinarily occupies in the quadrants. The whole is enclosed in a metal case permanently connected to earth to screen needle and quadrants from external electric fields. When it is required to compare the potentials of two conductors, the needle is kept at a high potential by being connected permanently to one pole of a battery the other pole of which is earthed (p. 360). One pair of quadrants is connected to earth, say to a water-pipe, the others are joined to one of the conductors in question. The needle is then repelled by those quadrants at the higher potential, and it can be shown that the deflexion is proportional to the potential difference between the quadrant pairs. Hence the potentials of the conductors can be compared directly. Attached to the suspending wire there is a small plane or concave mirror which comes opposite a glass window in the case, by means of this the deflexions can be read by one of the optical methods given on pp. 145 and 160.

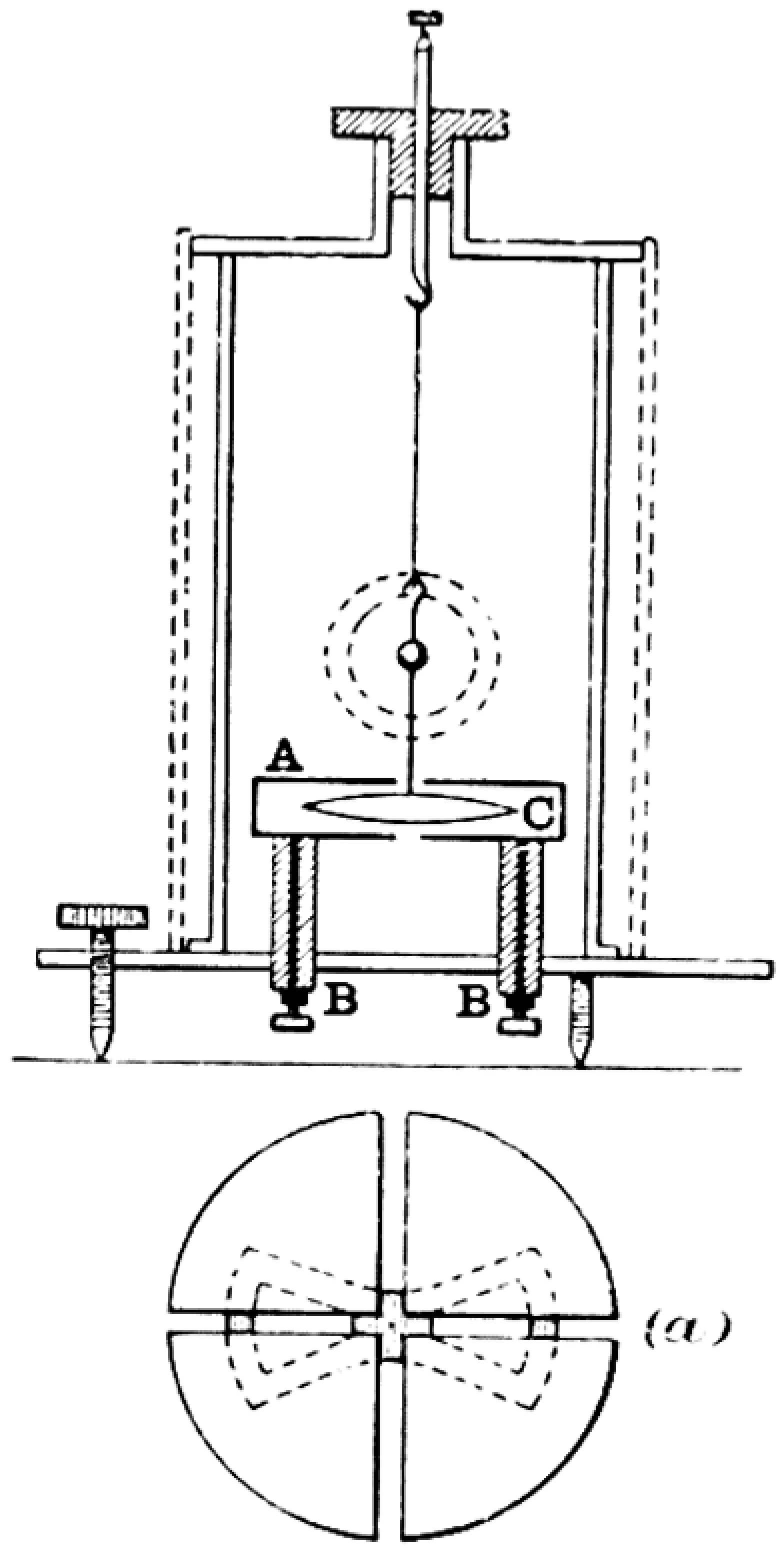


FIG. 213.—Quadrant Electrometer

Condensing Electroscope.—The condenser effect can be used to make an electroscope more sensitive. The knob of the electroscope takes the form of a large flat disc, which is covered on its upper surface with a thin layer of insulating varnish (Fig. 214). A is a similar disc which can be held by an insulating handle. The upper plate being removed suppose that B is connected to a large conductor whose potential is not very high. Electricity flows into the electroscope until it is at the same potential as the conductor, but the transfer may be so small that the leaves do not diverge appreciably. Let the plate A be now placed on B and earthed; the plates are separated by the varnish, and, as A is connected to earth, the arrangement acts like a condenser and the potential of B is lowered. More electricity therefore flows into this plate from the conductor under test until equality of potential is again restored. If now the conductor is disconnected from the electroscope we shall have succeeded in giving to the latter a much larger charge than would have been obtained in the usual method of testing. The plate A is now removed, the capacity of B decreases and therefore its potential rises, at the same time part of the charge on it runs into the leaves and causes them to diverge. By this means much lower potentials can be detected than is possible with the ordinary electroscope.

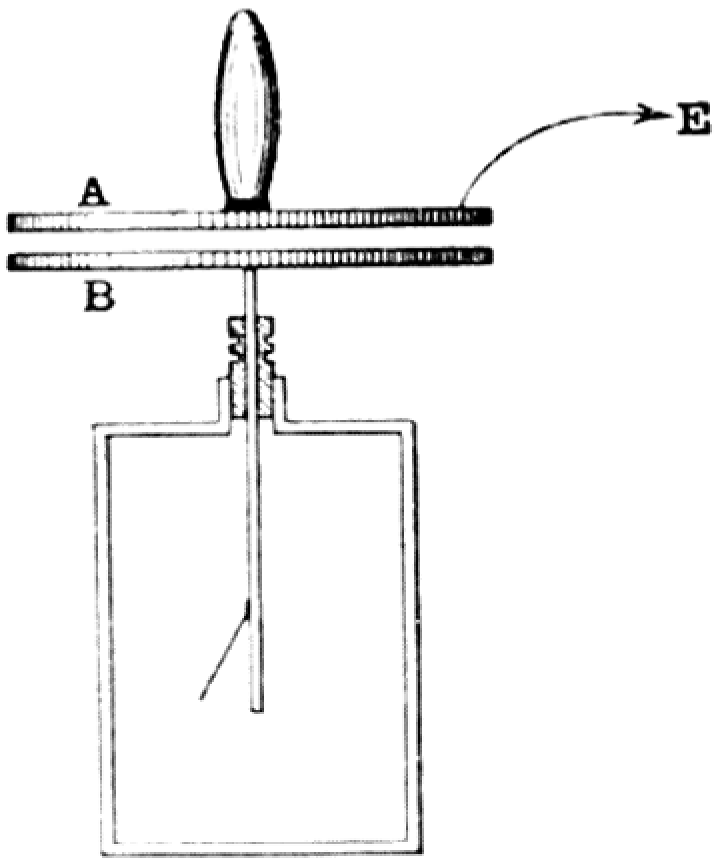


FIG. 214.—Condensing Electroscope.

EXAMPLES ON CHAPTER XXXII

1. How do you explain the fact that if a Leyden jar is placed on an insulator and the outside coating is not touched, the jar will not take so large a charge as when uninsulated? (L. '93.)

2. How much work is done in carrying a charge of 50 units from the earth to a point where the potential is 20?

3. The electrical field between two points 12 cms. apart is uniform, and 2880 ergs of work are done when a charge of 80 units is carried from one to the other. Find the intensity of the field and the density of the lines of force.

CHAPTER XXXIII

ELECTRIC CURRENTS AND THEIR MAGNETIC EFFECTS

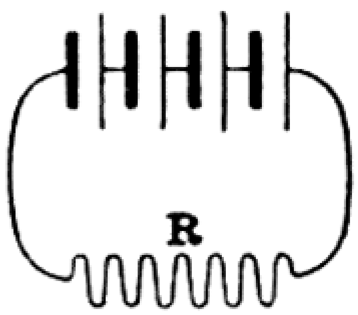
Current.—When two conductors at different potentials are joined by a wire, it has been shown that a transfer of electricity takes place until they are reduced to a common potential. This electricity in motion is called an electric current. In the cases we have dealt with hitherto, the currents have been so short in duration that the effects they produce, other than the equalisation of potential, would be difficult to observe. If by some means two conductors can be maintained at a steady difference of potential, even when they are joined by a wire, there will be a constant current flowing from one to the other and the accumulated effects can readily be studied. The direction of the current is defined as that in which the positive electricity flows. The current strength is equal to the number of units of electricity that pass a given point in the wire every second; if the unit is that of p. 345, the current is said to be measured in electrostatic units.

Voltaic Cells.—Voltaic cells give us the simplest means of maintaining a current. In its simplest form a cell consists of two plates, one of copper the other of zinc, dipping into dilute sulphuric acid. When the outer ends of these are joined to the terminals of an electrometer the needle is deflected in a direction which shows that the copper is at a higher potential than the zinc. The copper is called the positive and the zinc the negative pole of the battery. If they are joined by a wire a continuous current flows through it from the copper to the zinc, and, as we shall see later, through the cell from the zinc to the copper. When the poles are not connected by a conductor the cell is said to be on open circuit, and the difference of potential between the poles in these circumstances is called the **electromotive force of the cell.** This expression is usually abbreviated to E.M.F. This type of cell is found to be inefficient in practice, it

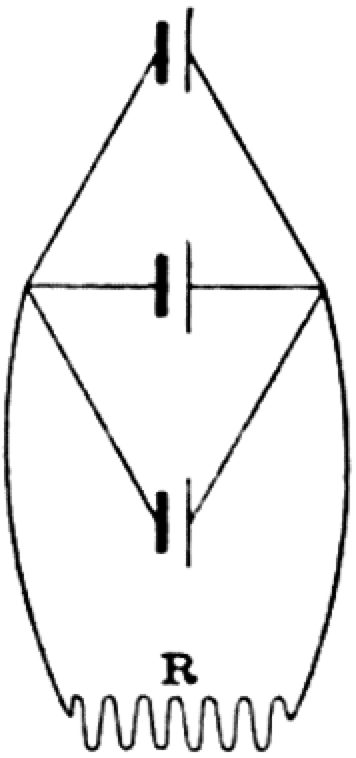
is therefore advantageously replaced by more elaborate forms of which a description is given in Chap. XXXV.

Cells in Series and in Parallel.—It may readily be shown that the E.M.F. of a cell does not depend upon the size of its plates.

EXPERIMENT.—Connect the poles of a cell on open circuit to the terminals of a quadrant electrometer, then gradually withdraw the plates from the liquid. The deflexion of the needle remains unchanged until contact between a plate and liquid is broken, so proving the statement just made.



(*a*) Cells in Series



(*b*) Cells in Parallel

FIG. 215.—(*a*) Cells in Series; (*b*) Cells in Parallel.

When the negative pole of one cell is joined to the positive of a second and the negative of this to the positive pole of a third, and so on, the cells are said to be joined in series; the free poles of the extreme cells are called the poles of the battery. If all the positive poles are joined together to form one pole and all the negatives to form another, the cells are said to be joined in parallel. These arrangements are indicated diagrammatically in Fig. 215, (*a*) and (*b*), where the thick and thin lines represent the different poles, and R represents the external circuit, *i.e.* the conductor through which the battery is required to send a current.

It may easily be proved, with the help of an electrometer, that the E.M.F. of a battery in the series arrangement is the sum of the E.M.F.'s of the separate cells, and that when a number of similar cells are connected in parallel the E.M.F. is that of a single cell. The student will shortly be in a position to prove these results for himself by other means. The E.M.F. of six cells in series can be detected by a condensing electroscope.

EXPERIMENT.—Connect the positive pole of the battery to the upper plate, hold the wire from the negative pole in an insulating covering and momentarily touch the lower plate with it. As the upper plate is now gradually removed the leaves diverge with negative electricity. The experiment shows that we have to do with very small differences of potential compared with those with which we dealt in electrostatics, in fact a battery of several hundred cells would be required to produce a deflexion of 1 cm. on a gold-leaf electroscope such as has been described in Fig. 347.

General Effects produced by Currents.—The effects produced by currents may be divided into three classes: (*a*) Thermal, (*b*) Chemical, (*c*) Magnetic.

Thermal Effects.—When a current flows through a conductor heat is generated. Well-known applications of this fact are the incandescent electric lamp and the electric arc. In each of these cases the conductor is heated to such a high temperature as to cause incandescence.

Chemical Effects.—Solutions of inorganic acids and salts in water and other liquids are conductors of electricity, but when a current is passed through them certain chemical changes result. Such liquids are called electrolytes. The liquids in voltaic cells must be electrolytes in order that the current may be able to pass through them. The conducting power of liquids other than electrolytes is usually very small. These effects are most strikingly shown by using lead acetate as the electrolyte.

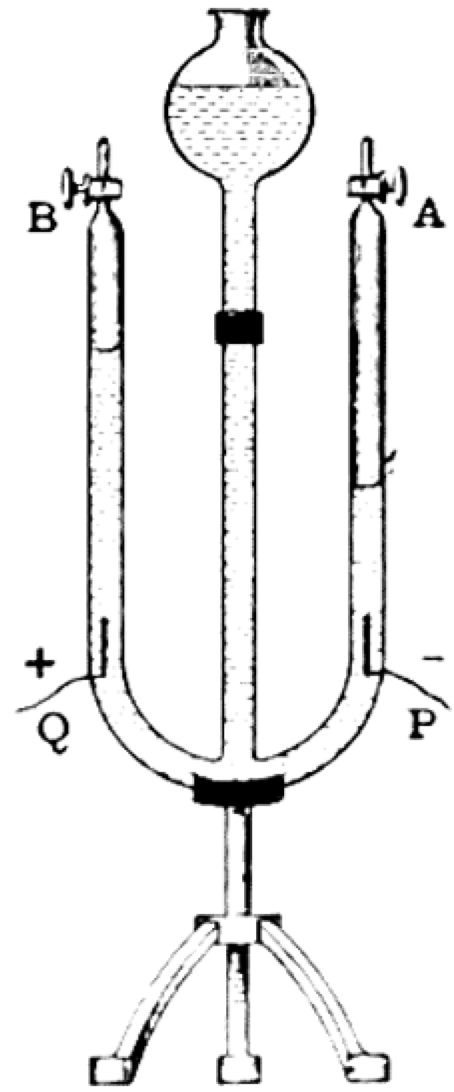


Fig. 216.—Apparatus for decomposing Sulphuric Acid.

Experiment.—Immerse a lead plate and a thick lead wire in an aqueous solution of lead acetate which is contained in a glass vessel. Connect the plate to the positive pole and the wire to the negative pole of a battery of a few Daniell cells in series. When the current has passed for a few minutes a deposit of lead will be seen on the wire, spreading out in all directions like the branches of a tree.

Experiment.—Pour some dilute sulphuric acid into the glass apparatus shown in Fig. 216 until it reaches the level of the taps A, B, then close the taps. Connect the two platinum plates P, Q, with the opposite poles of a battery of several cells, and send a current through the liquid for some minutes. Bubbles of gas rise from the plates and collect in the tubes above. These may be proved, by the usual chemical tests, to be hydrogen and oxygen; the former comes from the plate connected with the negative battery pole.

These experiments show that an electrolyte is decomposed when a current passes through it.

Magnetic Effects. Oersted's Experiment.—An important advance was made in 1820 when Oersted discovered that a wire carrying a current is surrounded by a magnetic field.

EXPERIMENT.—Hold a wire carrying a current just above and parallel to a compass needle (Fig. 217), the needle is deflected. When the direction of the current is reversed the deflexion of the needle is also reversed. Hold the wire below the needle, the deflexion is reversed.

In the remainder of this chapter we shall consider the magnetic effects in more detail; the heating and chemical effects will be dealt with in later chapters.

Magnetic Field due to a Current.—In the first place we must

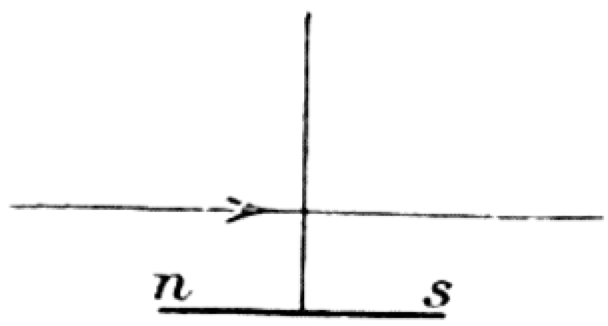


FIG. 217.—Oersted's Experiment.

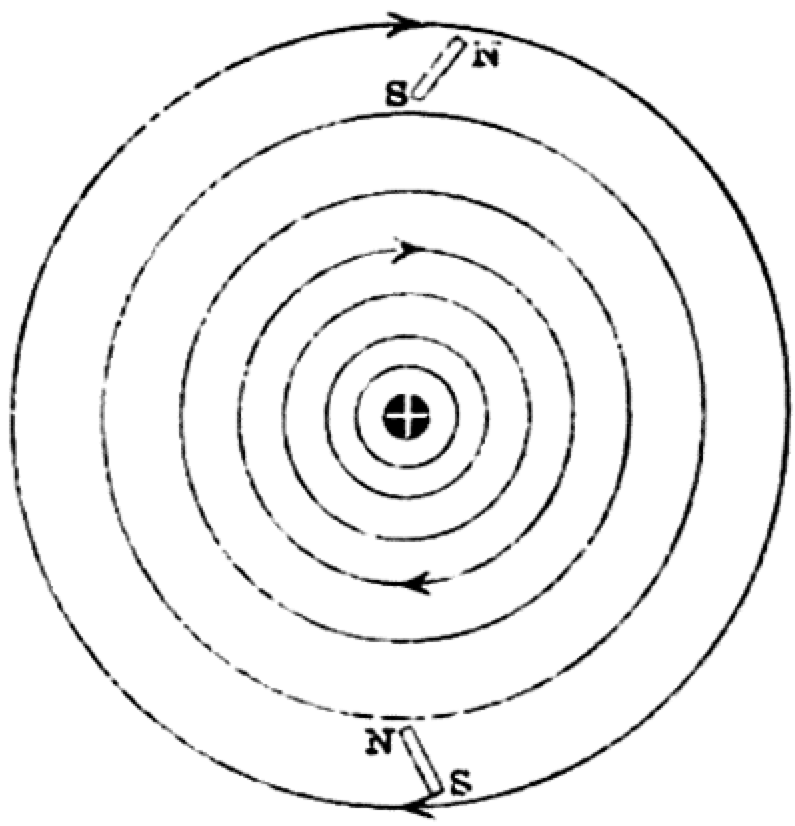


FIG. 218.—Magnetic Field due to a Straight Wire carrying a Current.

discover how the direction of the magnetic lines of force is related to that of the current.

EXPERIMENT.—Pass a long vertical wire through a sheet of cardboard on which iron filings are sprinkled and send a strong current through it. When the cardboard is tapped the filings arrange themselves in circles round the wire but are not attracted by the wire itself (Fig. 218). The lines of force are therefore circular and have neither beginning nor end. To discover the positive direction of the lines place a compass needle due S. of the wire. It will be found if the current is flowing vertically downwards that the N. pole is deflected to the W. If the needle is placed due N. of the wire the N. pole is urged to the E. The lines of force therefore run in the direction shown by the arrows in Fig. 218.

These results may be expressed by the following rule: **Look along the wire in the direction in which the current is travelling, then the lines of force go round in the direction of the hands of a watch.** The student will find it a great advantage to picture to himself these

lines surrounding the wire in the various cases that follow. When a magnet is suspended at the centre of a rectangular coil in which a current is running in the direction of the arrows (Fig. 219), the rule

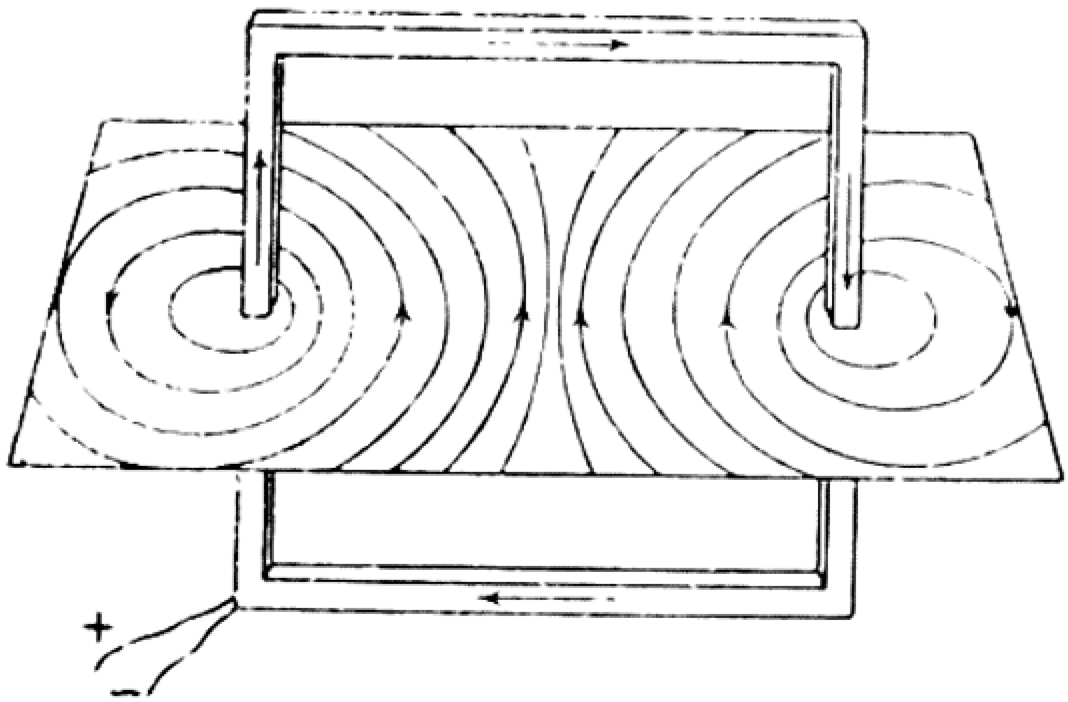


FIG. 219.—Magnetic Field due to a Rectangular Current.

just given shows that the magnetic field due to each side of the rectangle urges the N. pole into the paper. The forces on the magnet may therefore be largely increased by winding a number of turns round it; this is the principle of galvanometers, which are instruments for measuring current.

Field due to a Straight Wire.—We may investigate, either with a deflexion magnetometer or by an oscillation method, how the magnetic field varies at different distances from a long, straight, wire carrying a current. If the first method is used the needle must be placed due magnetic N. or S. of the wire, so that the field of the earth, H, is perpendicular to that produced by the current.

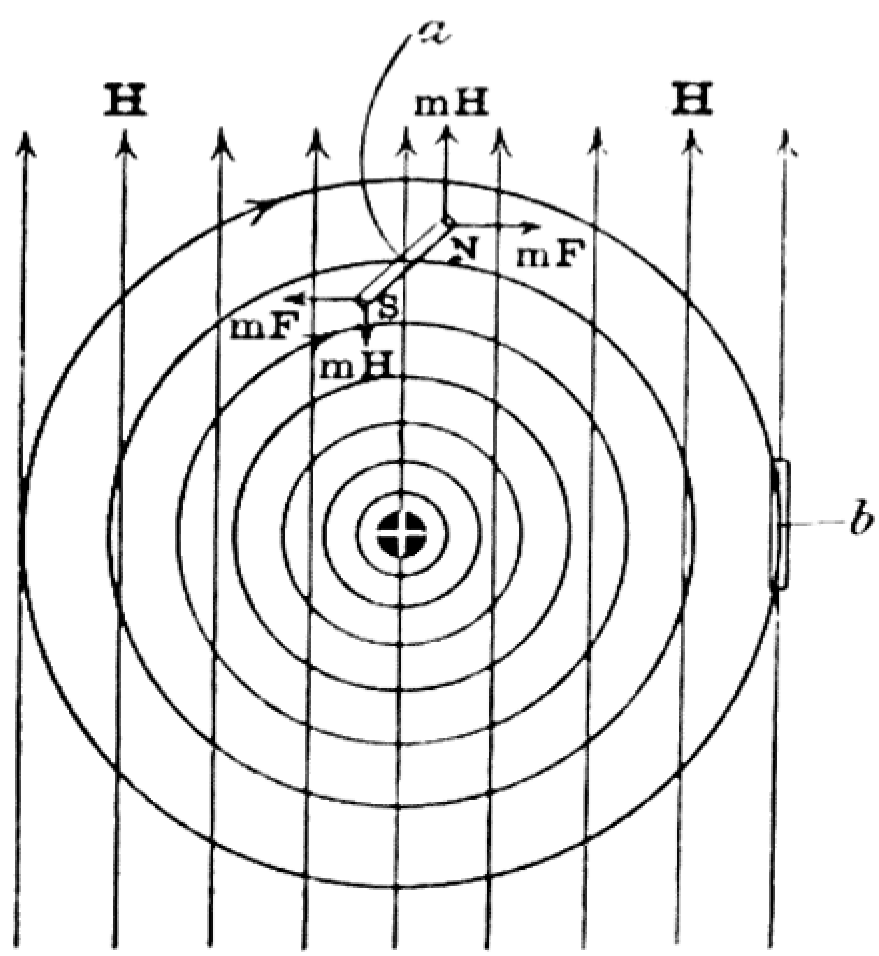


FIG 220.—Measurement of the Field of a Linear Current.

(See Fig. 220, *a*, in which the current is supposed to be flowing into the paper.) Then the field, F, of the current is found from the equation

$F = H \,.\, \tan\theta$, where θ is the needle deflexion (p. 319). If the oscillation method is used, the needle must be placed E. or W. of the wire (b, Fig. 220), so that the fields due to the earth and the current are parallel, and the method of calculation given on p. 323 is applied. Either method shows that the field varies inversely as the distance from the wire. Soon after the discovery of this law Laplace showed that the same result could be obtained by calculation, if it was assumed that the field at O due to each short element of length s varied as $As \,.\, \sin\theta/R^2$, where A is the current strength, R is the distance from the element of length s, and θ is the angle between R and s (Fig. 221). It is seen from the figure that $s \,.\, \sin\theta = ab$, which is the apparent length of s as seen by an observer at O. Hence the expression may be written Al/R^2, where l is the apparent length of the element. The total field at O is the sum of the fields due to the separate elements like s into which the wire may be divided.

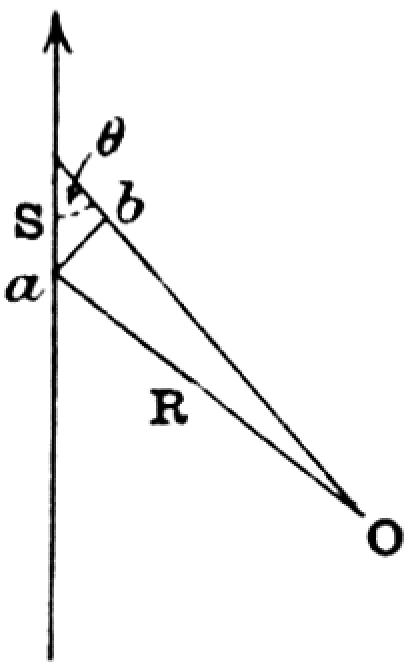


FIG. 221.—Laplace's Law of the Magnetic Field due to a Short Current.

Field due to a Circular Current. Unit Current.—Consider the magnetic field at the centre of a circular coil of radius R. To an observer at the centre the apparent length is the actual length, since every short piece is perpendicular to the radius, $\theta = \frac{\pi}{2}$ and $\sin\theta = 1$. Hence the field varies as Al/R^2, where l, the sum of the lengths, is $2\pi R$. Thus the field $F \propto \frac{2\pi R \,.\, A}{R^2}$, or $F = k \,.\, \frac{2\pi A}{R}$, where k is a constant depending on the units of current adopted. We have already defined the electrostatic unit of current strength on p. 359, but this unit is difficult to realise in practice. It is much simpler to define the unit of current from the magnetic field it produces, and then use this to define a new unit of quantity of electricity. For this purpose k in the last equation is put equal to unity in air, then $F = 2\pi A/R$ for the circular current. From this equation it is seen that if A and R are each unity $F = 2\pi$, we thus have our definition of unit current: **Unit current is that which, flowing in a circle of 1 cm. radius, exerts a force of 2π dynes on a unit magnetic pole placed at the centre.** This unit is called the C.G.S. electromagnetic unit of current or

briefly the E.M. unit. It is too large to be convenient for many purposes; hence a practical unit, called **the ampere,** which is $\frac{1}{10}$ of it, is generally employed. The magnetic field produced at the centre of a circular coil of n turns by a current of A amperes in each is therefore $2\pi n\mathrm{A}/10\mathrm{R}$. The practical unit of quantity of electricity is that conveyed by a current of one ampere running for one second; it is called **the coulomb.** The C.G.S. unit of quantity, to which no special name is given, is that carried by one electromagnetic unit of current in one second; it of course contains ten coulombs. If two points are at a difference of potential V, then (p. 353) VQ ergs of work are expended when a charge Q is carried from one to the other. As we have altered our unit of quantity, we must also change our unit of potential difference if this statement is still to be true. **Two points are said to differ in potential by one electromagnetic unit when one erg of work is done in carrying one electromagnetic unit of electricity between them.**[1] This unit proves to be much too small to be convenient; the practical unit of potential difference is 10^8 as large, and is called **the volt.** To give some idea of its magnitude it may be stated that the E.M.F. of a Daniell cell is 1·1 volts. When an electromagnetic unit of quantity is moved through a potential difference of one volt 10^8 ergs are expended; hence when one coulomb falls through a potential difference of one volt, 10^7 ergs are set free, or when Q coulombs fall through V volts, $\mathrm{QV} \times 10^7$ ergs of work are liberated.

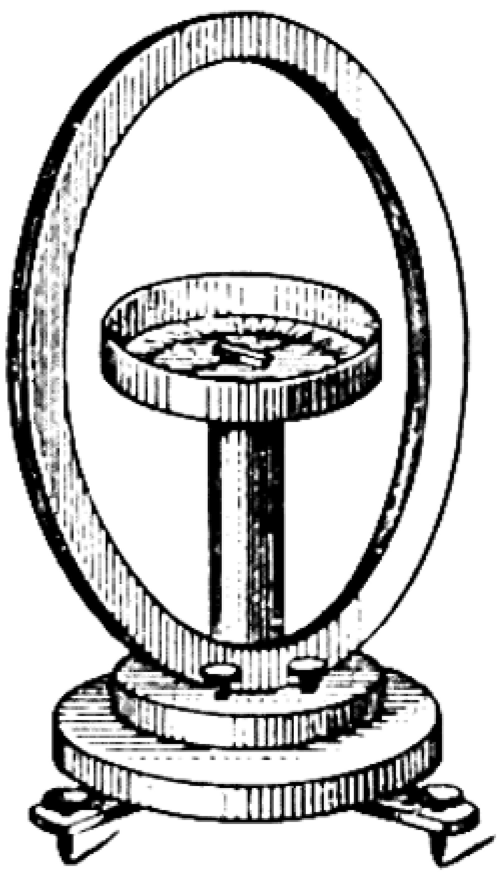


FIG. 222.—Tangent Galvanometer.

Tangent Galvanometer.—The simplest form of current measurer is the tangent galvanometer (Fig. 222). It consists of a short magnet, supported on a needle point, at the centre of a circular coil of wire of one or more turns wound on a wooden frame. The coil can be turned about a vertical axis and clamped in position. When a current is to be measured the plane of the coil is placed in the magnetic meridian. A long, light, pointer fixed at right angles to the needle

[1] Hence one erg of work will be expended when unit E.M. current flows between the points for one second.

moves over a graduated circle; it should stand at the zero when the needle is in the plane of the coil. The magnetic field F at the centre, due to a current A amperes circulating in the wire, is perpendicular to the plane of the coil, from the watch rule, and is equal to $2\pi n A/10R$. As the coil is in the meridian, F is perpendicular to the earth's horizontal field H; the needle is therefore deflected through an angle θ, given by $F = H . \tan \theta$ (p. 319).

$$\therefore \frac{2\pi n A}{10R} = H . \tan \theta$$

or

$$A = \frac{10RH}{2\pi n} . \tan \theta$$

If the instrument is always used in the same place we may treat $10RH/2\pi n$ as constant, say k, whence $A = k . \tan \theta$. When R, H, and n are known, the current is given in amperes at once. The constant k is called the reduction factor of the galvanometer; it is the number by which $\tan \theta$ must be multiplied to give the current in amperes. The formula just given is true only if the field F is uniform round the needle and is perpendicular to H (p. 319). The first condition is the more closely fulfilled if the needle is small compared with the coil diameter, the second is satisfied when the coil is in the meridian. To test the latter adjustment a current is reversed through the instrument, when the deflexion should also be exactly reversed; the coil must be twisted round a vertical axis until this is found to be the case, it is then in a suitable position for measuring currents.

EXPERIMENT.[1]—Place the coil of a tangent galvanometer perpendicular to the meridian so that the lines due to the current are parallel to the earth's field, H, and replace the compass box by a flat sheet of cardboard. Plot the lines of force when a current is running and examine over what distance they are parallel.

Proof of Laplace's Inverse Square Law.—A modified form of tangent galvanometer may be used to prove Laplace's inverse square law. A bit of magnetised watch-spring attached to the back of a small concave mirror is hung by a single silk fibre at the common centre of three circular coils, each consisting of a single turn (Fig. 223). One coil has a diameter of 15 cms., the other two, of which one only is shown in the figure, are 30 cms. in diameter. Current may be

[1] Barton and Black, "Practical Physics," pp. 144 and 148.

sent round each coil separately or they may be combined. A lamp and scale is used to measure the deflexion. It has been found (p. 364), assuming the law, that $F = 2\pi nA/10R$; if R is doubled the field is halved, but it may be brought back to its original value by doubling n the number of turns. The field due to the two large coils together should therefore be equal to that produced by the small coil. The coils are placed in the meridian and a current is sent first through the two large coils, connected together so that it circulates in the same direction in each, and the deflexion is observed; it is then sent through the small coil when the same deflexion should be

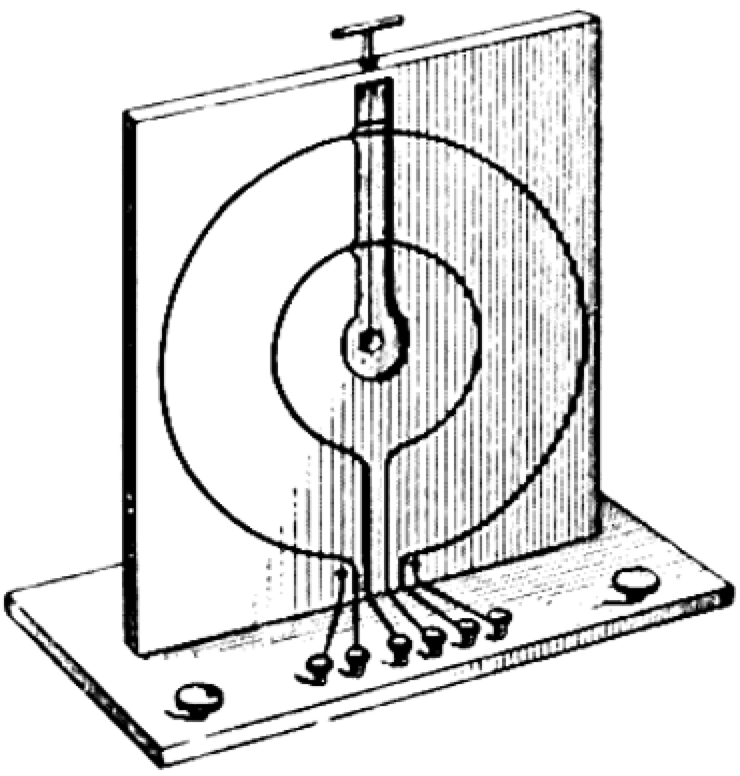


FIG. 223.—Apparatus to prove Laplace's Inverse Square Law.

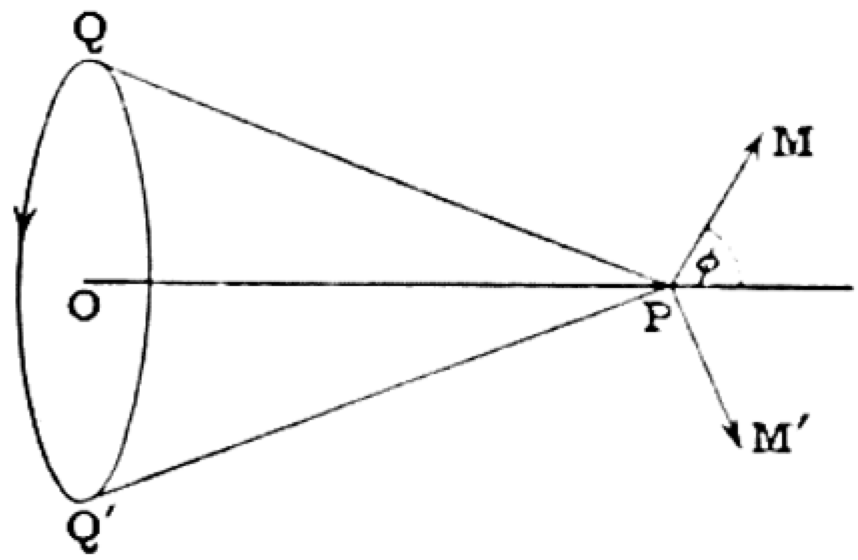


FIG. 224.—Field at any Point on the Axis of a Circular Coil.

produced. Or the coils may be so connected that the field of the outer pair is opposed to that of the inner; no deflexion should then occur.

Field due to a Circular Coil at any point on the Axis.—Let a current A electromagnetic units pass round a circular coil of radius R, it is required to calculate the magnetic field at any point, P, on the axis. The direction of the field due to a small length s at Q is along PM, and perpendicular to QP, for the lines of force are circles round the short length of wire at Q. Its magnitude is $As \,.\, \sin\theta/PQ^2$, where θ is the angle between PQ and s; but this angle is $\pi/2$, hence the field is As/PQ^2. If we take an equal length at Q′ at the opposite end of the diameter QOQ′ we have an equal force along PM′. The components perpendicular to OP cancel out and the resultant of the

two forces is along the axis OP. This will be true for every pair of small lengths at the opposite ends of a diameter, hence the resultant for the whole coil is merely the sum of the components along OP. The component of As/PQ^2 in this direction is $As \cdot \cos\phi/PQ^2$, and the field due to the whole coil is therefore $\frac{A}{PQ^2} \cdot \cos\phi \cdot \times$ (sum of the small lengths like s), *i.e.* $\frac{A}{PQ^2} \cdot \cos\phi \cdot 2\pi R$. But $\cos\phi = \sin OPQ = R/PQ$, and $PQ = (x^2 + R^2)^{\frac{1}{2}}$, where $OP = x$, or, finally, $F = 2\pi R^2 A/(x^2 + R^2)^{\frac{3}{2}}$. This result, as far as its variation with x is concerned, may be tested with a magnetometer or by an oscillation method.

EXPERIMENT.—Place the circular coil of a tangent galvanometer in the meridian and connect in series with it one or more Daniell cells. Note the deflexion of the needle at different points on the axis; the field of the coil is given by $F = H \cdot \tan\theta$. Plot $\tan\theta$ against $1/(x^2 + R^2)^{\frac{3}{2}}$, this should be a straight line. If the coil is fixed perpendicular to the meridian the field at different distances can be found by the method of p. 323.

Astatic Galvanometers.—It has already been stated that if a galvanometer is to obey the tangent law its coil radius must be large; as the field at the centre due to the current is $2\pi nA/R$ it is clear that a large radius means a weak deflecting field and therefore an instrument incapable of measuring small currents. For many purposes a very sensitive galvanometer is required and it is not so material that it should obey the tangent law. In order that the field due to the current shall be strong R must be small, and n, the number of turns of wire in the coil, large. Also the deflexion of the needle is opposed by the earth's field, hence the horizontal component H must be weakened or neutralised in some way. This is done, (1) By placing a magnet near the galvanometer so that its lines of force are opposed to the earth's; with a proper adjustment the resultant field can be made very weak and its direction any we please, hence we shall not be limited to placing the coil in the magnetic meridian. (2) By using an astatic needle; with one perfectly astatic the earth's field would exert *no* directive action, in practice all that can be done is to weaken its effect. A reference to Fig. 225 shows that the effect of the current is further increased if coils are wound round each of the needles composing the astatic pair, but in opposite directions so that they assist each other. Finally the needle is suspended by a very thin fibre, and a lamp and scale or a telescope (pp. 145 and 160) is

used to measure the deflexions. These arrangements are all combined in the astatic mirror galvanometer shown diagrammatically in Fig. 225. The bar magnet NS can be moved up or down and turned round a vertical axis to vary the controlling field, the small mirror C is shown between the two coils. The whole is enclosed in

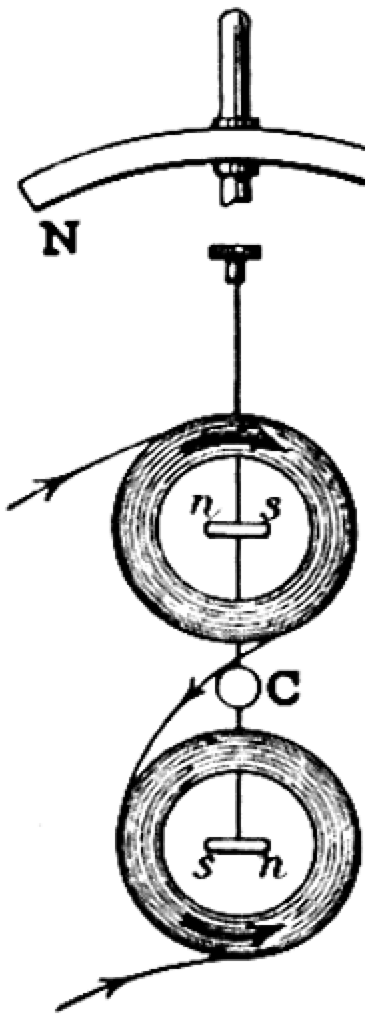


FIG. 225.—Astatic Mirror Galvanometer.

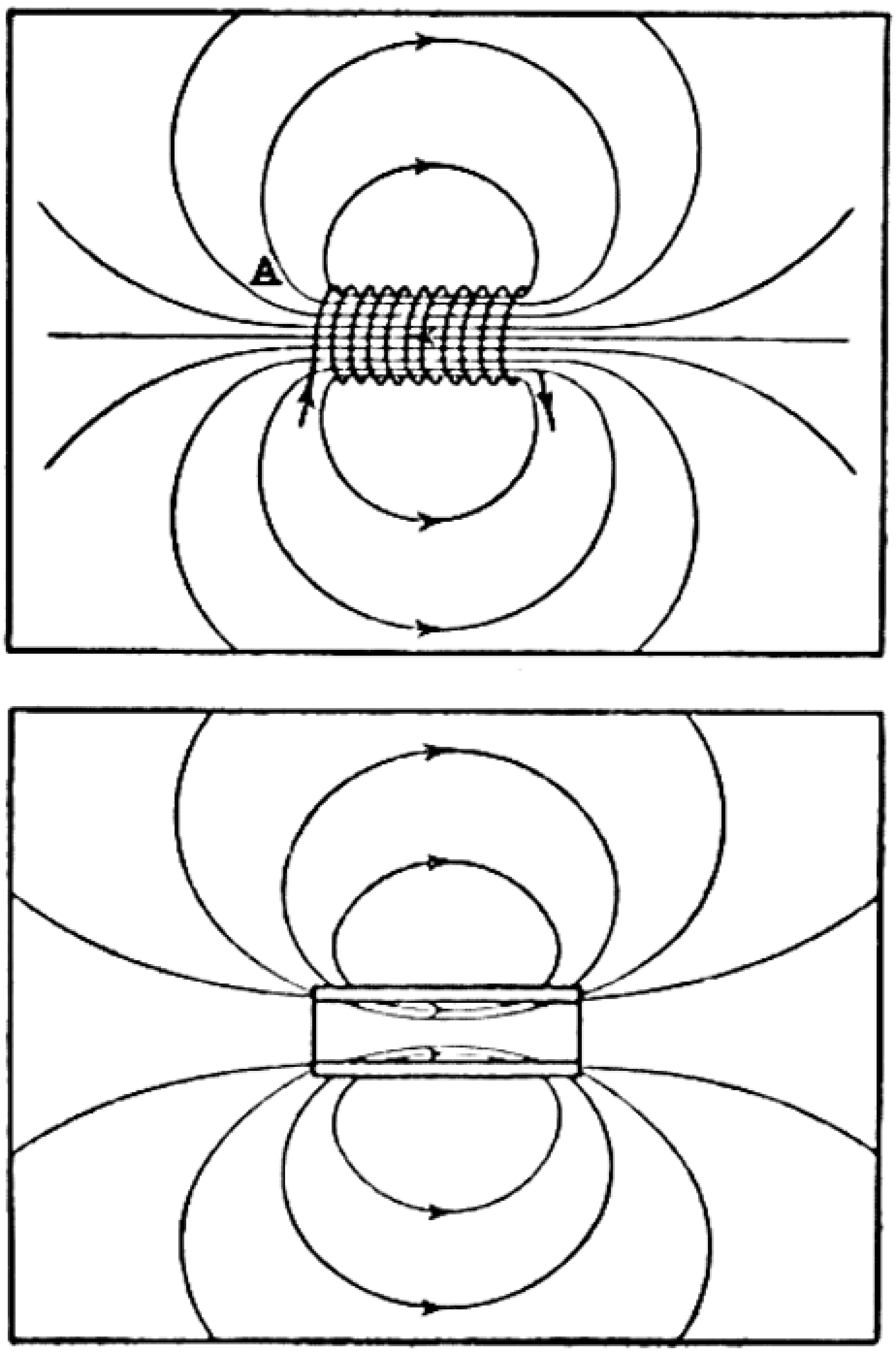


FIG. 226.—Field due to a Solenoid and a Hollow Magnet.

a case (not shown) to screen the needle from draughts. A method of determining how the deflexion varies with the current is given in the next chapter.

Solenoids and Electromagnets.—Let a current be sent through the solenoid shown in Fig. 226 in the direction of the arrows. Each turn gives rise to lines of magnetic force whose direction is given by the watch rule. Between the windings the lines cancel each other, but in the direction of the axis their effects are added and a strong

field results. By placing strips of cardboard round the coil the field can be plotted with a small compass needle; the figure shows the general direction of the lines. This should be compared with Fig. 187 *A*. For comparison, the lines of force due to a magnetised steel tube are also shown. It is seen that in the interior the lines run in opposite directions in the two cases. The end A of the solenoid is found to repel a N. and attract a S. pole, or the coil behaves as if it had a magnetic pole at each end. By applying the watch rule we get the following result, which it is often useful to remember. If on looking at one end of the solenoid the current appears to circulate in the direction of the hands of a watch, then we are looking at its S. pole.

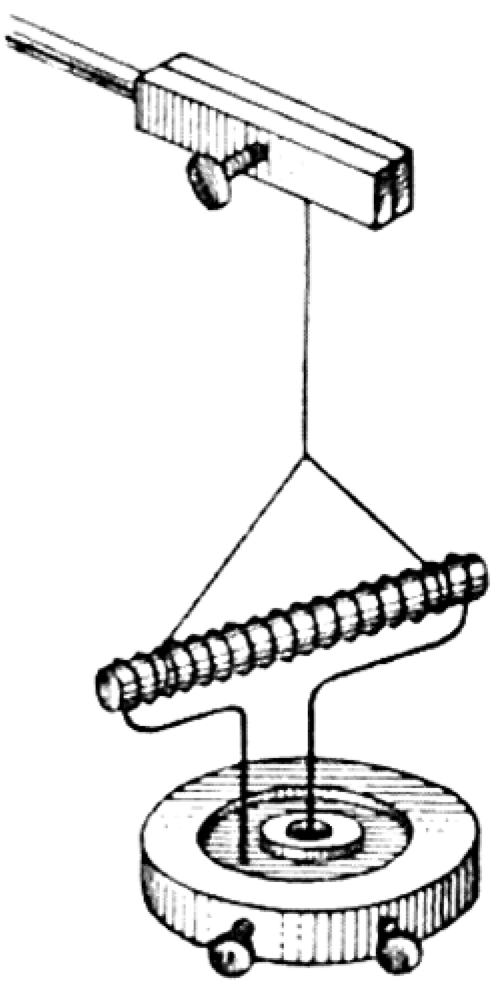


FIG. 227.—Straight Electro-magnet.

EXPERIMENT.—Push a strip of zinc and a strip of copper through a large cork bung, join their upper ends to a light solenoid and float the whole in dilute sulphuric acid. The arrangement constitutes a floating battery, and, as the current flows, the coil sets with its axis pointing N. and S. just like a compass needle does.

EXPERIMENT.—Hang the solenoid by a cotton thread with its ends dipping into separate pools of mercury (Fig. 227). Connect one pole of a battery to each pool. The solenoid sets N. and S. as before.

These magnetic effects are greatly increased if a bar of soft iron is placed inside the solenoid, for the iron being in a magnetic field becomes strongly magnetised. The density of the lines may be increased by this means in the ratio 2000 : 1; this renders it possible to construct very powerful electromagnets.

EXAMPLES ON CHAPTER XXXIII

1. Give definitions of electric force, potential, and current in the electrostatic system of units. (L. '08.)

2. A small magnet is placed 10 cms. due magnetic E. of a long vertical wire and is found to make 24 oscillations/min. under the influence of the earth's

horizontal field. How many oscillations per minute will it make when a current of 6 amperes runs (1) upwards, (2) downwards, in the wire ?

3. Suppose the magnet in the last question had been placed 10 cms. due N. of the wire, find the tangent of the angle through which it would be deflected by the current.

4. A tangent galvanometer has a coil of 10 turns, of average radius 20 cms. Find the current (*a*) in C.G.S. electromagnetic units, (*b*) in amperes, required to deflect the needle through 45° when the coil is in the magnetic meridian.

CHAPTER XXXIV

ELECTRICAL RESISTANCE. OHM'S LAW AND ITS APPLICATIONS

Ohm's Law.—No substance is a perfect conductor, when a current flows from one point to another it experiences a resistance to its passage, and this is found to vary with the nature and shape of the material. The relation between the potential difference at the ends of a wire and the current running through it was first discovered by Ohm. **Ohm's law** states that **the ratio of the potential difference to the current is constant while the physical state of the conductor remains the same.** If A is the current and E the potential difference, Ohm's law states that $E/A = R$, where R is a constant called the resistance of the conductor. Stated in this form the law does two things, it provides us with an exact definition of resistance, and further states that this resistance is independent of the current while the physical state of the conductor, as regards temperature, etc., remains unchanged. The resistance of a conductor is therefore a new physical constant. An experiment on the flow of water through a tube will perhaps bring out more clearly the significance of the latter statement.

EXPERIMENT.—Connect a piece of quill glass tube about 10 cms. long to the bottom of a glass reservoir which contains water. The pressure P forcing the water through the tube is that due to the height of the liquid surface in the reservoir above the lower end of the tube. Measure the quantity of water, Q, delivered in one minute for different pressures. We might, in analogy with the electrical problem, define the resistance of the tube to the flow of water as the ratio P/Q. The numbers obtained will show that this resistance *increases* as Q is increased, whereas electrical resistance is independent of the current. The curve connecting P and Q is shown in Fig. 228; if the resistance were constant it should be a straight line.

Ohm's law may be proved by the apparatus shown in Fig. 229. Current from a battery B is sent through a circuit made up of a manganin wire PQ, a tangent galvanometer G, and another manganin

wire S, whose length can be varied. A current A, measured by the galvanometer, flows from P to Q, and can be varied in strength by including more or less of the wire S in the circuit. At the same time the difference of potential E between P and Q is measured by connecting these points to the terminals of a quadrant electrometer C. The ratio E/A will be found to be constant. Ohm's law does not hold for gases.

The Ohm.—We can use the Ohm's law equation $E = AR$ to define the unit of resistance. When unit E.M.F. at the ends of a wire sends unit current through it the equation shows that the

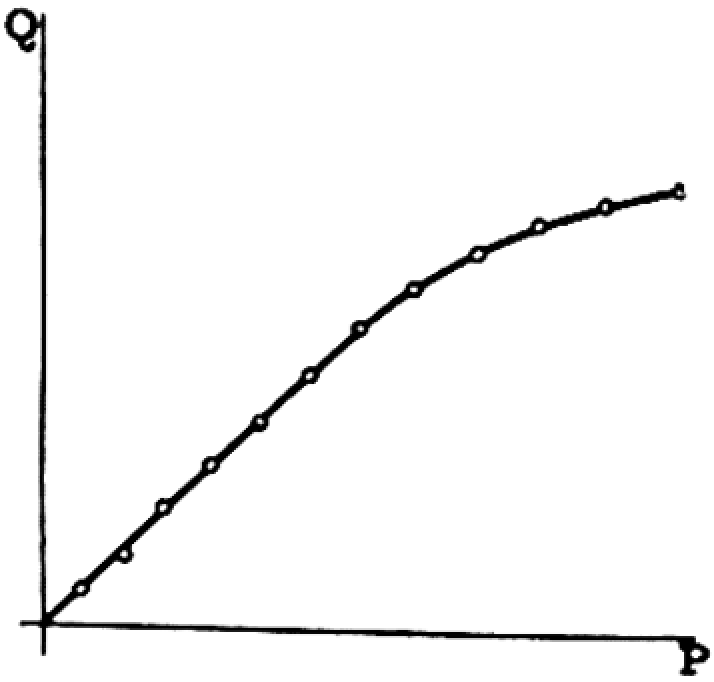


FIG. 228.—Curve showing Flow of Water through a Tube.

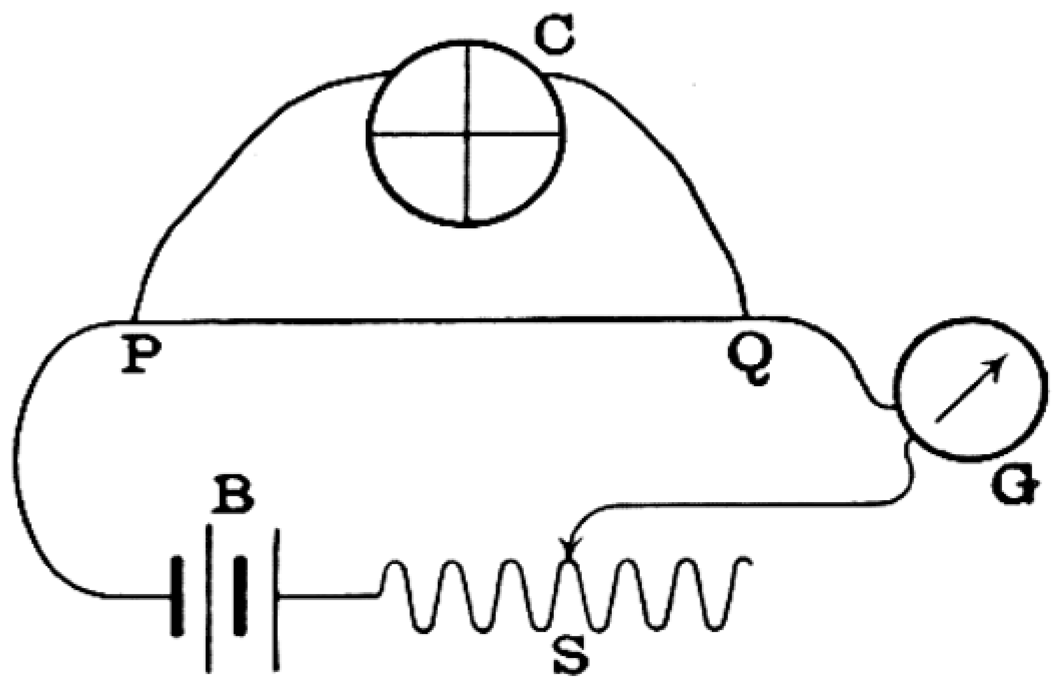


FIG. 229.—Apparatus for proving Ohm's Law.

resistance must also be unity. When E and A are in electromagnetic units the resistance R is also in these units; if E and A are given in volts and amperes respectively then the resistance is given in a new unit called **the ohm.** In defining the ohm it is advantageous to make it such a multiple of the electromagnetic unit of resistance that the equation expressing Ohm's law is true for both sets of units, without the introduction of a numerical factor. As the volt is 10^8 times the electromagnetic unit of potential difference, and the ampere 10^{-1} the electromagnetic unit of current, it follows from the equation that the ohm must be 10^9 the electromagnetic unit of resistance.[1] A conductor has a resistance of one ohm when an E.M.F. of one volt

[1] Suppose the equation is given in practical units, then reducing each quantity to the corresponding E.M. unit we have $10^8 E = \frac{A}{10} . R . 10^9$ or $E = AR$ as before.

sends through it a current of one ampere. A column of mercury 106·3 cms. long and 1 mm.^{2} section has a resistance of one ohm at 0° C.

Specific Resistance.—Experiments which the student will shortly be able to carry out for himself show that resistance of a wire of constant diameter varies directly as its length, and inversely as its section. If R is the resistance of a conductor whose length and section are l and S respectively, $R = kl/S$, where k is a constant for the given material. This constant is evidently the resistance of a piece of the conductor whose length and section are each unity, in other words, it is the resistance between the opposite faces of a centimetre cube; it is called the **specific resistance** or the **resistivity** of the material. In books of tables it is usually given in microhms (millionths of an ohm) per cm.^{3} The resistance of a metallic conductor generally increases with the temperature, if R_0 is the resistance at 0° C. that at another temperature $t°$ is found to be given by the equation $R = R_0(1 + at)$. The quantity a is called the temperature coefficient of resistance; for pure metals it is approximately equal to 0·00366, which is the coefficient of cubical expansion of a gas. The resistance of a pure metal wire is therefore as sensitive to changes of temperature as is the volume of a mass of gas at constant pressure. This property is made use of in the construction of electrical thermometers, p. 389. The resistivity of alloys is much greater than that of pure metals, while their change of resistance with temperature is much less. These properties make them very useful in the construction of standard resistance coils. An alloy of copper-nickel-manganese, called manganin, has a temperature coefficient which is practically zero for small temperature changes.

EXPERIMENT.—Arrange in series 2 m. of thin iron wire coiled in a helix, a cell, and a low resistance galvanometer. Heat the wire by a Bunsen flame, the current decreases, showing that the resistance has been increased by heating.

Resistance Boxes.—For purposes of measurement it is convenient to have a set of coils of known resistance arranged in a box so that they may readily be put into or cut out of a circuit. A manganin wire is doubled on itself at the middle, so as to get rid of induction effects (p. 418), and is then wound on a bobbin. The ends are soldered to brass blocks (Fig. 230), which can be connected when necessary by well-fitting brass plugs. When the plugs are in position the current flows through the blocks, as they have a negligible resistance, but when a plug is removed the only path is through the wire.

Any desired resistance can thus be thrown into the circuit by the removal of appropriate plugs.

Ohm's Law applied to a Complete Circuit.—When a simple cell is on open circuit the P.D.[1] between the poles is, as already defined, the E.M.F. of the cell; but when the poles are joined by a wire this P.D. is less than the E.M.F., for positive electricity flowing in the external circuit from the copper to the zinc tends to decrease the potential of the former and raise that of the latter. In fact, owing to the chemical actions going on in the cell, which are the cause of the E.M.F. (see Chap. XXXV), the current is forced through the cell from the zinc to the copper. The E.M.F. has therefore to drive the current through two resistances one after the other, viz. the external resistance R, and the internal resistance of the cell B. Now, from Ohm's law, the P.D. producing the current is equal to the product of the current into the resistance, the total resistance is $(R + B)$, hence $E = A(R + B)$, where E is the E.M.F. and A is the current. The part AR represents that part of E which is used in driving the current through the external circuit, the remainder AB is spent in forcing it through the cell. We are now in a position to compare resistances or E.M.F.'s by simple experiments.

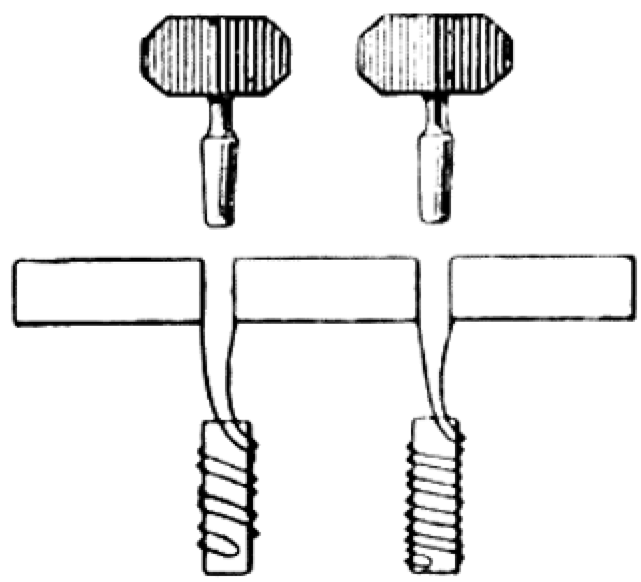


FIG. 230.—Construction of a Resistance Box.

Measurement of Resistances by Substitution.—A resistance can be measured by a substitution method if we have a resistance box at our disposal.

EXPERIMENT.—Join the ends of the resistance R to be measured to two brass blocks which can be connected when necessary by a brass plug (such an arrangement is called a plug key) (Fig. 231). In series with this connect a constant cell C, a resistance box P, and a suitable galvanometer G. Note the deflexion when the plug is out, then put it in, so that R is cut out of the circuit, and take plugs out of the box until the deflexion is the same as before. The resistance in the box is then equal to the unknown resistance R.

Comparison of E.M.F.'s.—To compare the E.M.F.'s of two cells several methods are available.[2]

[1] We will in future use the abbreviation P.D. for potential difference.
[2] Barton and Black, "Practical Physics," p. 154.

EXPERIMENT.—Place one of the cells in series with a resistance box and a tangent galvanometer, take out resistance in the box, *i.e.* pull out the necessary plugs, until a convenient deflexion is produced. Replace the first cell by the second and again note the deflexion, the resistance being kept constant. Let E_1 and E_2 be the E.M.F.'s of the cells, R the resistance of the box and galvanometer taken together, and let us assume that the battery resistance is so small that it can be neglected. The currents are proportional to the tangents of the deflexion,

$$\therefore \ A_1 = \frac{E_1}{R} = k \,.\, \tan \theta_1$$

and

$$A_2 = \frac{E_2}{R} = k \,.\, \tan \theta_2$$

$$\therefore \ \frac{E_1}{E_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

In order to make the battery resistance negligible a sensitive galvanometer should be used, when R will require to be large.

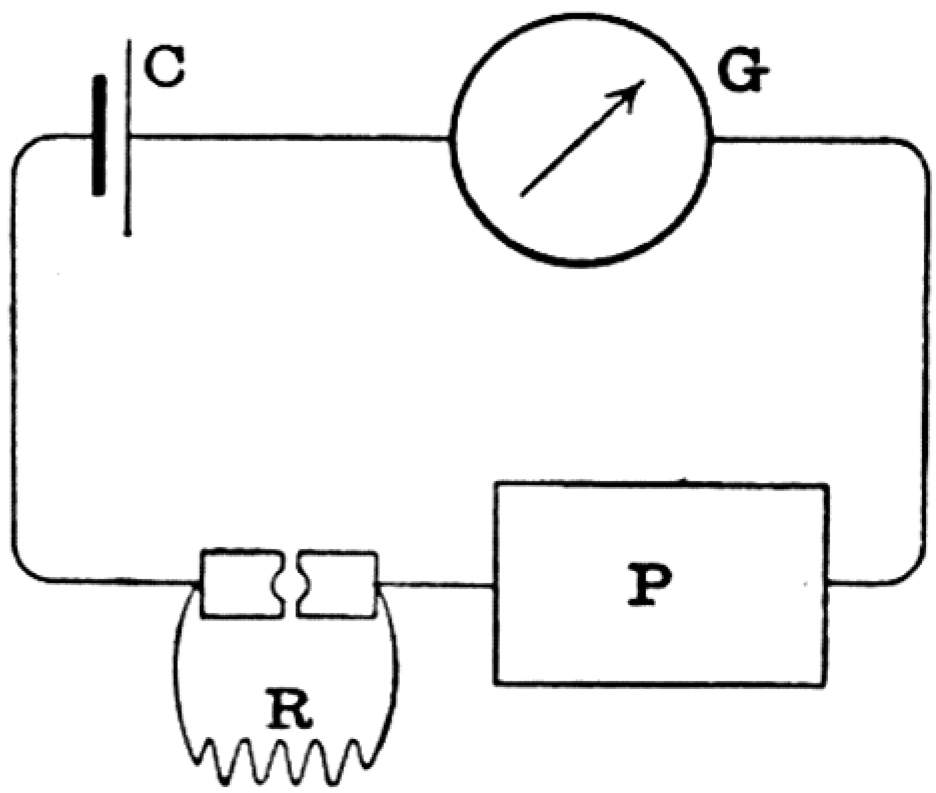


FIG. 231.—Measurement of Resistance by Substitution.

EXPERIMENT.—The method may be varied by altering the resistance in the box until the same deflexion is obtained with each cell. Let R_1 be the resistance of box and galvanometer when the first cell is used, and B_1 the internal resistance of the cell, R_2 and B_2 the corresponding quantities for the second cell. The current A is the same in each case, as the deflexions are equal

$$\therefore \ A = \frac{E_1}{R_1 + B_1} = \frac{E_2}{R_2 + B_2}$$

or

$$\frac{E_1}{E_2} = \frac{R_1 + B_1}{R_2 + B_2}$$

If B_1 and B_2 are negligible, $E_1/E_2 = R_1/R_2$.

The advantage of the method is that we are independent of the

law connecting the current and the deflexion of the needle, since the deflexion is the same for each cell, but the galvanometer resistance is required. The resistances of the galvanometer and cells may be eliminated by altering the resistances in the box to get another deflexion, which is the same for each cell. Let r_1 and r_2 be the additional resistances taken out in the box for the first and second cell respectively, then

$$\frac{E_1}{E_2} = \frac{R_1 + B_1}{R_2 + B_2} = \frac{R_1 + r_1 + B_1}{R_2 + r_2 + B_2}$$

$$\therefore \frac{E_1}{E_2} = \frac{r_1}{r_2}$$

since the last two fractions are equal to the new fraction obtained by subtracting the numerators to form a new numerator and the denominators to form a new denominator (note on p. 189). The resistances r_1 and r_2 are known accurately.

Sum and Difference Method.—The two cells are first connected in series so that they send a current in the same direction through a tangent galvanometer and a resistance box, the E.M.F. in the circuit is then E_1+E_2. The resistance is adjusted until a suitable deflexion of 40°–50° is obtained. The cells are next connected so that they tend to send currents in opposite directions through the circuit; the E.M.F. is then E_1-E_2, if E_2 is the E.M.F. of the cell that has been reversed, and the current is in the same direction as in the first case. If R is the total constant resistance of the circuit,

$$\frac{E_1 + E_2}{R} = k \cdot \tan \theta_1$$

$$\frac{E_1 - E_2}{R} = k \cdot \tan \theta_2$$

$$\therefore \frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

whence

$$\frac{E_1}{E_2} = \frac{\tan \theta_1 + \tan \theta_2}{\tan \theta_1 - \tan \theta_2}$$

For putting the cells in series and opposition alternately a commutator is convenient; this is simply a key by means of which the current can be reversed in a circuit without disconnecting any wires. The arrangement of the circuit is shown in Fig. 232. The commutator PQRS consists of four brass blocks each of which can be connected by a plug to the block on either side. The cell C, which is to be reversed, is joined to a diagonal pair of blocks and the remainder of the circuit to the other pair. Suppose R is joined to the

positive pole of C, then when P and S are connected and also Q and R the cells are in series, and current flows in the direction of the arrows. If the plugs are shifted so as to connect R and S and also P and Q the cells are in opposition, and the current from cell C tends to flow through the galvanometer in the opposite direction.

EXPERIMENT.—Compare the E.M.F. of one cell with that of a battery composed successively of 1, 2, and 3, similar cells in series, show that the E.M.F. of the battery is the sum of the E.M.F.'s of the cells. Also place the battery cells in parallel and show that the E.M.F. is that of one cell.

To test whether a Galvanometer obeys the Tangent Law.—For this purpose we shall suppose that the resistance G of the galvano-

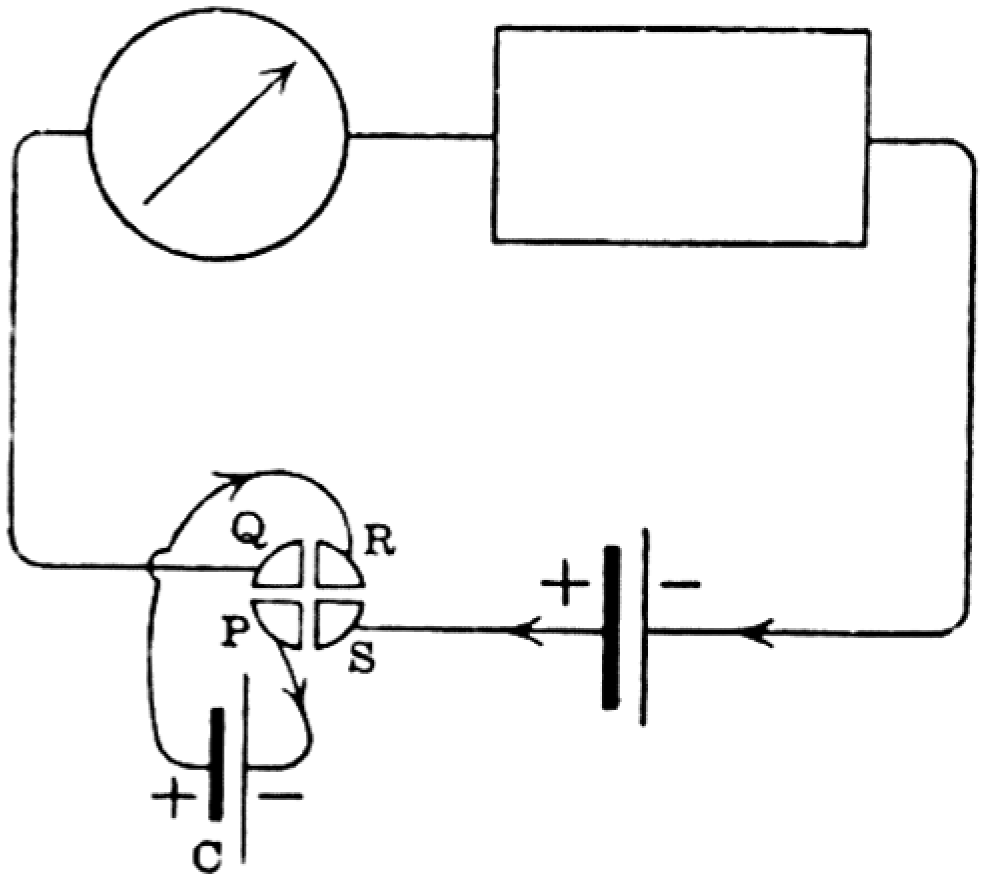


FIG. 232.—Sum and Difference Method of comparing E.M.F.'s.

meter coil is known and that the resistance of the battery is negligible, the latter will usually be true.

EXPERIMENT.—Arrange in series a constant cell, galvanometer, and resistance box. Put a series of resistances in the box and note the corresponding deflexions, reading each end of the needle and taking the mean (p. 330). Let R be the resistance in the box when the deflexion is θ, then the current in the circuit is $A = \dfrac{E}{R + G}$. If the tangent law is followed, $A = k \, . \tan\theta$

$$\therefore \frac{E}{R+G} = k \, . \tan\theta$$

hence
$$(R + G)\tan\theta = \frac{E}{k} = \text{a constant.}$$

Find whether $(R + G) \, . \tan\theta$ is constant for different values of R and θ. Or

since $A \propto 1/(R + G)$ we may plot $1/(R + G)$ against $\tan\theta$, when a straight line should be obtained. Usually it will be found that the tangent law is departed from when the deflexions are large.

Resistances in Series and in Parallel.—When resistances are connected so that the current has to flow through each in succession they are said to be joined in series; the total resistance to be overcome is evidently the sum of the separate resistances. We have already made use of this principle in the resistance box. If the resistances are arranged as in Fig. 233, so that the current has a choice of paths, they are said to be arranged in parallel. Let us calculate the resistance R of the single wire that can replace the three resistances and keep the total current between P and Q unaltered; this is called the equivalent resistance of the system. Let

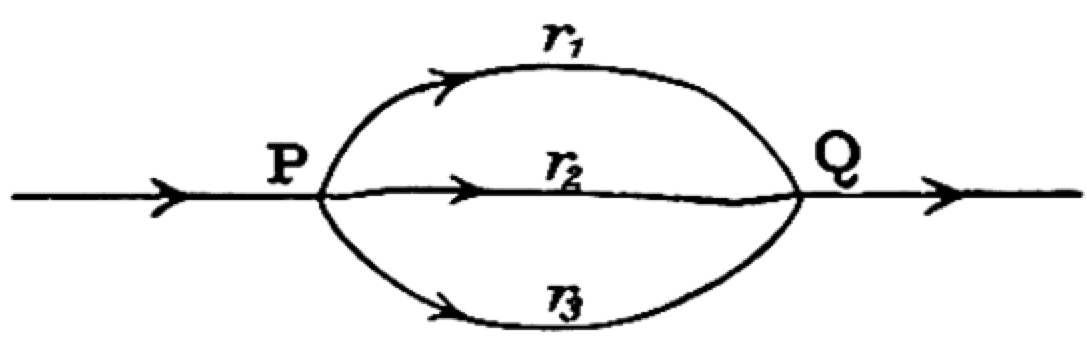


FIG. 233.—Resistances in Parallel.

r_1, r_2, r_3 be the resistances, a_1, a_2, a_3 the corresponding currents in each, A the current in the main circuit, *i.e.* in the wire leading to P or from Q.

Then $$A = a_1 + a_2 + a_3$$

Also if E is the P.D. between P and Q we have, by applying Ohm's law separately to each resistance,

$$a_1 = E/r_1$$
$$a_2 = E/r_2$$
$$a_3 = E/r_3$$

$$\therefore\ A = a_1 + a_2 + a_3 = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

For the equivalent resistance, $A = E/R$

$$\therefore\ \frac{E}{R} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

or $$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

If the resistance of a wire is R then 1/R is called its **conductance**; the equation therefore says that the equivalent conductance of a set of resistances in parallel is the sum of the conductances of each branch. When there are only two resistances in parallel it is seen from above that the ratio of the currents $a_1/a_2 = r_2/r_1$, *i.e.* they are inversely as the resistances. Adding unity to each side of this equation, we get

$$\frac{a_1 + a_2}{a_2} = \frac{r_1 + r_2}{r_1}$$

or, since now

$$\mathrm{A} = a_1 + a_2$$

$$\frac{\mathrm{A}}{a_2} = \frac{r_1 + r_2}{r_1}$$

and

$$a_2 = \frac{r_1}{r_1 + r_2} \cdot \mathrm{A}$$

showing that the fraction of the main current A which goes through r_2 is $r_1/(r_1 + r_2)$. The resistance equivalent to r_1 and r_2 in parallel is given by $\frac{1}{\mathrm{R}} = \frac{1}{r_1} + \frac{1}{r_2}$ or $\mathrm{R} = \frac{r_1 r_2}{r_1 + r_2}$. That equivalent to n equal resistances in parallel, each equal to r, is r/n. These results should be remembered.

Shunts.—When a current which it is desired to measure is too large to pass through a galvanometer the terminals of the instrument are joined by a suitable resistance called a shunt. The galvanometer and shunt resistance are then in parallel and only a fraction of the current goes through the former. This fraction, from the last paragraph, is seen to be $s/(s + g)$, if s is the shunt resistance and g that of the galvanometer. For example, if $s = g/9$ the current in the galvanometer is $\frac{1}{10}$ of the main current; by using a suitable shunt much larger currents may be measured than can be passed through the galvanometer directly. This is the principle of amperemeters or ammeters, instruments constructed to measure currents directly in amperes or fractions of an ampere. An ammeter consists of some form of galvanometer, most frequently a modification of the one described on p. 413, which is permanently shunted by a low resistance. A pointer attached to the needle moves over a graduated circle and indicates directly the amperes running in the circuit.

It should be noticed that by shunting a galvanometer the main

current is altered, for the resistance between the galvanometer terminals was originally g, after shunting it is $sg/(s+g)$. Hence a compensating resistance equal to this difference $g\left(1-\frac{s}{s+g}\right)$ must be placed in the circuit if the main current is to be unaltered.[1]

Voltmeters.—Voltmeters are instruments designed to measure directly in volts the P.D. between two points in a circuit. For example, a quadrant electrometer might be arranged so that the scale divisions gave, say, the E.M.F. of a battery directly in volts, it would then be called an electrostatic voltmeter. It possesses the advantage that it requires no current to produce the deflexion, consequently when it is joined to the poles of a battery the P.D. between these is unaltered (see p. 375). An electrometer is, however, a difficult instrument to use; for many purposes it is replaced by a galvanometer placed in series with a high resistance, of such amount that the current which passes is very small. A pointer attached to the needle gives directly the voltage applied to the terminals. The resistance and galvanometer can then be used as a voltmeter. With the help of an ammeter and voltmeter a resistance can be measured directly. A battery is caused to send a current through the resistance and an ammeter in series with it, while a voltmeter joined to the *ends* of the resistance gives the P.D. that exists between these points. Knowing the current in amperes and the P.D. in volts the resistance can be calculated from Ohm's law. By this means the resistance of an incandescent lamp can be measured when it is glowing.[2]

Reduction Factor of a Tangent Galvanometer.—The reduction factor of a tangent galvanometer can be measured if, by means of a voltmeter or otherwise, we know the E.M.F. of a cell; conversely if the reduction factor is known an E.M.F. can be measured.

EXPERIMENT.—Place a cell of E.M.F. E in series with the galvanometer and a resistance. Adjust the latter until a deflexion θ_1 in the neighbourhood of 45° is obtained; suppose the total resistance in the circuit is R. Take out further resistance r in the box until the deflexion is reduced to θ_2. Then the currents

$$A_1 = k \,.\, \tan\theta_1 = \frac{E}{R}$$

and

$$A_2 = k \,.\, \tan\theta_2 = \frac{E}{R+r}$$

---

[1] Barton and Black, "Practical Physics," p. 160. [2] *Ibid.*, pp. 146 and 174.

from which the unknown resistance R, which includes the battery and galvanometer, must be eliminated.

We have
$$\frac{R}{E} = \frac{1}{k} . \cot \theta_1$$

$$\frac{R + r}{E} = \frac{1}{k} . \cot \theta_2$$

subtracting,
$$\frac{r}{E} = \frac{1}{k}(\cot \theta_2 - \cot \theta_1)$$

whence k can be found since r and E are known. If, on the other hand, k is known, E can be found.

Wheatstone's Bridge.—We will now describe the commonest and most accurate method of measuring resistances, called after its inventor the Wheatstone's bridge. Let two points A, C (Fig. 234), be joined by two resistances ABC, ADC in parallel and let currents be sent through these. As we go from A to C by either path the potential falls, it is therefore possible to find a point D on one branch which has the same potential as a point B on the other. This condition will be fulfilled when the resistance AB is the same fraction of ABC as AD is of ADC. Let the resistances be P, Q, R, and S, as in the figure, then when B and D are at the same potential

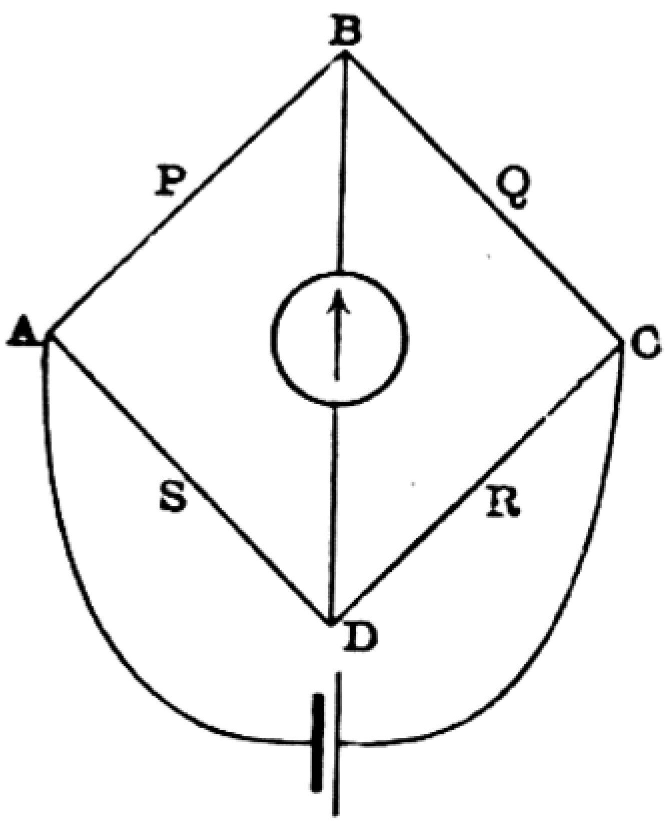


FIG. 234.—Diagram of Wheatstone's Bridge.

$$\frac{P}{P + Q} = \frac{S}{R + S}$$

or
$$\frac{P + Q}{P} = \frac{R + S}{S}$$

i.e
$$1 + \frac{Q}{P} = 1 + \frac{R}{S}$$

$$\frac{Q}{P} = \frac{R}{S}$$

or
$$QS = RP$$

Hence $Q = P \cdot \frac{R}{S}$, showing that if one resistance P and the *ratio* of the others R and S are known, Q can be calculated. Suppose ADC is a uniform wire, P a coil whose resistance is known, and Q one whose resistance is required. One terminal of a galvanometer is joined to B, and a wire from the other terminal is made to touch ADC at different points, when a point is found which has the same potential as B no current runs through the galvanometer. Thus D can be found experimentally and the ratio of the resistances R, S is simply the ratio of the lengths CD, AD into which the wire is divided, hence Q can be found. A convenient apparatus with which to carry out the measurement is that known as the metre bridge (Fig. 235). The metre wire ADC is stretched above a divided scale,

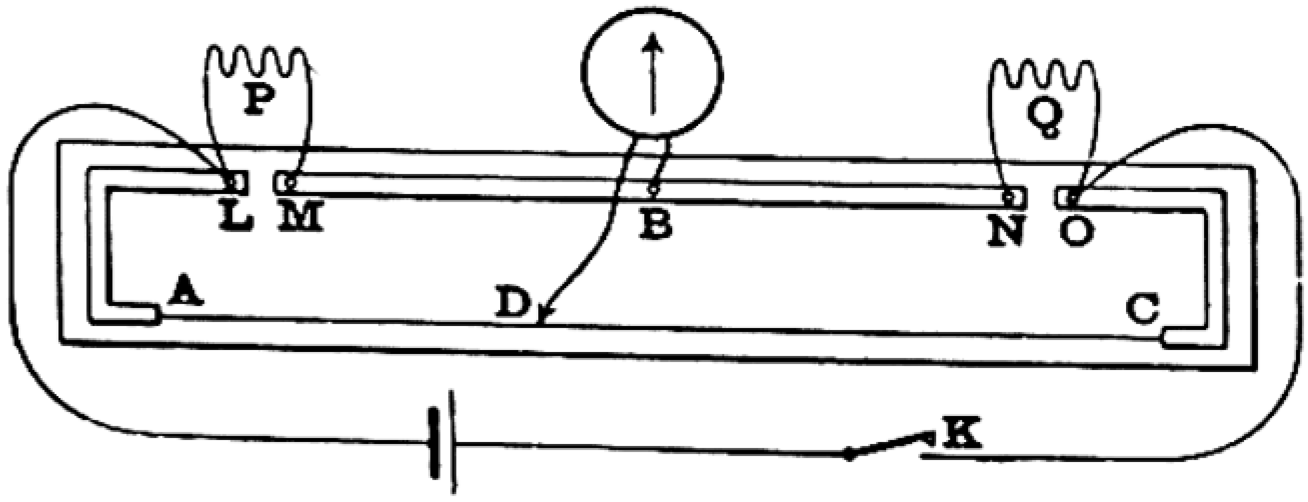


FIG. 235.—Metre Bridge.

thick copper strips of negligible resistance connect A to L and C to O, MN is another such strip. The known resistance P is inserted in the gap LM and the unknown resistance Q in the gap ON. One galvanometer terminal is joined to a screw at B, the other is connected to a key which slides along the wire and can be depressed so as to touch it when required. A battery is joined through a tapping key to L, O. To make a measurement the battery key is *first* depressed, then the galvanometer is put in circuit through its key; if the needle moves the point of contact D is shifted until, on repeating the experiment, no deflexion is obtained, the bridge is then said to be balanced. The advantages of the method are: (1) It is a null method, we have merely to see whether there is a deflexion, not to read one accurately, we are therefore independent of the law of the galvanometer. (2) It is independent of the constancy of the battery, if the current is halved it will be halved in each branch and the balance will be undisturbed. (3) It has a wide range, by altering the ratio

CD/AD resistances much greater or less than P can be found. (4) The resistances are not heated, as the current flows only while the battery key is down.

EXPERIMENT.—Verify with the metre bridge that the resistance of a wire varies directly as its length and inversely as its section. As $R = kl/S$ find the resistivity (p. 374) of the material.

We make the very important deduction from the last experiment that a steady current distributes itself uniformly across the section of the wire, if it were only found at the surface, as in electrostatics, the resistance would vary inversely as the circumference.

EXPERIMENT.—Verify the formulæ already given for resistances in series and parallel.

Thomson's Galvanometer Test.[1]—A galvanometer may be used as the indicating instrument while its own resistance is being measured. For this purpose it takes the place of the resistance to be measured in the arm Q (Fig. 234), and B, D are joined by a wire through a tapping key. When current flows in the bridge the galvanometer shows a steady deflexion, the key at D is then depressed. If B and D are at the same potential no current flows in the wire and the galvanometer deflexion is *unchanged;* as before $Q = RP/S$.

Mance's Battery Test.—The resistance of the battery supplying the current may also be measured. It is placed in one of the arms, say Q (Fig. 234), and A, C are joined by a wire through a tapping key; the galvanometer occupies its usual position between B and D. The battery sends a current through the bridge which produces a steady galvanometer deflexion when the key at D is depressed, this point of contact is shifted until the deflexion is *unchanged* at the moment when A and C are connected through the second key. When this condition is fulfilled, $Q = RP/S$. For various reasons the method is not a good one.

The Post Office Box.—This is another form of Wheatstone's bridge. It is shown in Fig. 236 with the letters to correspond with those of Fig. 234. The three arms AB, AD, and DC are here made of resistance coils, Q is the resistance to be measured. AD and AB usually contain three or more coils whose resistances are 10, 100 and 1000 ohms which can be unplugged in the usual manner; they are

[1] Barton and Black, "Practical Physics," p. 169.

called the ratio arms. The arm DC is altered until a balance is obtained. If AD $= 1000$ and AB $= 10$ ohms, $Q = R \cdot P/S = R/100$, so that the arm CD is 100 times as large as Q, hence the latter can be determined to $\frac{1}{100}$ of an ohm. Similarly if $P = 100\,S$ the largest resistance that can be measured is 100 R.

EXPERIMENT.—Use the P.O. box for Mance's battery test and for Thomson's galvanometer test.

The Potentiometer.—In all the methods given for comparing the E.M.F.'s of cells, except that in which an electrometer is used,

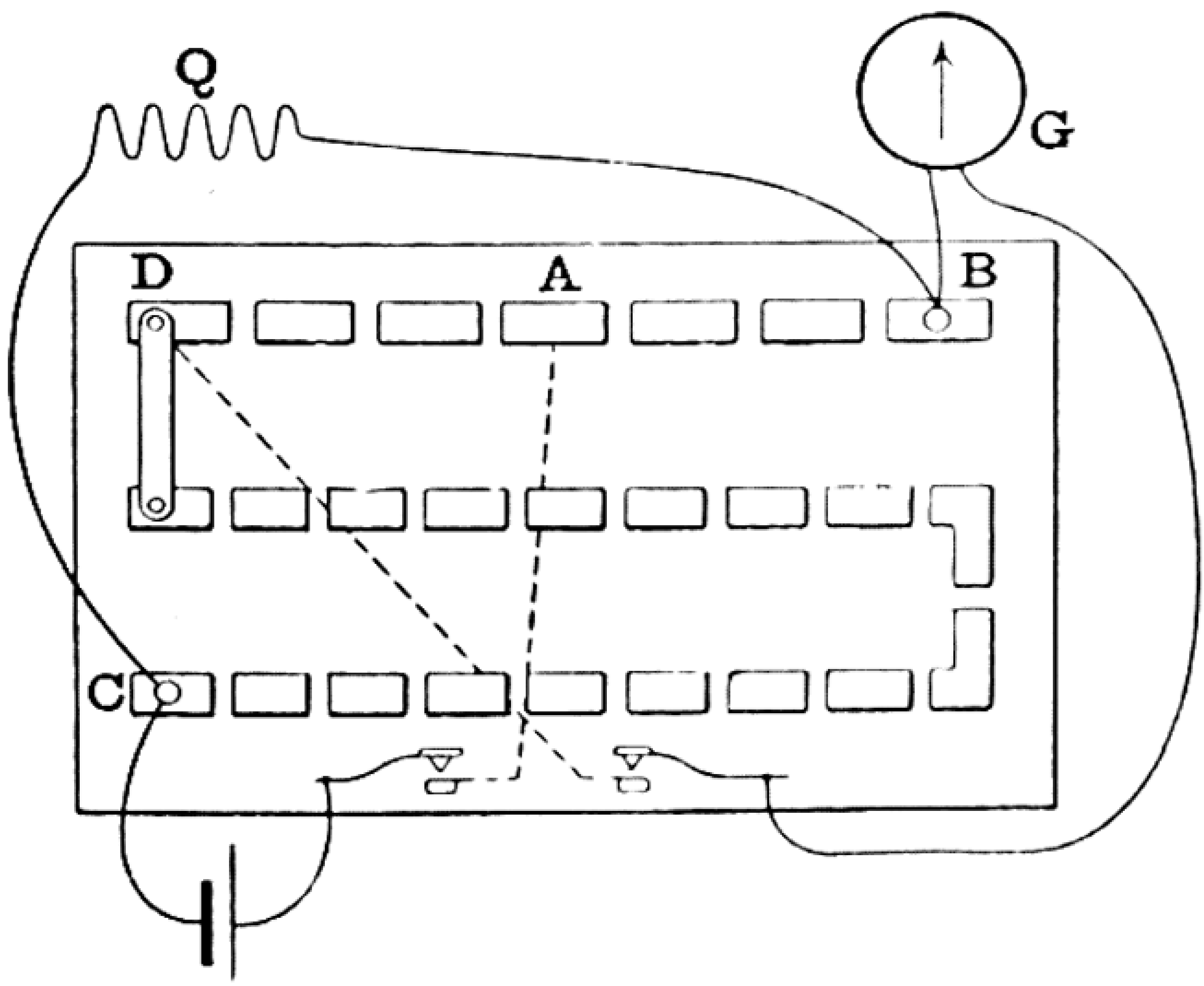


FIG. 236.—Post Office Box (diagrammatic).

it has been assumed either that the battery resistance is so small that it may be neglected, or that it is constant, when it may be eliminated. The potentiometer method of comparing E.M.F.'s, which we will now describe, gets rid of these difficulties and possesses the further advantage that it is a null method. Let a constant cell B (Fig. 237), such as an accumulator (p. 402), of E.M.F. higher than any of the cells to be compared, send a current through an adjustable resistance R and a straight uniform wire PQ. Suppose the positive pole is joined to P. Connect the positive pole of one of the cells whose E.M.F. is required also to P, and join in series with it a galvanometer G and a tapping key to make contact with

the wire at C. On account of the current in the wire P is at a higher potential than C, hence a current tends to flow through the galvanometer in the direction of the arrow; this is opposed by the E.M.F. of the cell S, if its E.M.F. E_1 is equal to the P.D. between P and C, no current will flow when the tapping key is depressed. Hence a point C is found at which the galvanometer shows no deflexion and the length PC, say l_1, is measured on a scale fixed below the wire. S is then replaced by a second cell of E.M.F. E_2, and a new length l_2 is found at which a balance occurs; then $E_1/E_2 = l_1/l_2$. As no

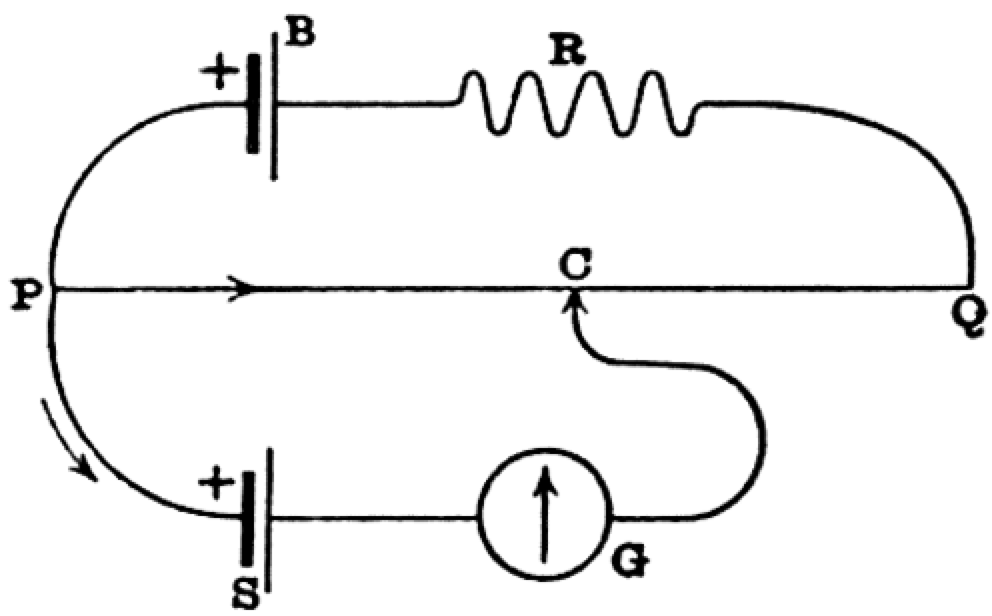


FIG. 237.—Potentiometer Method of comparing E.M.F.'s.

current flows through the cell at the final adjustment the result is independent of its internal resistance. Evidently the P.D. between P and Q must be larger than any of the E.M.F.'s to be compared.

EXPERIMENT.—Use the potentiometer to show that the E.M.F. of a cell does not depend on the size of its plates. To do this make S a Daniell cell, obtain a balance, then withdraw the plates by a certain amount and show the balance is undisturbed.

EXPERIMENT.—Arrange the Daniell cell of the last experiment in series with a tangent galvanometer; if the plates are partially withdrawn or moved further apart the current diminishes. As the E.M.F. is unaltered, the internal resistance must have increased.

These experiments show that the resistance of a cell is increased when the path the current has to traverse in the liquid is made longer or of smaller section. Similar results have already been found for a wire (p. 384).

Cells in Series and Parallel.—The current produced in a circuit depends not only on the E.M.F. of the battery, but also on its internal resistance, and it is possible that an increase in the number of cells may not result in an increase of current. For example, if

the external resistance is very low, the main resistance in the circuit may be that of the battery; if the number of cells in series is doubled, we shall then double both the E.M.F. and the resistance, and the current will be unchanged. In such cases it may be advantageous to group the cells in parallel, or to arrange a number of rows made up of cells in series and then connect these in parallel. Let us calculate the current sent through an external resistance R by a number of cells, the E.M.F. of each being E and the internal resistance B. When n cells are in series the E.M.F. is nE, and the internal resistance of the battery is nB; hence the current

$$A = \frac{nE}{R + nB}$$

If the cells are in parallel, the E.M.F. is E (p. 378), and the internal resistance is B/n, since we have now n resistances in parallel. The current is therefore

$$A = \frac{E}{R + B/n} = \frac{nE}{nR + B}$$

Next suppose there are n rows of cells each consisting of m in series, and suppose these rows are connected in parallel and joined to the external resistance R. The E.M.F. of a row is mE and its resistance is mB, when the rows are in parallel the E.M.F. is unchanged but the resistance is mB/n. Hence the current

$$A = \frac{mE}{R + mB/n} = \frac{mnE}{nR + mB}$$

The number of cells is mn, hence the numerator is constant, and the current is largest when the arrangement is such as to make the denominator a minimum.

Now $$nR + mB = (\sqrt{nR} - \sqrt{mB})^2 + 2\sqrt{mnRB}$$

The last term is constant, hence A is greatest when $(\sqrt{nR} - \sqrt{mB})^2$ is least; since this is a square it cannot be negative, and its minimum value is therefore zero. Hence A is a maximum when

$$(\sqrt{nR} - \sqrt{mB})^2 = 0$$

i.e. when $$nR = mB$$

or $$R = \frac{mB}{n}$$

Thus the largest current is obtained when the cells are arranged in such a way as to make the external resistance equal to the internal resistance of the battery. A useful rule to remember is that the cells should be placed in series when the external resistance is large.

Ballistic Galvanometer and Comparison of Capacities.—For many purposes galvanometers are required to measure the total quantity of electricity that passes through them during a very short discharge; instruments of this type are called ballistic galvanometers. They differ from the ordinary astatic galvanometer in having a heavy needle, which swings to and fro very slowly with very little decrease in the amplitude of its oscillations. Provided (1) that the deflexion is small, and (2) that the whole discharge passes through the coils before the needle moves appreciably, it can be shown that the total quantity is proportional to the first swing of the needle. Such a galvanometer can be used to compare the capacities of condensers and also E.M.F.'s of cells. Let a condenser of capacity C be charged by being connected to the poles of a battery whose E.M.F. is E; the charge is $Q = EC$ (p. 355). Hence if the same battery is used to charge different condensers, whose capacities are C_1 and C_2, and they are then discharged through a ballistic galvanometer, producing deflexions d_1 and d_2,

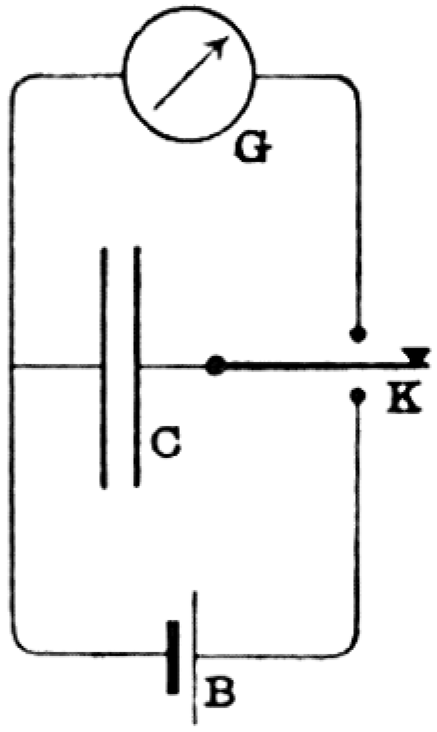


FIG. 238.—Comparison of Capacities.

$$\frac{Q_1}{Q_2} = \frac{EC_1}{EC_2} = \frac{d_1}{d_2}$$

or

$$\frac{C_1}{C_2} = \frac{d_1}{d_2}.$$

If the same condenser is used, but different batteries, we can compare the E.M.F.'s, getting, as the result, $E_1/E_2 = d_1/d_2$. Fig. 238 shows the apparatus used. When the Morse key K is depressed the condenser C is charged from the battery B. If the key is then released the poles of the condenser are joined through the galvanometer G and a "throw" is produced. The practical unit of capacity is the micro-farad. **When one coulomb of electricity raises the P.D. between the plates of a condenser by one volt, the capacity is called a farad.** The micro-farad is one-millionth of this.

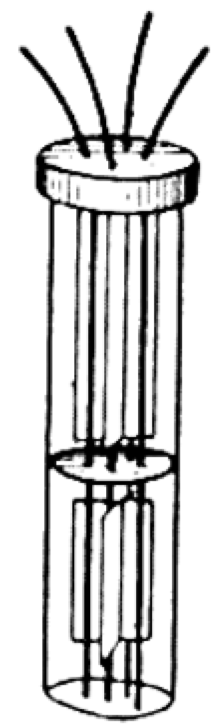
Volt-ammeter Method of measuring Battery Resistance.—This method has the advantage of giving results under definite conditions with regard to the current that is passing. Connect the battery in series with a regulating resistance and an ammeter (or a tangent galvanometer of known reduction factor). Let R be the resistance of the external circuit, B the battery resistance, and A the current.

Then $$E = A(B + R)$$

where E is the E.M.F. of the cell. Now, this E.M.F. is spent partly in driving the current through the resistance R and partly in sending it through the resistance B. The P.D. at the ends of the external resistance is AR, by Ohm's law, calling this e

we have $$e = AR$$
hence $$E - e = AB$$

If a voltmeter V is also joined to the poles it reads e when the current is running. When the resistance R is broken the cell is on open circuit, as the voltmeter has such a high resistance that very little current passes, the voltage reading is therefore E (p. 359). Also in the first case the ammeter gave the current A, hence B can be calculated from the last equation.[1]



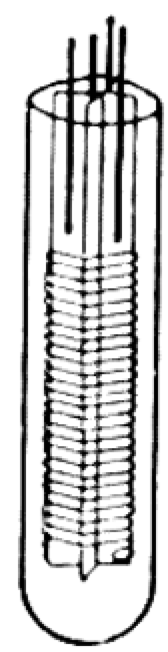


FIG. 239.—Platinum Thermometer.

Platinum Thermometer.—It has been stated that the variation of the resistance of a pure metal with temperature is used for purposes of temperature measurement. A piece of pure, well-annealed, platinum wire is fused to thick copper leads and is then wound in a coil on a framework of mica or quartz (Fig. 239). This is enclosed in a protective tube of quartz or porcelain which is immersed in the substance whose temperature is required; the resistance of the platinum is measured in a Wheatstone's bridge. By special means the resistance of the leads can be eliminated. Assuming (p. 374) that the resistance is given by the formula $R = R_0(1 + \alpha t)$, the constant α can be obtained by measuring the resistance at the two fixed temperatures 0° C. and 100° C. If then the resistance is measured at

[1] Barton and Black, "Practical Physics," p. 170, for another method

some unknown temperature t, this temperature can be calculated. The advantages of resistance thermometers are the constancy of the zero (p. 24), the ease and accuracy with which the measurements can be made, the wide range of temperature over which they can be used, and their adaptability to different conditions. With respect to the last point, for example, the temperatures of a number of works furnaces can be measured in a central office, and, by photographic means, a continuous record can be obtained.

EXPERIMENT.—Measure the resistance of a coil at 0° and at a series of other temperatures. The Wheatstone bridge method should be used. Calculate the temperature coefficient from the equation $R = R_0(1 + at)$.

EXAMPLES ON CHAPTER XXXIV

1. How would you arrange 36 cells, each having a resistance 1·6 ohms, so as to send the strongest possible current through an external resistance of 5·6 ohms? (L. '82.)

2. A cell of 40 ohms internal resistance is connected by thick wires with the terminals of a tangent galvanometer, formed by a single ring of stout copper wire. The deflexion is 45°. Three similar cells are then connected in series with the first. What is the deflexion? Would any other arrangement of the four cells give a stronger current, and why? (L. '83.)

3. Three cells, A, B, C, whose E.M.F.'s are 1·07, 1·54, and 1·9 volts, and resistances 0·72, 2·3, and 0·1 ohms respectively, are connected in series, and the circuit is completed by a resistance of 5·9 ohms. Find the current. If the cell B were reversed, what would be the current? (L. '85.)

4. The P.D. between the poles of a battery (of 1·2 ohms resistance) is 6 volts when the poles are insulated, and 4·5 volts when they are joined by a wire. What is the resistance of the wire? (L. '86.)

5. There are 25 turns of wire in a galvanometer coil, the mean radius of which is 150 cms. Assuming H to be 0·18, find the current which will deflect a magnet, placed at the centre of the coil, 45°. If the resistance of the circuit, including the battery, is 3 ohms, find the E.M.F. necessary to produce the current. (L. '88.)

6. A length of uniform wire, of resistance 12 ohms, is bent into a circle, and two points at a quarter the circumference apart are connected with a battery whose resistance is 1 ohm and E.M.F. 3 volts. Find the current in the different parts of the circuit. (L. '89.)

7. What resistance should a wire have, which, when connected across the terminals of a galvanometer whose resistance is 3663 ohms, would let $\frac{1}{100}$ of the current pass through the galvanometer? (L. '91.).

8. Given an ammeter and voltmeter, explain how you would find the resistance of a glow lamp while glowing. Give a diagram of the arrangements. (L. '95.)

9. A tangent galvanometer in series with a battery shows a deflexion of 60° ; on introducing a resistance of 15 ohms the deflexion falls to 45°. Calculate the resistance of the circuit. (L. '05.)

10. A battery, of negligible resistance and an E.M.F. of 4 volts, is connected to the opposite corners A and C of a quadrilateral wire frame ABCD. The resistance of the side AB = 90 ohms, of BC = 110 ohms, of CD = 60 ohms, and of DA = 40 ohms. Calculate the P.D. between the points B and D. (L. '06.)

CHAPTER XXXV

CHEMICAL EFFECTS OF CURRENTS

Terms Used.—It has been shown by the experiments on p. 361 that certain liquids, called electrolytes, are conductors of electricity and that the passage of a current through them causes decomposition. We will now consider these cases in more detail. A number of new terms will be required, most of which were first used by Faraday. The apparatus in which the decomposition takes place is called a voltameter or electrolytic cell; the conducting plates by means of which the current is led into and out of the liquid are called electrodes, the plate by which the current enters is called the anode or positive electrode, that by which it leaves is called the kathode or negative electrode. The decomposition is termed electrolysis and its products are called ions. The ions are released only at the electrodes; those liberated at the anode are named anions, those released at the kathode kations. It does not follow, however, that the ions will accumulate in the form in which they are released, for further chemical reactions may take place between them and the electrodes or the undecomposed electrolyte; these are called secondary reactions. Thus when sodium sulphate solution is electrolysed, the anion is the SO_4 group, and the kation is sodium, but each of these reacts with water according to the equations

$$2Na + 2H_2O = 2NaHO + H_2$$
$$SO_4 + H_2O = H_2SO_4 + O$$

The final products are therefore hydrogen and oxygen in the proportions to form water, while caustic soda and sulphuric acid accumulate at the kathode and anode respectively, as may be shown by appropriate chemical tests. The acid radical of the compound forms the anion and the base the kation.

Faraday's Laws of Electrolysis.—In order to study the quantitative laws, various types of voltameter are required.

(*a*) *Copper voltameter.* EXPERIMENT.—Clean and weigh two copper plates 10×5 cms. in area, and suspend them in a 20 per cent. solution of copper sulphate to which 1 per cent. of sulphuric acid has been added. Send a current of one ampere through the solution for at least 30 minutes; remove the plates and at once wash them in distilled water and dry them, first with blotting paper, and then at some distance above a gas flame. Now reweigh them; the kathode shows an increase and the anode an almost exactly equal decrease in weight.

Decomposition has taken place in accordance with the equation $CuSO_4 = Cu + SO_4$, copper is deposited on the kathode while the SO_4 attacks the anode and reforms copper sulphate. The copper released is thus found by direct weighing of the kathode; this is found to give more consistent results than the decrease in weight of the anode. A more adherent layer of copper is obtained if the current entering each cm.^{2} of the kathode is small, the anode is therefore usually formed of two plates joined together, and the kathode is placed between them so that each side receives current.

(*b*) *Silver voltameter.*—This is preferable for accurate work as the weight deposited is greater (p. 395). The electrolyte, a neutral solution of silver nitrate, is placed in the kathode, which takes the form of a platinum bowl; a horizontal disc of silver forms the anode. To prevent particles of silver falling into the bowl the anode is wrapped in filter paper.

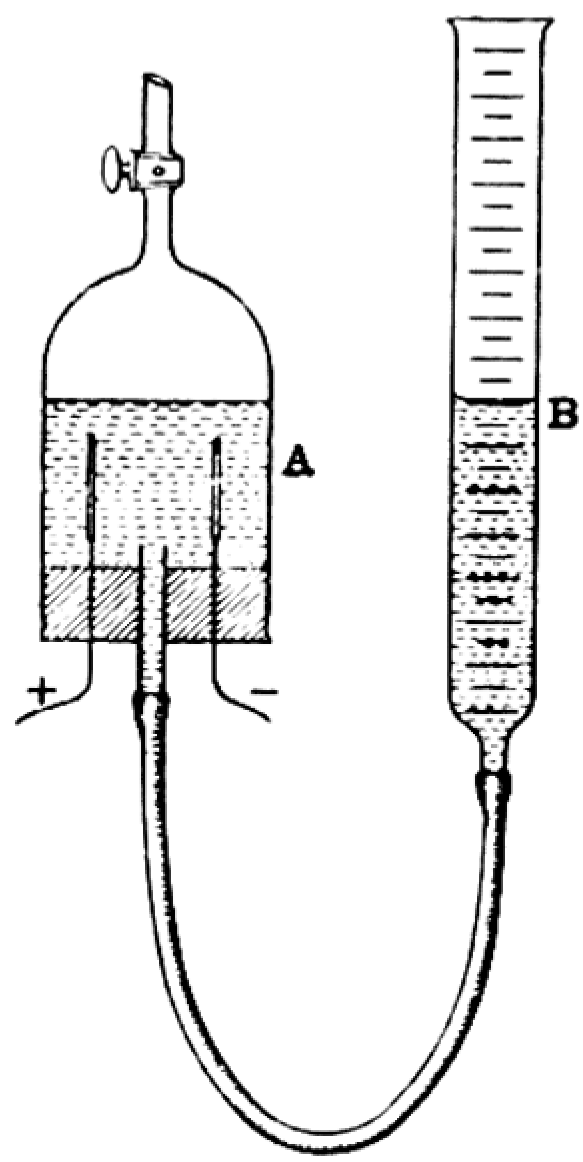


FIG. 240.—Water Voltameter.

(*c*) *Water voltameter.*—A form which can be easily set up is shown in Fig. 240. The glass vessel A, closed at the bottom by a rubber stopper, contains a 15 per cent. solution of caustic potash in which two nickel electrodes are immersed. It communicates by rubber tubing with the graduated tube B. When the liquid surfaces are at the same level the gas in A is at atmospheric pressure. If a current is passed

hydrogen and oxygen are released, the ions being potassium and (HO). At the electrodes the following reactions take place :—

$$2K + 2H_2O = 2KHO + H_2$$
$$2HO + H_2O = 2H_2O + O$$

Tube B is lowered until the gas in A is again at atmospheric pressure, when the liquid displaced by the evolved gas runs into B and may be measured. The temperature and pressure being observed, the mass of either gas can be found as in Chap. VI, since a litre of hydrogen at N.T.P. weighs 0·0896 gm.

When a number of voltameters are placed in series in a circuit it is found that **the masses of the ions deposited are proportional to their chemical equivalents.** Thus in the three voltameters just described, the amounts of H, O, Cu and Ag released by the current are as $1 : \frac{16}{2} : \frac{63}{2} : 108$. It should be remembered that the chemical equivalent of an element is its atomic weight divided by its valency, and that the valencies of oxygen and copper in water and cupric sulphate respectively are each two. The amount of copper deposited from a cuprous salt is twice that released from a cupric salt, since the valency of copper is then unity. It is important to discover how the mass of the substance deposited varies with the current.

EXPERIMENT.—Set up in series a tangent galvanometer, shunted if necessary, a copper voltameter, regulating resistance, and cells capable of sending 1–2 amps. round the circuit. Keep the current constant and determine how much copper is deposited in 30 minutes. Repeat the experiment using a current half as large and running it twice as long. It will be found that the same mass is deposited in the two cases. The measurements can be made more quickly with a water voltameter.

The experiment shows that the mass released is proportional to the quantity of electricity that has passed. If $m =$ mass deposited by a current of A amperes in t secs., $m \propto At$ or $m = Awt$, where w is a constant for the given substance. This constant is evidently the mass deposited by 1 ampere running for 1 sec., *i.e.* by 1 coulomb of electricity ; it is called the ampere-electro-chemical equivalent of the element in question. It follows from what has been written above that the electro-chemical equivalents of the elements are proportional to their chemical equivalents.

These results are embodied in Faraday's laws of electrolysis : **The masses of the ions released are proportional (1) to the quantity of**

electricity that passes round the circuit, and (2) to the chemical equivalents of the ions.

The ampere-electro-chemical equivalent of silver has been found to be 0·001118 gm., hence that of any other element can be calculated from (2). Thus if x is the electro-chemical equivalent of copper in a cupric salt $x : 0{\cdot}001118 = \frac{63}{2} : 108$. If follows from (2) that the quantity of electricity necessary to separate a gram-equivalent [1] is the same for every element and is equal to the number of coulombs required to deposit 108 gms. of silver. Let it be represented by y.

Then to deposit 0·001118 gms. of silver requires 1 coulomb

and to deposit 108 ,, ,, $\frac{108}{0{\cdot}001118}$ coulombs,

or $y = 96{,}500$ coulombs approximately.

Experiment.—Assuming the electro-chemical equivalent of copper, the laws of electrolysis can be used to find the reduction factor of a tangent galvanometer. The copper voltameter of the last experiment is joined in series with a resistance and a battery, and a constant current of about 1 ampere is passed for 30 minutes. The $m = A wt = k \tan\theta \, . \, wt$ or $k = \frac{m}{\tan\theta \, . \, wt}$.

Conversely if k is known the electro-chemical equivalent w can be found.

Dissociation Theory.—To explain the facts set out above it is assumed that each ion carries with it a definite positive or negative charge, and that current is conveyed through the liquid by the movement of these charged ions, the kation carrying a positive charge in the direction of the current and the anion an equal negative charge in the opposite direction. If it is assumed that the hydrogen and copper ions are the same as the hydrogen and copper atoms respectively, then since 63 gms. of copper contain as many atoms as 1 gm. of hydrogen, while 31·5 gms. of copper and 1 gm. of hydrogen each carry 96,500 coulombs, it follows that 63 gms. of copper carry twice the amount of electricity carried by 1 gm. of hydrogen, and therefore the charge on a divalent copper atom is twice that on an atom of hydrogen. Similarly a trivalent ion carries a charge three times as large and so on. Since the elements deposited at the kathode carry a positive charge they are called electro-positive, likewise those separating at the anode are negatively charged and

[1] A gram-equivalent of a substance is its chemical equivalent expressed in grams. Thus the gram equivalents of silver and copper are 108 and 31·5 gms. respectively.

are called electro-negative. The question arises, How do the ions acquire their freedom to move? How, for instance, does the copper ion become separated from the SO_4 ion? Older theories supposed that the electric field between the electrodes pulled the oppositely charged ions apart. If that were the case no current should pass until a certain minimum E.M.F. is reached, large enough to cause this disruption of the molecule, when a vigorous decomposition should begin accompanied by a strong current. Actually the electrolyte obeys Ohm's law and the current is proportional to the E.M.F. for all values of the latter. The modern dissociation theory supposes that when the salt is dissolved some, or all, of its molecules are dissociated into $Cu^{\cdot\cdot}$ and SO_4'' ions,[1] which move freely in the liquid; the molecules in this condition are said to be ionised. The function of the applied E.M.F. is then merely to cause a drift of the ions up to the electrodes, where their charges are either given up or neutralised. In the case of such an ion as $Na^{\cdot}$ it may be asked why it does not react with the water and form caustic soda. The theory supposes that the ordinary chemical properties of substances are modified by the presence of the charges on the ions, directly these are given up to the kathode the sodium resumes its usual chemical activities and decomposes water. Measurements of the electrical conductivity of strong and weak solutions show that a larger proportion of the salt molecules are ionised as the solution becomes more dilute. Only those molecules which are ionised contribute to the conductivity of the liquid.

Voltaic Cells.—When a simple cell, such as that on p. 359, supplies current A (E.M. units) for a time t, a quantity of electricity At falls through a P.D. equal to the E.M.F. of the cell; energy equal to AEt ergs is therefore furnished by the cell, E being in E.M. units (p. 365).

Experiment.—Place a rod of pure zinc in dilute sulphuric acid; very little, if any, chemical action takes place. Let a piece of copper touch it in the midst of the liquid, the zinc is now vigorously dissolved and heat is evolved. If impure zinc is used solution begins at once near the impurities; this is called local action, and may be stopped by amalgamating the zinc with mercury.

Experiment.—Separate the metals and the action ceases, but it can be restarted by joining the two metals by a wire *outside* the liquid. In this case we know a current flows along the wire and heats it.

---

[1] The dots are used to indicate that the Cu ion carries two positive charges, the dashes denote negative charges.

Careful calorimetric measurements show that the total quantities of heat developed in these two experiments are equal, if the same masses of zinc are dissolved in each case. In the first experiment the heat appears as a rise in temperature of the cell alone; in the second it is initially converted into electrical energy, which, in turn, reappears as heat in the cell and the *connecting wire*. According to this view the electrical energy supplied by the battery arises from the heat liberated when zinc is oxidised (see p. 400). Now, the current passes through the cell by the movement of ions exactly as in a voltameter, the ions being $H^{\cdot}$ and SO_4'', hydrogen therefore appears at the copper plate, where the current leaves the liquid. If it is allowed to collect there it causes a decrease in the current, for

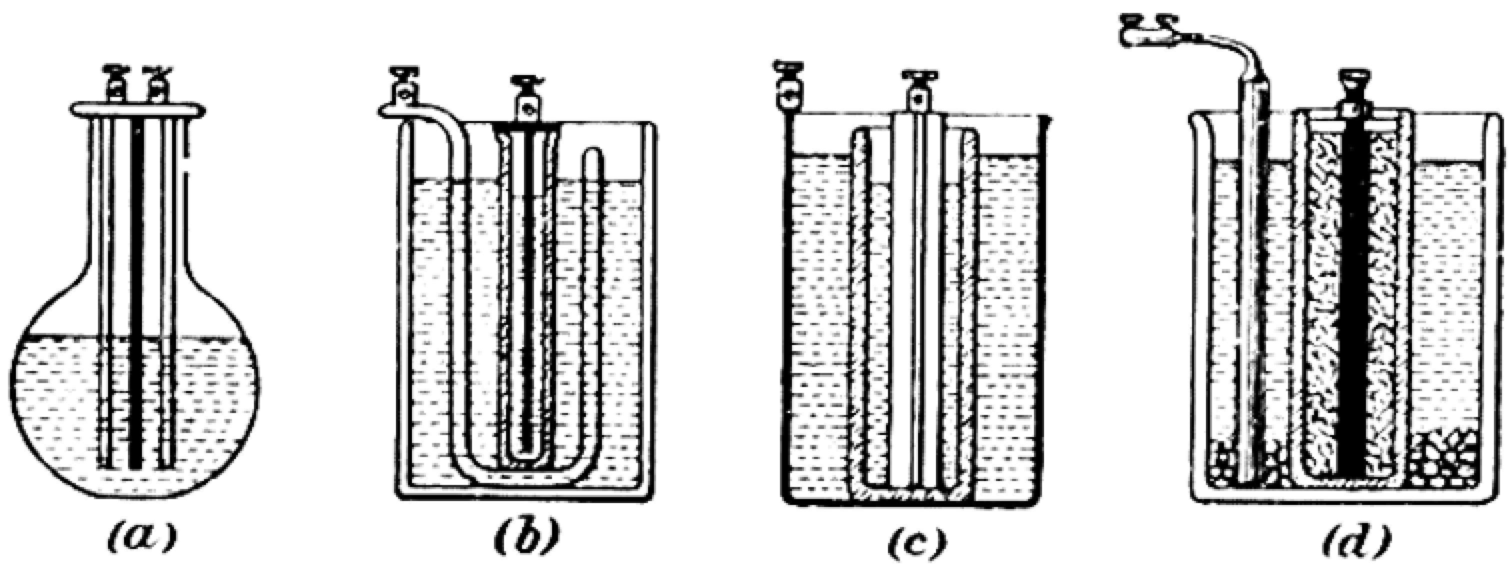


FIG. 241.—Various Types of Cells.

(1) this layer of gas has a large resistance, and (2) it is readily oxidisable and tends to set up an E.M.F. in a direction opposite to that of the cell. The cell in this condition is said to be polarised. This back E.M.F. may be readily shown.

EXPERIMENT.—Quickly replace the zinc of a polarised cell by a clean sheet of copper, and include in the circuit a galvanometer. A current flows from the new to the original copper through the connecting wire, *i.e.* in the opposite direction to the original current.

In designing a voltaic cell means must be taken to remove this hydrogen layer, and the plates should be large and close together so that the internal resistance of the cell may be small.

Bichromate Cell.—The positive pole here consists of two carbon plates joined together, between them is placed the negative zinc plate; the current in the cell has therefore a choice of two short, wide paths and the internal resistance is small (Fig. 241, *a*). The

electrolyte is a solution of chromic and sulphuric acids, sometimes the former is replaced by potassium bichromate. Zinc is dissolved according to the equation $Zn + H_2SO_4 = ZnSO_4 + H_2$. The released hydrogen travels with the current to the carbon plates, where it is oxidised by the chromic acid to form water. The E.M.F. is high, about 2 volts, but it is inconstant as the depolarising action is not very efficient.

The Grove Cell.—This is shown in section in Fig. 241, *b*. A bent plate of zinc, giving a large surface, is immersed in dilute sulphuric acid; standing on this is a vessel of porous earthenware which contains strong nitric acid. A sheet of platinum, forming the positive pole, is immersed in the latter liquid. In this case the hydrogen travels from the zinc through the porous pot and is oxidised by the nitric acid when it reaches the platinum. The function of the porous pot is to keep the two liquids separate. The E.M.F. of the cell is 1·9 volts and is fairly constant; its disadvantages are the costliness of the platinum and the disagreeable fumes from the nitric acid.

The Bunsen Cell.—The components are the same as for the Grove cell, except that the platinum is replaced by a stick of carbon thereby reducing the cost.

The Daniell Cell.—This also is a two-fluid cell. The negative pole is a zinc cylinder which is immersed in dilute sulphuric acid contained in a porous pot (Fig. 241, *c*). This pot stands in an outer vessel which contains a strong solution of copper sulphate, the positive pole, consisting of a sheet of copper, is immersed in the latter liquid. When the external circuit is closed zinc goes into solution as zinc sulphate, the hydrogen ion travels through the cell and finally replaces the copper of the copper sulphate, thus forming sulphuric acid. The replaced copper is deposited on the positive plate of the cell. The outer vessel may be made of copper, it then forms the positive pole. The E.M.F. is very constant at about 1·1 volts provided the current taken is very small, but, like all cells containing a porous partition, the resistance is high and may amount to several ohms. In all the above cells the zinc should be well amalgamated to prevent its useless solution by local action.

The Leclanché Cell.—This type is shown in Fig. 241, *d*. It contains only one electrolyte, a solution of ammonium chloride (NH_4Cl). The negative pole is a zinc rod, the positive consists of a plate or rod of carbon packed round in a porous vessel with a mixture of

manganese dioxide and powdered carbon. The purpose of the carbon is to increase the conductance of the mixture. When a current passes through the circuit Cl′ moves to the zinc and forms zinc chloride, while NH_4 travels to the carbon where it breaks up into ammonia (NH_3) and hydrogen. Ammonia goes into solution and the hydrogen is oxidised to form water. The cell has an E.M.F. of 1·4 volts, but it polarises rather rapidly; after a short rest, however, it recovers. Its great advantage over the preceding forms, in addition to its cheapness, is the fact that no consumption of zinc takes place except when a current is passing, it is therefore greatly used for testing purposes in the laboratory and for house bells.

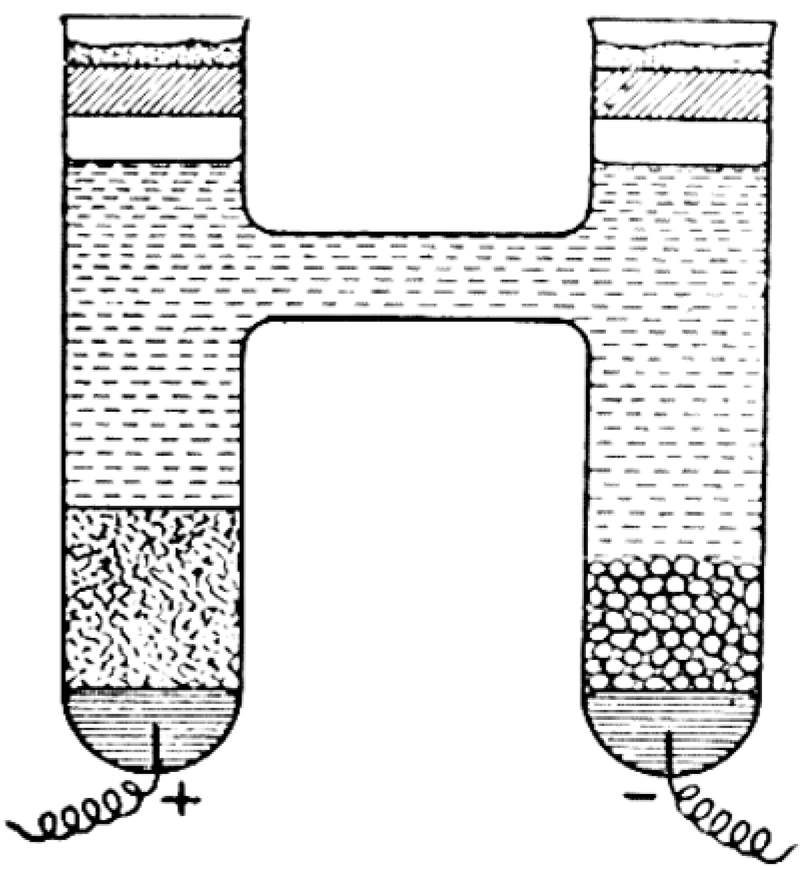


FIG. 242.—H-form of Clark Cell.

The Clark Cell.—This cell is used as a standard of E.M.F. and not as a source of current. The H-form is shown in Fig. 242. The glass tube on the left contains a small quantity of mercury, which forms the positive pole. Above this is a paste, made by mixing mercurous sulphate with a saturated solution of zinc sulphate, and above this again is a further quantity of the zinc sulphate solution. At the bottom of the other limb is the negative pole, consisting of an amalgam containing 10 per cent. of zinc; this is covered with a layer of zinc sulphate crystals. Connection is made with the outside by platinum wires fused through the glass, and the cell is closed above with corks which are covered with layers of marine glue. It is found that the zinc amalgam gives the same E.M.F. as a rod of pure zinc, the latter is used in some forms. At 15° C. the E.M.F. is 1·4322 volts.[1] Its disadvantage as a standard is its somewhat rapid variation with temperature, for this reason it is gradually being replaced by the cadmium cell. This is made in the same form as the Clark, but the zinc amalgam is replaced by one of cadmium, and cadmium sulphate is used instead of zinc sulphate.

[1] The *legal* voltage, which is taken as the standard of E.M.F. for commercial purposes, is 1·434 volts at 15° C.

Its E.M.F. is 1·0183 volts at 20° C. and it varies little with temperature.

Calculation of the E.M.F. of a Daniell Cell.—On the assumption that the energy of a cell arises from the chemical actions going on in it, the E.M.F. can be calculated. When a charge Q, E.M. units, falls through a potential V, E.M. units, the work done by the field is QV ergs. Suppose that 96,500 coulombs are driven round a circuit by a Daniell cell whose E.M.F. is E volts. Expressing these in E.M. units the work done is $96{,}500 \times 10^{-1} \times E \times 10^{8}$ ergs. But the passage of this electricity is accompanied by the solution of 1 gm.-equivalent of zinc and the deposition of 1 gm.-equivalent of copper. Now, it is known from thermo-chemical data that the former causes an evolution of heat amounting to 19,000 cals., while the replacement of the copper by hydrogen and its consequent deposition on the copper plate involves a further evolution of 6200 cals. The total heat available for conversion into electrical energy is thus 25,200 cals. or $25{,}200 \times 42 \times 10^{6}$ ergs. This must equal the work done by the cell during the passage of 96,500 coulombs.

Hence $$96{,}500 \times 10^{-1} \times E \times 10^{8} = 25{,}200 \times 42 \times 10^{6}$$
or $$E = 1·09 \text{ volts}$$

This is practically equal to the E.M.F. as found experimentally. A similar calculation for other cells is not so satisfactory; Helmholtz has shown that other factors must be taken into consideration in these cases. The above calculation shows that part of the E.M.F. arises from the heat evolved during depolarisation; this partly explains the high E.M.F. of Grove and Bunsen cells, since a large amount of heat is evolved when hydrogen is oxidised to form water. It can now be shown that a single Daniell cell is incapable of decomposing water. Suppose such a cell in series with a water voltameter, and let, if possible, 96,500 coulombs pass round the circuit. This will be accompanied by the solution of a gm.-equivalent of zinc and the release of a gm.-equivalent of hydrogen. Thermo-chemical experiments tell us that the release of the hydrogen will require energy amounting to 34,000 cals., while, as we have just seen, the cell can only supply 25,200 cals.; hence it cannot supply the necessary energy, and decomposition of the water will not take place.

Polarisation E.M.F. in Voltameters.—Whenever chemical work is done in a voltameter, as in the separation of hydrogen and oxygen,

the separated products tend to recombine again; this gives rise to a back E.M.F., tending to send a current in the reverse direction through the solution. The electrodes are then said to be polarised.

EXPERIMENT.—In Fig. 243 A represents a water voltameter, B a battery of two accumulators, K a Morse key, and G a voltmeter or high resistance galvanometer. First depress the key for a few seconds to connect the battery with the voltameter; now release K, the electrodes are then connected through the galvanometer and a current passes in the direction of the arrow C if the arrow D represents the direction of the original current. Hence the second current passes through the voltameter in the opposite direction to the current from the accumulator.

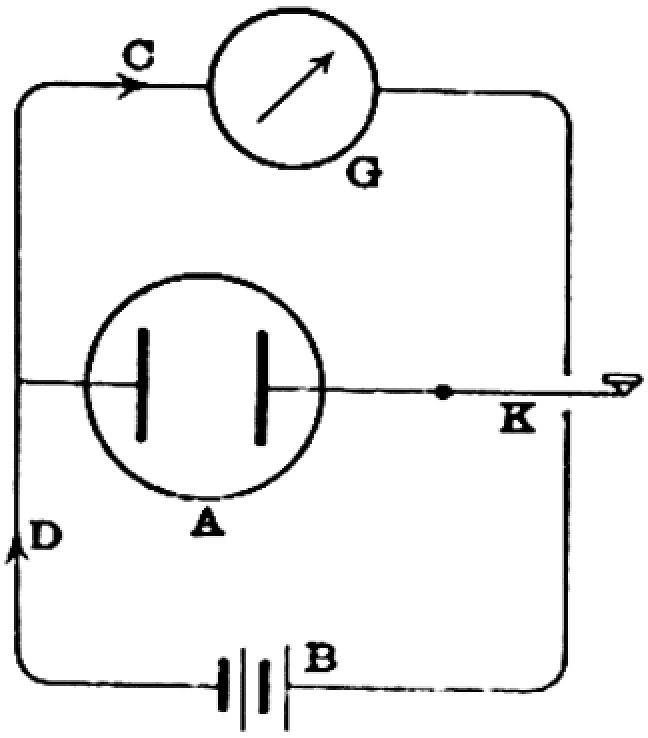


FIG. 243.—Apparatus to show back E.M.F. in a Voltameter.

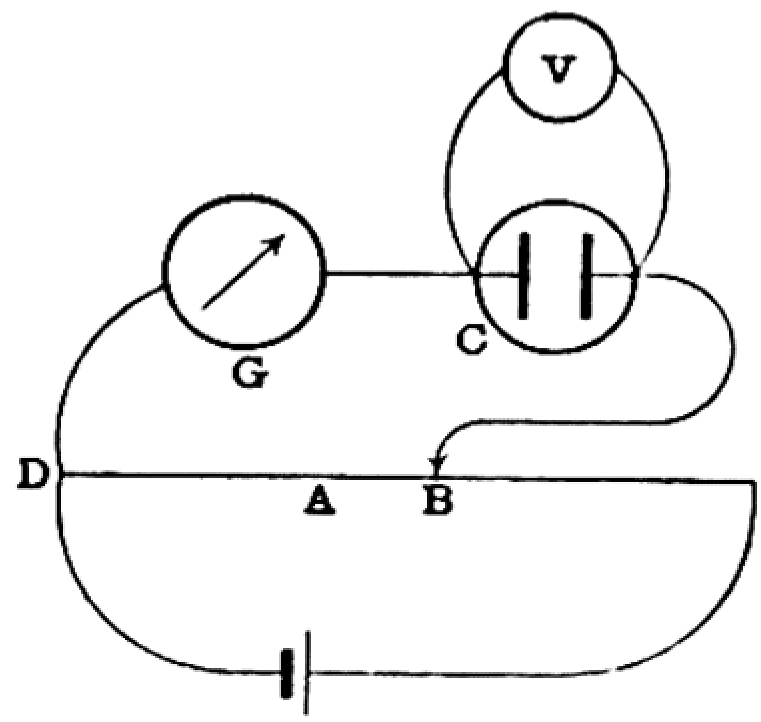


FIG. 244.—Apparatus to determine the E.M.F. required to decompose Water.

If an E.M.F. less than 1·67 volts is applied to a water voltameter in series with a galvanometer a current at first flows, but it rapidly falls away to a very small value and no bubbles of gas can be seen at the electrodes. The small amount of gas that is produced is absorbed by the electrodes, and its tendency to go into solution again creates the back E.M.F. If the applied E.M.F. is greater than 1·67 volts the current may decrease at first, but it will never become practically zero. This can be shown as follows:—

EXPERIMENT.—Join the poles of an accumulator to the ends of a thin manganin wire, A, 2 m. long (Fig. 244). B is a movable contact piece, G a galvanometer, C a water voltameter with clean platinum electrodes, and V a voltmeter. Place B near D, note the small current in G after 1 min., and also the voltmeter reading; the latter will be small since B and D are at nearly the same potential. Repeat these observations as B is gradually moved to the right, so increasing the E.M.F. applied to the voltameter. The voltmeter should be cut out by

a key when the galvanometer is being read, otherwise the latter instrument will measure the joint current going through the branches V, C. If the voltmeter readings are plotted as abscissæ, with the currents as ordinates, the curve will be found to bend upwards sharply at about 1·67 volts, showing that this is the minimum E.M.F. required to decompose water.

No chemical work is done in a copper voltameter with copper plates, for a certain mass of copper is dissolved from the anode and an equal mass is deposited on the kathode; no back E.M.F. exists in such cases, and the smallest E.M.F. produces a current.

Applications.—The principles explained in this chapter have numerous applications. For example, articles made of a baser metal can be electro-plated, *i.e.* covered with a layer of silver, by placing them in a silver nitrate solution and making them the kathode of an electrolytic cell. Impure copper is purified in a similar manner. An ingot of impure copper, obtained by smelting the ore, is made the anode of a copper voltameter, the kathode is formed from a sheet of pure copper; the metal is carried through the solution by the current and deposited as pure copper on the kathode (p. 393).

Caustic soda is obtained from common salt by similar means. A solution of salt is electrolysed using a mercury kathode, the sodium forms an amalgam with the mercury, and this is allowed to act upon water, forming caustic soda. In one form of commercial current meter the current passes through a solution of a mercury salt, the mercury released at the kathode falls into a measuring vessel and hence the total electricity passed in a given time can be found. This apparatus is used to measure the current supplied in lighting a building.

The Accumulator or Storage Cell.—The chemical energy stored in an electrolytic cell can be regained as electrical energy; this is the principle of the storage cell. Two lead plates are formed into grids and the interstices in them are packed with a paste of lead sulphate, formed by mixing litharge (PbO) with dilute sulphuric acid. They are then immersed in dilute sulphuric acid and a current is sent through from an external source, *e.g.* a dynamo (p. 458). The hydrogen released at the kathode reduces the sulphate to a mass of spongy metallic lead and sulphuric acid is formed. The SO_4'' ions travel to the anode and form lead peroxide according to the equation

$$PbSO_4 + 2H_2O + SO_4 = PbO_2 + 2H_2SO_4$$

When all the lead sulphate has been changed in this manner hydrogen

and oxygen will be released, as in the water voltameter, and will escape. At this stage the plates are covered with Pb and PbO_2 respectively and the cell is fully charged. If it is now connected to a conducting circuit a current will flow through the cell in the opposite direction to that used in charging it, and the chemical changes will also proceed in the reverse direction, lead sulphate being formed on each plate. The cell can be recharged as often as is desired. Its E.M.F. is about 2 volts and is very steady, while its internal resistance is low. It is most commonly used when a constant current is required.

EXAMPLES ON CHAPTER XXXV

1. Ten cells, each of internal resistance 2 ohms and E.M.F. 1·5 volts, are connected (*a*) in series, (*b*) in two series of 5 each, the like ends of the two series being joined together. The external resistance is 10 ohms; find what is the strength of the current in this resistance in each case and compare the rates of consumption of zinc. (L. '84.)

2. A certain tangent galvanometer has a current passed through it which deflects its needle 45°. The same current passes through a copper sulphate cell, where it deposits 0·3 gm. of copper in 30 minutes. Taking the electro-chemical equivalent of copper as 0·00033 gm./coulomb, find the value of the current, and show how to determine the current for any other reading of the galvanometer. (L. '95.)

3. A small accumulator has a "capacity" of 14 ampere-hours. [That is, it can give A amps. for t hrs., where $At = 14$.] What is theoretically the least weight of PbO_2 on its positive plates; given that the PbO_2 is reduced to PbO and that (*a*) the electro-chemical equivalent of H is 0·00001038 gm./coulomb, (*b*) atomic weight of lead is 207 and of oxygen 16? Calculate also how much heat is evolved during the whole discharge of this cell through a total resistance of 10 ohms, the average E.M.F. being 2 volts. (L. 1900.) [For heating effects see next chapter.]

4. Five cells, each of E.M.F. 2 volts and resistance 0·04 ohm, are arranged in series, and drive a current between platinum electrodes immersed in dilute sulphuric acid. The resistance of the acid between the electrodes is 4 ohms, and the E.M.F. of polarisation is 1·5 volts. Calculate the mass of water decomposed in 1 hour. Elec.-chem.-equiv. of hydrogen is 10^{-5} gms./coulomb. (L. '05.)

5. A battery of 6 Daniell cells in series sends a current through a solution of silver nitrate. Find the amount of zinc dissolved in each cell while 1 grm. of silver is deposited. (At. wt. of silver = 108, of zinc = 65.) (L. '10.)

6. In question 5 calculate the amount of zinc dissolved in each cell when the cells are arranged (1) in parallel, (2) in two sets of 3 in series, the two sets being connected in parallel.

CHAPTER XXXVI

HEATING EFFECTS OF CURRENTS

Joule's Law.—Instances have been given on p. 361 of the heating effect of a current on the conductor through which it is passing; we will now consider these thermal effects more fully.

EXPERIMENT.—Join thin copper, iron, and manganin wires, of equal diameters, in series with a suitable battery and regulating resistance. As the current is increased by reducing the resistance the wires become red hot, the manganin wire glows first and the copper last of all. Wheatstone bridge measurements show that the specific resistance of manganin is the largest and that of copper the least of the three given materials, the experiment therefore demonstrates that, for the same current, the greatest development of heat takes place at those places in the circuit where the resistance is highest.

The laws relating to the generation of heat were discovered experimentally by Joule. When a current A flowed for a time t through a resistance R, Joule found that the heat developed was proportional to the product A^2Rt. This result, known as Joule's law, can be established theoretically. Let the P.D. at the ends of a wire resistance be E volts, and suppose the current produced is A amperes. Then in a time t seconds, At coulombs of electricity fall through a P.D. of E volts, hence the electrical energy given to this part of the circuit is $E \,.\, At \times 10^7$ ergs (p. 365). Since no other work is done this energy is converted into heat. Thus the calories developed $H = EAt \times 10^7/J$, J being the mechanical equivalent 42×10^6. Hence $H = 0{\cdot}24EAt$ cals. But R being the resistance of the wire in ohms, $E = AR$; substituting for E, $H = 0{\cdot}24A^2Rt$ cals., which is the numerical expression of Joule's law. We might instead substitute for A, when $H = 0{\cdot}24E^2t/R$. Generally the resistance will alter with the temperature, hence if we require to know the amount of heat developed in any part of a circuit it is best to use the expression $H = 0{\cdot}24EAt$. since E and A can be measured directly. Joule's

law is equally true for electrolytes, since, as we have seen, the action of the current is merely directive and no chemical work is done in the body of the solution. If the total resistance of a circuit containing a voltameter is R ohms, and the back E.M.F. in the voltameter is e, then the current is $A = (E - e)/R$; multiplying by At, this becomes $EAt = A^2Rt + eAt$.

The first term gives the total energy supplied by the battery (the factor 10^7 being omitted on both sides of the equation), the second gives the energy dissipated as heat, while the third gives the chemical work done in the voltameter by forcing through it a quantity of electricity At against an E.M.F. e. If the voltameter were replaced by a wire having the same resistance, and an equal current A were sent round the circuit (requiring of course a smaller E.M.F.), exactly the same quantity of heat would be generated in the two cases, but the energy supplied by the source would be less in the second by an amount eAt. In each case the energy of the current dissipated as heat is the same.

EXPERIMENT.—Joule's results may be verified by measuring the heat developed in a calorimeter as shown in Fig. 245. A resistance coil of 1–2 ohms is immersed in a quantity of non-conducting liquid, such as paraffin oil, whose mass and specific heat are known. Current is led in by thick copper wires and is measured by the tangent galvanometer or ammeter A; it is kept constant by varying the resistance S. The liquid is thoroughly stirred and its rise of temperature is measured with a thermometer. The heat developed can then be measured as in Chap. III. The student should verify that the heat H varies (1) as the resistance of the coil if the current is unchanged, (2) as the square of the current. The apparatus can also be used to determine J. For this purpose it is best, for the reason given above, to measure the current and the P.D. at the ends of the coil. The latter quantity is obtained by a voltmeter V, placed as in the figure, then $H = EAt \times 10^7/J$, from which J can be found.[1]

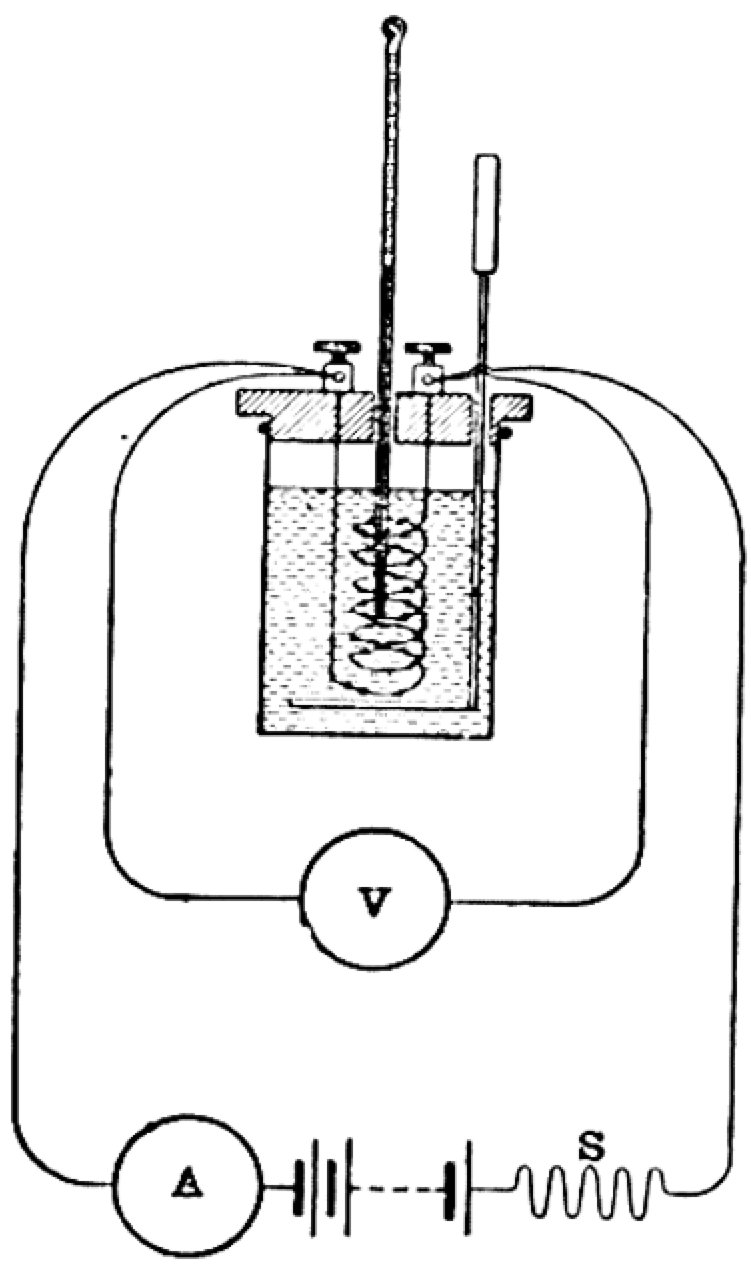


FIG. 245.—Apparatus to prove Joule's law.

[1] Barton and Black, "Practical Physics," p. 175.

Measurement of J by an Electrical Method.—Callendar and Barnes have made a very careful measurement of the mechanical equivalent of heat by the electrical method. Their calorimeter is shown in Fig. 246. Heat is developed by passing current through a platinum wire AB stretched along the axis of a capillary tube. The tube is surrounded by a double-walled space C from which the air has been exhausted, and this is further enclosed in a water-jacket D. An accurately measured current passes along the wire through thick leads at either end, and the P.D. between A and B is measured by comparing it with the E.M.F. of a standard cell by means of a potentiometer. At the same time a steady current of water flows

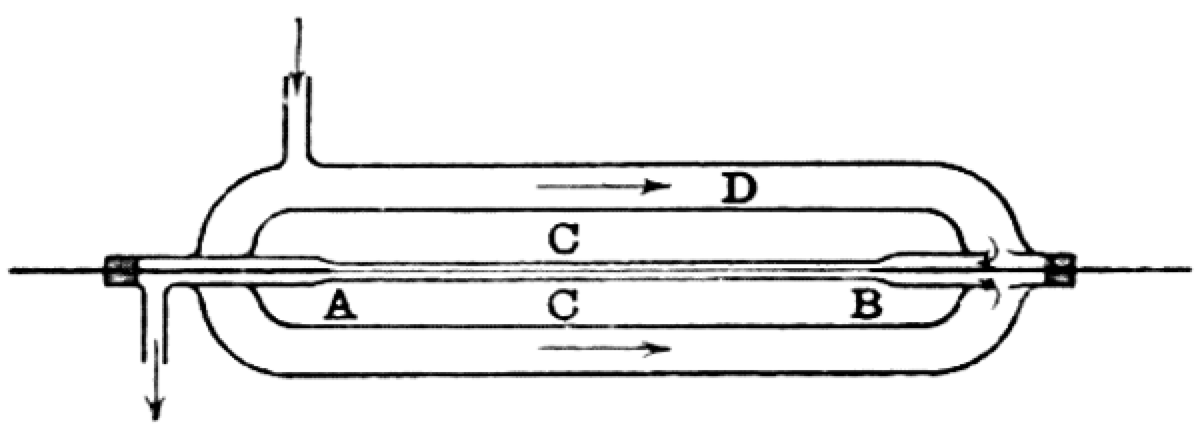


FIG. 246.—Callendar and Barnes' Apparatus.

through the capillary tube and absorbs the heat developed in the wire ; its difference of temperature at entrance and exit is measured by means of two platinum thermometers (not shown in the figure). Let m gms. of water pass through in t secs., θ_1 and θ_2 be its temperatures at entrance and exit respectively, s be the specific heat of water. The temperature difference increases at the beginning of the experiment, but after a time it becomes steady and the heat absorbed by the water in t secs. is $ms(\theta_2 - \theta_1)$ cals. The evacuated space C reduces the heat lost by convection currents, but there is a small amount lost by radiation, this is very constant, and may be allowed for, as the temperature of the water-jacket is steady. If E is the P.D. in volts between the ends of the wire and A the current in amperes, the energy supplied electrically in a time t is $EAt \times 10^7$ ergs; reducing to calories through division by J we have

$$\frac{EAt \times 10^7}{J} = ms(\theta_2 - \theta_1)$$

from which J can be found. Evidently if we assume the value of J the specific heat of the liquid used can be found. In the latter form the experiment has been largely employed in recent years. Prof.

Barnes has used it to find how the specific heat of water varies with temperature (p. 30). Swan and others have replaced the water by a current of gas, and by this means have determined the specific heat C_p of air and other gases at constant pressure. This method is of far greater accuracy than Regnault's (p. 36), as the error due to radiation losses is much smaller.

Electric Power.—Power or activity is defined as the rate of working, thus if a source supplies A amperes at E volts its activity is $EA \times 10^7$ ergs. For practical purposes the erg is replaced by a new unit of energy named **the joule**; this is equal to 10^7 ergs. The practical unit of activity is **the watt**, which is defined as the rate of working necessary to supply energy at the rate of one joule per second. Thus if a source supplies A amperes at E volts the energy per second is EA joules, and the activity is EA watts. Another unit of power frequently used in engineering is called the horse-power; this is defined as a rate of working equivalent to 550 ft.-lbs./sec. The relation between the horse-power and watt is found as follows :—

$$
\begin{aligned}
1 \text{ lb. weight} &= 453{\cdot}6 \text{ gms. wt.} = 453{\cdot}6 \times 981 \text{ dynes} \\
1 \text{ foot} &= 30{\cdot}48 \text{ cms.} \\
\therefore\ 1 \text{ ft.-lb./sec.} &= 453{\cdot}6 \times 981 \times 30{\cdot}48 \text{ ergs/sec.} \\
&= 1{\cdot}356 \times 10^7 \text{ ergs/sec.} \\
\therefore\ 1 \text{ horse-power} &= 550 \times 1{\cdot}356 \times 10^7 \text{ ergs/sec.} \\
&= 550 \times 1{\cdot}356 \text{ joules/sec.} \\
&= 746 \text{ watts.}
\end{aligned}
$$

Applications.—The heating effects of currents have numerous industrial applications, of which the commonest are electrical cooking appliances, the incandescent electric lamp, the arc light, and the electric furnace. These are described in Chap. XLIII.

For some purposes currents are employed whose direction is rapidly reversed, these are called alternating currents. It is evident that if such a current is sent through any of the galvanometers or ammeters previously described the needle will not be deflected, as the magnetic field changes its direction to and fro very quickly. Currents of this nature are frequently measured by their heating effects, as these are independent of the direction of flow. Fig. 247 shows diagrammatically a hot-wire ammeter. A thin wire AB, fixed at each end, has fastened to its mid-point a thin wire which passes round an axle C and is kept taut by means of a spring D. The

axle carries a pointer which moves over a graduated scale. When current in either direction flows through AB expansion occurs, the

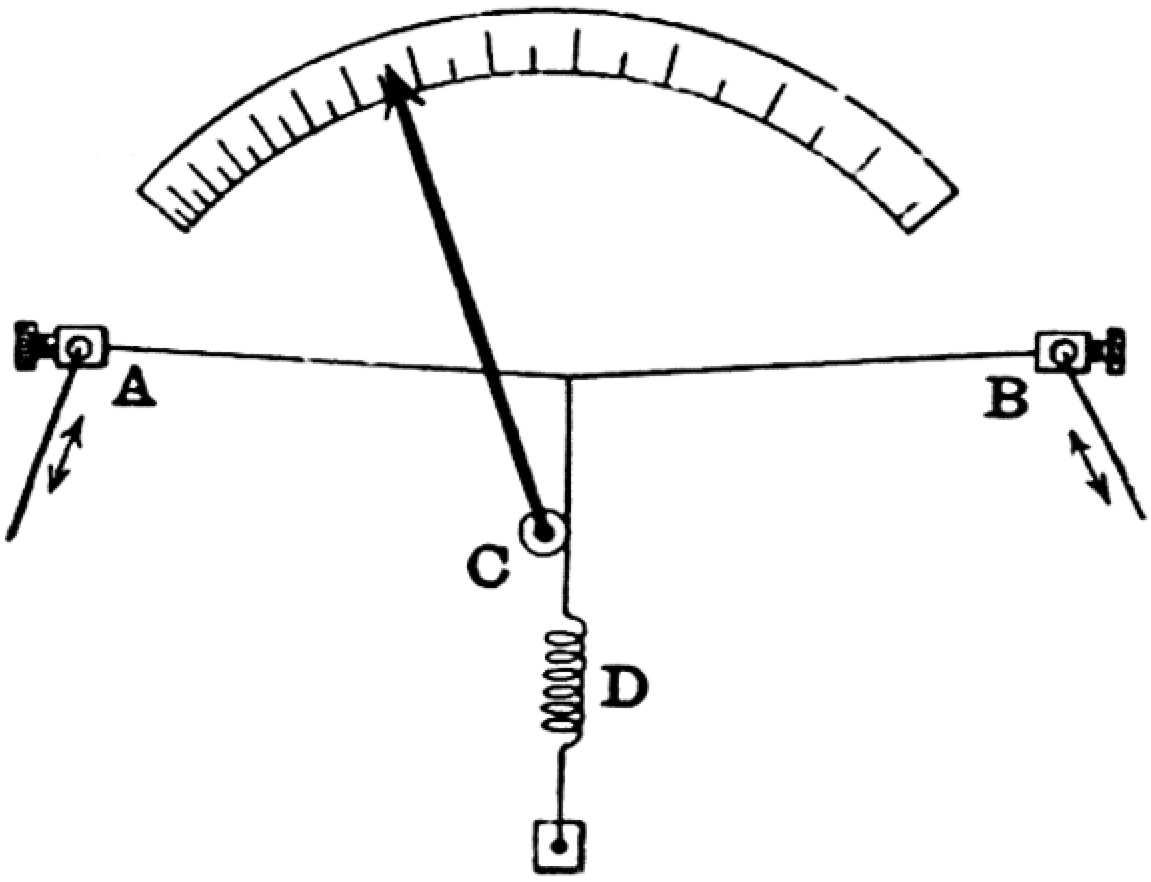


FIG. 247.—Hot-wire Ammeter.

sag is taken up by the spring D and the axle revolves. The instrument can be graduated by using known direct currents.

EXAMPLES ON CHAPTER XXXVI

1. Two wires, whose resistances are 3 : 5, are connected (*a*) in series, (*b*) in parallel, and a current of the same total strength is sent through each combination. Compare the quantities of heat produced per second in each wire in the two cases. (L. '84.)

2. A Daniell's cell has an internal resistance of 2 ohms. Compare the amounts of heat produced in the cell for each gram of zinc consumed in the battery (1) when the cell is short-circuited, (2) when the terminals are connected by a resistance of 2 ohms, (3) when they are connected by 200 ohms. (L. '94)

3. How do you account for the fact that an E.M.F. of about $1\frac{1}{2}$ volts is needed to electrolyse water at an appreciable rate? An accumulator, E.M.F. 2 volts, maintains a current in a circuit of total resistance 2 ohms. An electrolytic cell, with back E.M.F. 1·5 volts, is then inserted, the resistance again being adjusted to 2 ohms. Compare the currents and the rates of working in the two cases. (L. '96.)

4. A Daniell cell has an E.M.F. 1·08 volts and $\frac{1}{2}$ an ohm internal resistance. Its terminals are connected by two wires in parallel of 1 and 2 ohms resistance respectively. What is the current in each and what is the ratio of the heats developed in each? (L. '02.)

5. Compare the amounts of heat developed in the four arms of a balanced

Wheatstone bridge when the arms have the resistances 100 : 10 : 300 : 30 ohms respectively. (L. '05.)

6. What is the advantage of using currents of high E.M.F. for the transmission of power over long distances? (L. '06.)

7. On passing a current of 1 amp. through a piece of platinum wire its temperature rises 10° above that of surrounding objects, which are at 0°. Assuming the loss of heat is proportional to the difference of temperature, calculate the temperature of the wire when a current of 2 amps. is passed through it. The temperature coefficient of resistance of the wire may be taken as 0·004. (L. '07.)

8. Lamps aggregating 1 ohm resistance are supplied through leads of 0·02 ohm from a source at 51 volts. The voltage is subsequently raised to 255, and the lamps replaced by high-voltage lamps consuming the same total energy. Calculate the saving per 1000 hours at fourpence per kilowatt-hour. (L. '08.)

9. Two wires of the same material, but of different lengths and diameters, are joined in parallel to the poles of a battery so that they become heated to a high temperature. What must be the relation between the lengths and diameters in order that the two wires may have the same temperature? (L. '09.)

CHAPTER XXXVII

FORCES ACTING UPON CURRENTS IN MAGNETIC FIELDS

Force on Straight Conductor in a Uniform Magnetic Field.—In Chap. XXXIII. the forces exerted by currents on magnetic poles have been considered, but in every case, by Newton's third law of motion, there is an equal and opposite force acting upon the conductor carrying the current. Thus in the case of Fig. 221, the force on a pole at O of strength m, due to a small element s of the wire, is $mAs \,.\, \sin\theta/R^2$, A being the current in E.M. units (p. 364). The direction of this force is into the paper in the figure, *i.e.* it is perpendicular to the plane containing the current and the lines of force at s due to the pole. The force on the short length s of the current is equal and opposite to this. But m/R^2 is the magnetic field at s due to the pole m; calling this F, the force on the current element is $AFs \,.\, \sin\theta$. Hence the force per cm. acting on a current A, E.M. units, placed in a magnetic field of intensity F is $AF \,.\, \sin\theta$, where θ is the angle between A and F.

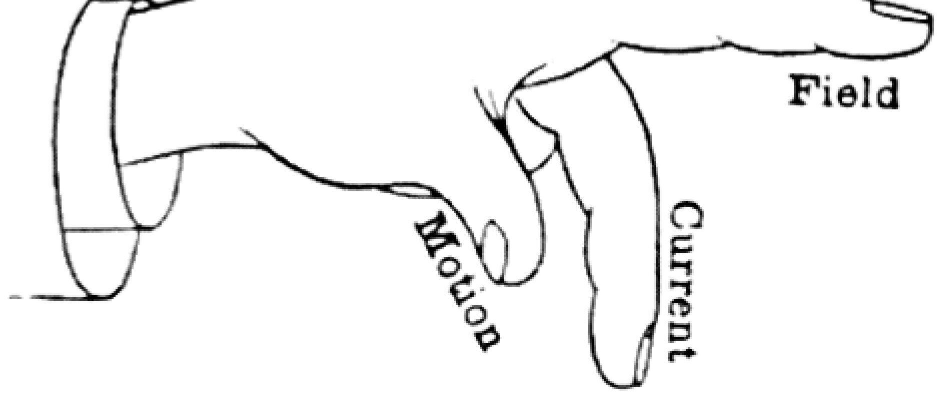


FIG. 248.—Fleming's Rule.

The direction of this force may always be obtained by supposing F to arise from a N. pole and finding the relative motion from the watch rule of p. 362; the following rule due to Fleming is often convenient: **Point the thumb and first two fingers of the left hand in three directions at right angles to each other, if the 1st finger points in the direction of the field, and the 2nd in the direction of the current, then the thumb shows the direction of the force acting upon the current** (Fig. 248).

The force may be ascribed to the joint action of the lines of force of the current and the original field. Thus in Fig. 249, (A) represents

a current flowing downwards into the paper, the dotted circles show the direction of its lines of force, and the straight dotted lines a uniform magnetic field. The resultant of these two fields is roughly shown by the thick lines (B). Above, the lines are in the same

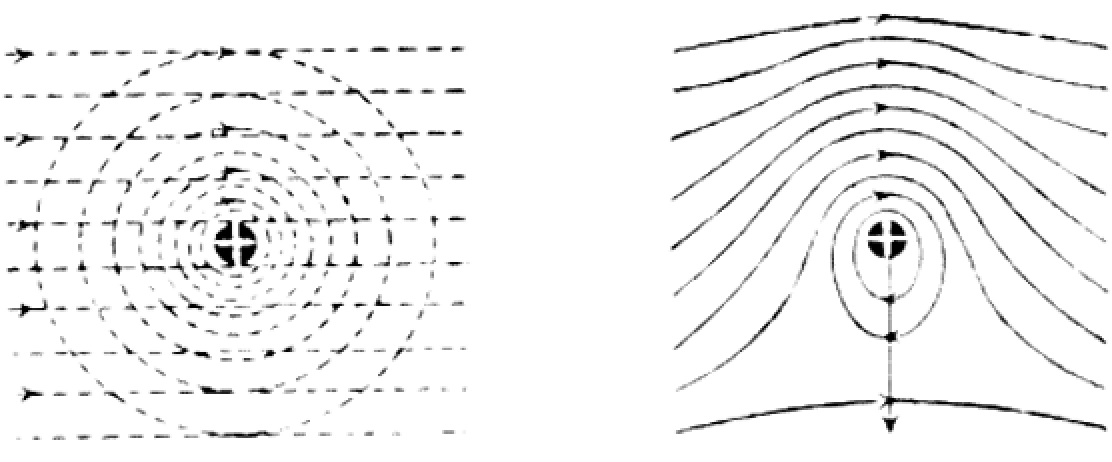


FIG. 249.—Showing Interaction of the Two Sets of Lines of Force.

direction and their resultant is large ; below, they are opposed and the resultant is weak. Evidently the mutual repulsion and the tension in the lines will tend to move the conductor downwards in the figure. These results may be illustrated by Barlow's wheel.

EXPERIMENT.—The axis of a metal wheel rests on two conducting pillars and its lower edge just touches a pool of mercury (Fig. 250). It is placed between

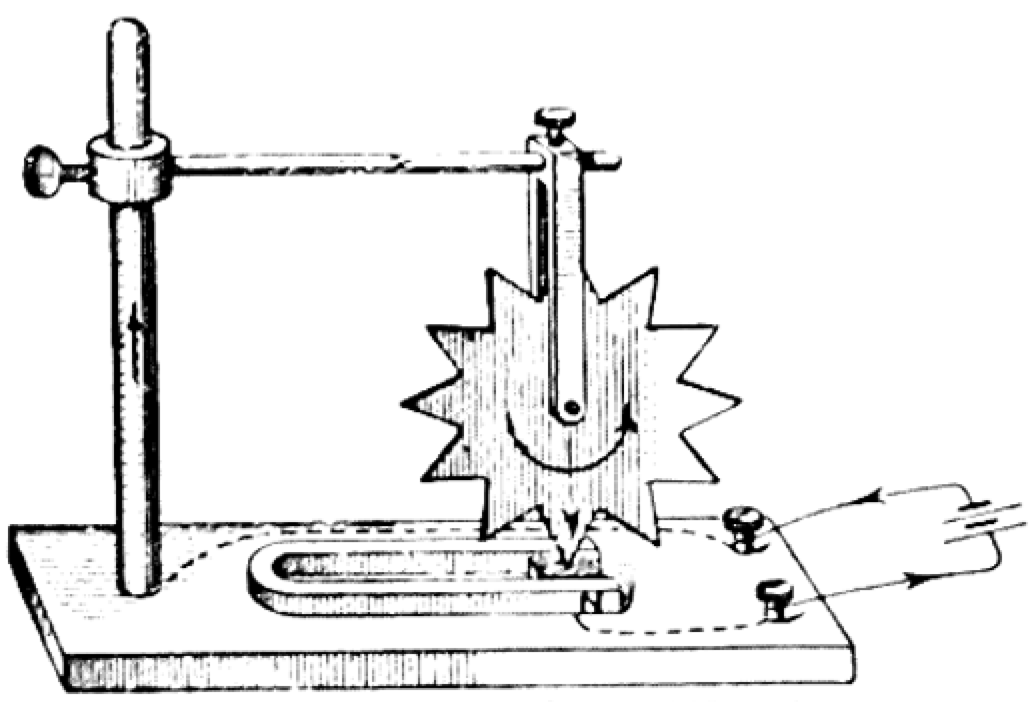


FIG. 250.—Barlow's Wheel.

the poles of a strong magnet and a current is passed from the centre to the rim ; the radius carrying the current is thus subjected to a force and the wheel turns in an anti-clockwise direction if the N. pole of the magnet is nearer to the observer. (Cp. with Fleming's rule.)

EXPERIMENT.—Hang a U-shaped conductor vertically between the poles of an electromagnet and weigh it in this position. When a current is sent through it its weight is apparently altered owing to the electro-magnetic force that comes into play.

It is often simpler to think of the circuit as a whole. Thus we have seen (p. 370) that a current-carrying solenoid sets with its axis parallel to the magnetic field in which it is placed, or regarding it as a magnet, so that the maximum number of lines enters its S. pole or negative face. This is still true if the circuit is reduced to a single turn. Hence a movable circuit sets itself so that the maximum number of lines enters its negative face, or so that the minimum number go in at its positive face. For example, in Barlow's wheel with the directions of current and field we have assumed, the lines of force due to the current in the radius are entering the disc from the near side of the figure over the left portion; we are therefore looking at the negative face of the circuit made up of disc and supports, and the radius moves to the right so as to include more lines from the magnet.

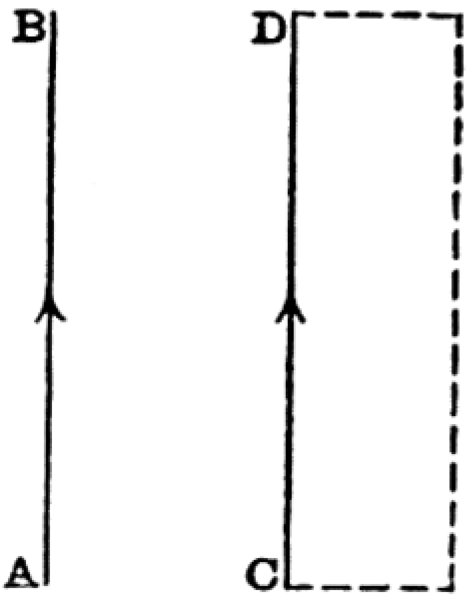


FIG. 251.—Action of Parallel Currents on each other.

Action of Currents on Currents.—Suppose AB, CD (Fig. 251) represent two long, straight conductors carrying currents in the direction of the arrows. The current in CD must have a return conductor somewhere; suppose it is represented by the dotted lines which extend a great distance to the right. We are then looking at the negative face of this circuit,[1] and since the lines due to the current in AB enter at this face the circuit will alter itself, if possible, so as to include more of these lines. This it can do only by CD approaching AB, hence the parallel currents attract each other. Similarly, currents flowing in opposite directions repel each other. The same result can be foreseen by noticing in the figure that, in the space between the conductors, the field of one is partially neutralised by that of the other, while beyond this both fields are in the same direction. There is thus a crowding of the lines on the outer side of each conductor and they are pulled together by the tensions in these lines. This attraction can be very elegantly shown by Roget's spiral (Fig. 252).

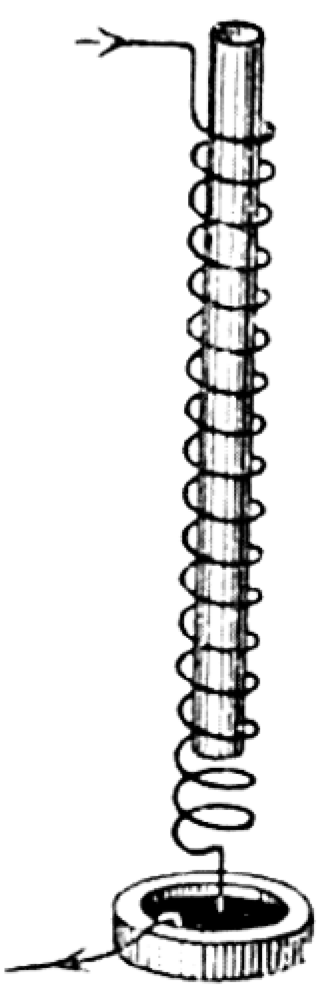


FIG. 252.—Roget's Spiral.

[1] See rule, p. 370.

EXPERIMENT.—Send a current through the iron spiral whose lower end just dips into a pool of mercury; the adjacent turns attract each other, since they carry parallel currents, hence the spiral shortens itself and breaks the circuit at the mercury. The contractile force being now removed the spiral lengthens again and the process is constantly repeated. If a soft iron core is placed as in the figure, lines of force radiate from its lower end, and the currents being now in a stronger field the vibrations take place more readily.

Applications.—The electro-motor and the moving coil galvanometer are direct applications of the preceding principles. In the latter instrument (Fig. 253) a small coil having a large number of turns is suspended between the poles of a strong horseshoe magnet, with its plane parallel with the lines of force; the ends of the coil are fastened to two thin phosphor-bronze strips, the upper of which serves as the means of suspension. When a current is passed through *viâ* the strips, the coil turns so as to include the maximum number of lines in its negative face. The actual deflexion, which may be measured by a lamp and scale, depends on the torsional couple of the suspending strip. This is now the commonest form of galvanometer; its great advantage is that as the coil is already in a strong magnetic field any small variations in the external field will be without appreciable effect. An iron core is usually fixed at the centre of the coil so that more lines pass across the gap in the magnet.

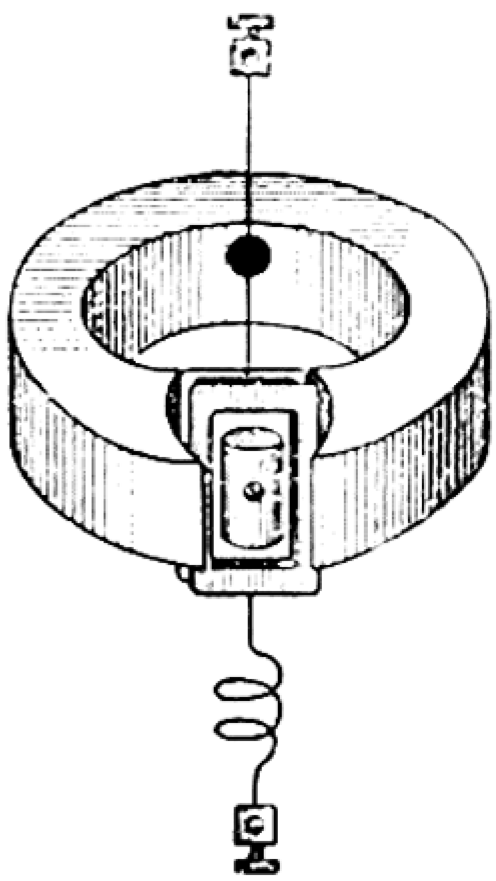


FIG. 253.—Moving Coil Galvanometer.

EXAMPLES ON CHAPTER XXXVII

1. A rectangular coil of wire of height 15 cms. and width 8 cms. is suspended in a horizontal magnetic field of strength 2·5. Calculate the couple acting upon it when its plane makes an angle of 45° with the magnetic field and a current of 20 C.G.S. units flows in the wire. (L. '08.)

2. The two wires, each 4 metres long, from the ceiling to a suspended incandescent lamp taking 1 ampere tend to set E. and W. of each other when the lamp is alight. Explain why this is so, and calculate the force on each wire, given the horizontal component of the earth's magnetic field = 0·18. (L. '12.)

3. A straight copper wire, 3 cms. long, is suspended horizontally between the poles of an electromagnet, and perpendicular to the lines of force, by means of conducting strips attached to its ends. The strips are hung from the arm of a balance. When a current of 2 amperes is sent along the wire it is found that an additional weight of 1 gm. must be placed on the balance to keep it in equilibrium. Find the field of the electromagnet.

CHAPTER XXXVIII

ELECTROMAGNETIC INDUCTION

Faraday's Law.—Oersted's discovery that a current gives rise to a magnetic field led Faraday to investigate the converse problem, whether a magnetic field could produce a current. After numerous failures he found that whenever the number of lines of magnetic force going through a circuit was varied an E.M.F. was produced. The currents and E.M.F.'s produced under such conditions are said to be induced or to arise from electro-magnetic induction.

EXPERIMENT.—Connect a solenoid in series with a galvanometer (which had better be of the moving coil type, for the reason given on p. 413), and thrust into one end of it the N. pole of a bar magnet (Fig. 254). While the motion is taking place a current flows through the instrument, but ceases directly the magnet comes to rest. Pull out the pole, the induced current is in the opposite direction. The currents are reversed if the S. pole is used instead of the N. If instead of moving the magnet to the coil the coil is moved towards the magnet it is found that an induced current is produced as before.

Since solenoid acts like a magnet we may replace the magnet of the last experiments by a solenoid carrying a current.

EXPERIMENT.—Thrust a current-carrying solenoid into the coil which is connected to the galvanometer, induced currents are produced as before, the N. pole of the solenoid can replace the N. pole of the magnet.

That coil in which the induced current flows is called the secondary, the other is the primary coil. The effects are much greater if a rod of soft iron is placed within the primary, but the currents are diminished when the resistance of the secondary is increased. When we come to quantitative laws it is best to speak of the induced E.M.F. rather than the current, as the former is found to be independent of the resistance of the secondary circuit. Relative motion of the two circuits is not necessary for the production of current.

Experiment.—Place one coil inside the other, then start the current in the primary, the needle deflects in the same direction as if the current-carrying solenoid had been thrust into the secondary. Break the primary circuit, the induced current is in the opposite direction.[1]

An examination of these results shows that whenever there is an alteration in the number of lines of magnetic force going through a circuit, an induced current is set up. Faraday drew the following conclusion from his experiments: **The induced E.M.F. is proportional to the rate of change of the number of lines passing through the circuit.** The modern units were not in use in Faraday's day,

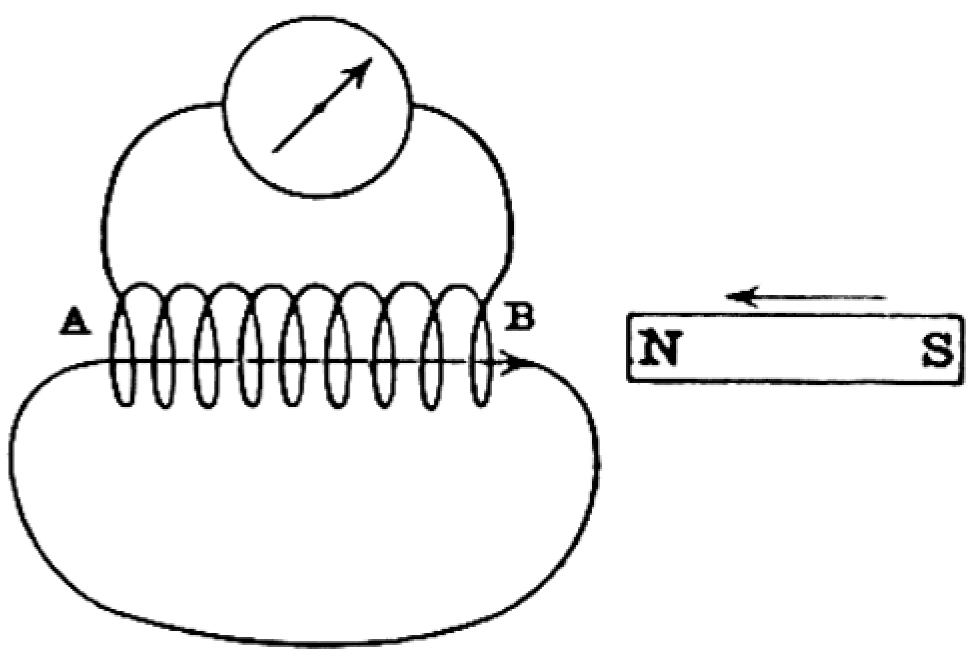


Fig. 254.—Induction of Currents by a Magnet.

but it can be shown that the induced E.M.F. measured in E.M. units (p. 365) is not merely proportional but actually *equal* to the rate of change, provided the lines are drawn according to the rule on p. 314. If the number changes from N_1 to N_2 at a uniform speed in t secs. the induced E.M.F. in E.M. units is $E = (N_2 - N_1)/t$.

Lenz's Law.—The rule as to the direction of the E.M.F. was given by Lenz shortly after Faraday's discovery. Lenz's law states that **the induced E.M.F. is in such a direction as to oppose the change which produces it.** Let us apply this to Fig. 254. Suppose the N. pole to be approaching the solenoid, the induced current causes the latter to act as if it had poles at its extremities, and it can be shown (see below) that B is a N. pole. The repulsion between the like poles then tends to arrest the motion. Similarly when the magnet is withdrawn, B becomes a S. pole and the attraction between the unlike poles again opposes the change. These results can be put in a slightly different but more useful form; in each case the induced

[1] Barton and Black, "Practical Physics," p. 178.

current is in such a direction as tends to keep constant the number of lines passing through the circuit. Thus when the N. pole is approaching the solenoid more lines are being thrust through from right to left; as B is a N. pole the lines from the induced current pass through the coil from left to right (see Fig. 254), thereby tending to neutralise the change produced by the motion. Similarly when the magnet retreats there are fewer lines passing through the circuit from right to left, but, as B is now a S. pole, the lines from the induced current enter at B and emerge at A, thus tending to keep the number enclosed by the coil constant. Take next the last experiment of the previous paragraph. From the principle just given we see that the moment after the primary circuit is completed the lines due to the primary and secondary currents are opposed, so as to keep the number threading the secondary circuit constant, hence if the main current appears to circulate clockwise the induced current is anti-clockwise, and *vice versâ*. Such a current is said to be *inverse*. When the primary current is decreasing, both sets of lines are in the same direction and the induced current is called *direct*.

Experiment.—Shunt a galvanometer with a low resistance and connect its terminals to the poles of a cell; note the direction in which the needle is deflected and mark the terminal connected to the positive pole. Remove cell and shunt and connect the secondary of a pair of coils in series with the galvanometer. When the induced currents produce a deflexion we now know at which terminal they enter the galvanometer, hence their direction in the solenoid can be found. Verify Lenz's law in each of the experiments given in the last paragraph.

Fig. 187 shows that lines of force radiate from the pole of a magnet, hence when the N. pole in Fig. 254 is moved to the left the coil cuts lines of force, the increase in the number embraced by the circuit being equal to the number that are cut. Faraday's law can thus be put in the following form: **The induced E.M.F. in a conductor is proportional to the rate at which it cuts lines of force.** Taking into account Lenz's law, the previous equation becomes $E = -\frac{N_2 - N_1}{t}$, the minus sign indicating that the induced E.M.F. opposes the change. In this form the law can be proved experimentally.

Experiment.—Replace the battery in the Barlow's wheel experiment (Fig. 250) by a galvanometer, and attach a speed counter to the axle so that its rate of revolution can be measured. If the wheel is now turned in an anti-clockwise direction its vertical radius cuts the lines of force of the magnet.

Show that the induced current, and therefore the E.M.F., is proportional to the rate of revolution. During the motion the radius moves so as to include more external lines of force in the circuit made up of the vertical radius, the galvanometer and connecting supports, hence the induced current flows so as to neutralise this change, *i.e.* in the opposite direction to the arrows in the figure.

In this form the apparatus is called a Faraday's wheel; it is really a simple form of dynamo or means to produce a current continuously by mechanical effort, just as Barlow's wheel is a simple kind of motor in which a current is used to produce motion in a mechanism.

Quantity of Electricity induced in any Change.—Let n be the change in a very short time t of the number of lines of force going through a circuit, then the induced E.M.F. e is n/t, and the current is e/R or $n/\mathrm{R}t$, where R is the resistance of the circuit. If A is this current, which we may suppose steady during the short interval, then $\mathrm{A} = n/\mathrm{R}t$ or $\mathrm{A}t = n/\mathrm{R}$. But $\mathrm{A}t$ is the quantity of electricity q that passes round the circuit in the time t, hence $q = n/\mathrm{R}$. Now we may suppose any change in the number of lines takes place in a series of small steps like that just considered, hence the total quantity of electricity put in motion is proportional to the total change in the lines divided by the resistance of the circuit. If Q is this quantity and N the total change in the lines passing through the circuit, $\mathrm{Q} = \mathrm{N}/\mathrm{R}$. It should be noticed that this result is independent of the *rate* at which the change is produced; if, however, we want to measure Q by a ballistic galvanometer the change must be completed before the needle has moved appreciably (p. 388). Other things being equal, N will be proportional to the number of turns on the coil.

Experiment.—Slip a small coil over a bar magnet to its middle point and connect the terminals to a ballistic galvanometer. Now pull the coil off quickly; a charge goes through the galvanometer which is proportional to N. As N is proportional to the number of turns we may, by varying the latter, plot a curve showing how Q varies with N and so verify the law just given. But the formula $\mathrm{Q} = \mathrm{N}/\mathrm{R}$ is based on Faraday's law, we must therefore look upon the experiment as a verification of the law.

Eddy Currents.—A changing field of magnetic force induces currents not only in neighbouring circuits formed of wires but also in any conductor, such as a mass of metal, which may be near. These are called eddy currents, and their direction is given by Lenz's law.

EXPERIMENT.—Suspend a bit of magnetised knitting needle by a silk fibre, and allow it to oscillate torsionally (1) over a sheet of glass, (2) over a sheet of copper. Note that the oscillations die away much more rapidly in the second case owing to the eddy currents produced in different portions of the copper.

EXPERIMENT.—Suspend a small metal cube by a cotton thread between the poles of an electromagnet. Twist the cube round several times and then release it; the spinning motion is checked at once when the magnetic field is put on, owing to the eddy currents in the metal.

If the lower magnet of an astatic galvanometer is enclosed in a small copper chamber its oscillations are rapidly decreased on account of the eddy currents in the copper, and readings of the deflexion can be taken much quicker. The needle is then said to be damped. Similar effects are shown by Arago's disc (Fig. 255). A horizontal circular copper sheet is spun round a vertical axis in a shallow box; a magnet is supported on a needle point on the top of the box and is thus screened from air currents set up by the motion. Eddy currents in the copper drag the magnet round in the same direction as the disc. The experiment may be varied by interchanging disc and magnet.

Self-inductance.—Induced E.M.F.'s may be caused not only by the varying fields arising from external magnets or circuits but also by the current in the circuit itself. Thus when a current is started in a solenoid lines of force are thrust through it, and, on the principles already given, this will generate an E.M.F. whose direction will be such as to oppose the change. There will thus be an inverse, or back, E.M.F. which will last as long as the current is growing; on account of this the full value of the current may not be reached for several seconds in the case of a large electromagnet. Similarly when a current is stopped lines of force disappear, and a direct E.M.F. is created which tends to keep the current flowing. This is the cause of the strong spark which is seen at the point of rupture when the circuit of an electromagnet is broken. The spark shows that the E.M.F. of this extra current may be very considerable. Circuits in which such E.M.F.'s are appreciable are called inductive, and the E.M.F. is said to be due to the **self-inductance** of the circuit. The self-inductance is increased by winding more turns in the same direction on the coil; it will also be greatly enhanced by the presence of a soft iron core, owing to the large number of lines arising from the magnetisation of the iron. The direction of this self-induced E.M.F. is shown by the following experiment (Fig. 256):—

EXPERIMENT.—Place in parallel an electromagnet A, and a pointer galvanometer G of the moving coil type, send a current through them in the direction to the arrows. Fix a cork behind the needle pointer so as to hinder its return of zero when the circuit is broken. At the moment of restarting the current the galvanometer gives a large deflexion and the pointer then falls back to the cork stop. This cannot be due to the needle swinging from zero, since it was held in its final position by the cork; it arises from the back E.M.F. in the magnet coils which, as is seen from the figure, sends a current through the galvanometer from left to right. While the steady current is running push the needle back to zero and hold it there by the cork stop. At the instant the circuit is broken at C the needle swings vigorously to the other side of the zero;

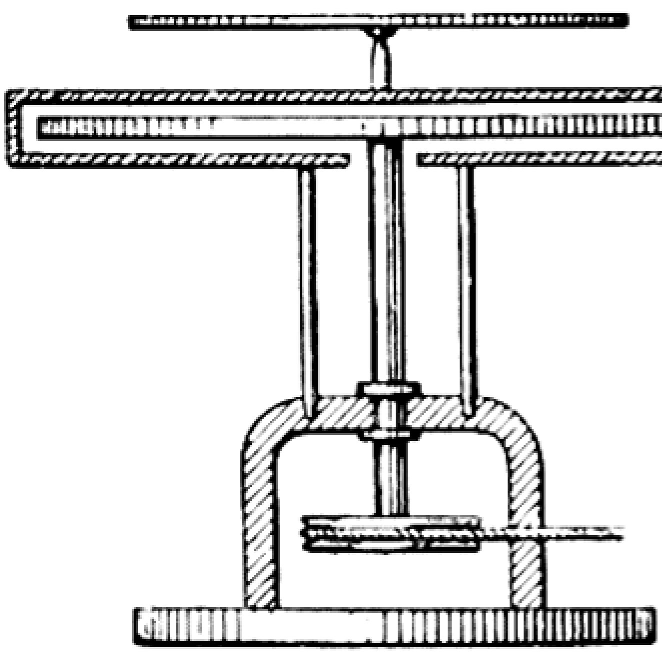


FIG. 255.—Arago's Disc.

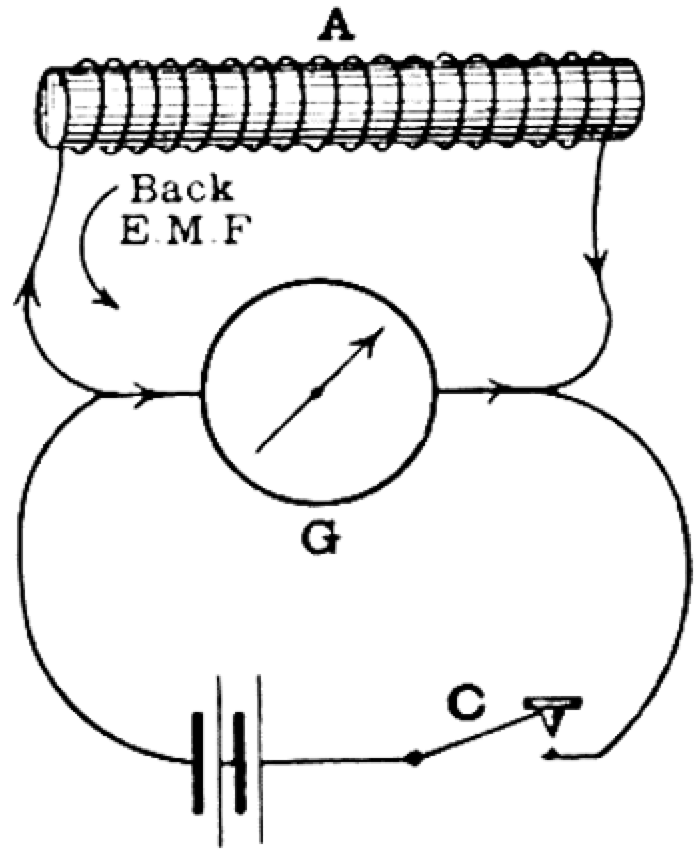


FIG. 256.—Showing Effect of Self-inductance.

as this cannot arise from its momentum carrying it past the position of rest it must be caused by the direct inductive E.M.F. in the magnet coils, this produces a current circulating in the clockwise direction in the circuit composed of magnet and galvanometer, and tends to keep the current running through the magnet in its original direction.

EXPERIMENT.—Replace the galvanometer by a small glow-lamp, its resistance is much higher than that of the magnet, hence it may be arranged that most of the current flows through the latter and the lamp glows feebly. At the moment the current is started the lamp flashes out with great brilliance, because the induced E.M.F. in the branch A opposes the change and the currents in the two branches are not inversely as their resistances. Similarly when the circuit is broken at C the induced E.M.F. sends an extra current through A in the same direction as the original, this circulates in a clockwise direction in the circuit AG.

In the second and third experiments (p. 414) the induced currents are said to be due to the mutual inductance of the two circuits.

If half the total turns on a coil are wound in a right-handed and the remaining half in a left-handed direction, then no magnetic lines will be included by the coil when carrying a current and inductive effects will be absent. This is the reason for winding resistance coils, as described on p. 375. Also when a resistance is being measured by a Wheatstone bridge, the battery key is first depressed so that the current may become steady before the galvanometer is put in circuit, in spite of any inductance that may be present.

The Earth Coil.—Suppose a coil held in a vertical position with its plane perpendicular to the earth's horizontal field, so that it embraces the maximum number, N, of the lines of this field. Let us call that face by which the lines enter the "marked face." When the coil is turned at a uniform speed round a vertical axis the number of lines included will alter and an induced E.M.F. will be set up. While the coil is turning from 0° to 90° the number entering the marked face is decreased from N to zero; the lines from the induced current will tend to neutralise this change and must therefore enter by the marked face. Hence to an observer looking from S. to N. the induced current will appear to circulate clockwise. (See rule on p. 370.) From 90° to 180° the lines entering the marked face are further decreased from 0 to $-$N, since they now enter at the other side, and the current is in the same direction in the *coil* as before. During the remaining half revolution the lines entering the marked face are increased from $-$N to $+$N and the current in the coil is reversed. In the 90° and 270° positions the edges of the coil are moving perpendicularly across the lines of the field, the rate at which the lines are cut is therefore the largest possible and the E.M.F. is a maximum; at the 0° and 180° positions the coil edges are moving parallel with the earth's field, so that no lines are cut, and the E.M.F. is zero. The number of lines which enter the marked face at different stages of a revolution is shown in the continuous curve (Fig. 257). At A and C the rate of change is zero, at B and D it is a maximum; but the induced E.M.F. is proportional to the rate of change, it is therefore represented by the dotted curve which shows that the E.M.F. is greatest when the number of lines included is zero. If the coil is joined to a ballistic galvanometer the electricity passing through the instrument during the first half of the revolution is proportional to (change in the number of lines)/resistance (p. 417), *i.e.* to 2N/R. If n is the number of turns of wire on the coil, S the mean area of a turn, and H the earth's horizontal field, then $N = n$SH.

The discharge Q_1 through the galvanometer is thus proportional to $2n\mathrm{SH/R}$. We can measure the angle of dip by induced currents; for suppose the coil is now placed with its plane horizontal and its axis of revolution in the magnetic meridian, it then embraces nSV lines, where V is the vertical component of the earth's field. If it is turned suddenly round a horizontal axis through 180° a quantity Q_2 passes through the galvanometer, and Q_2 is proportional to $2n\mathrm{SV/R}$. Hence $Q_2/Q_1 = \mathrm{V/H} = \tan\theta$, where θ is the angle of dip. A similar method can be used to compare any two magnetic fields by

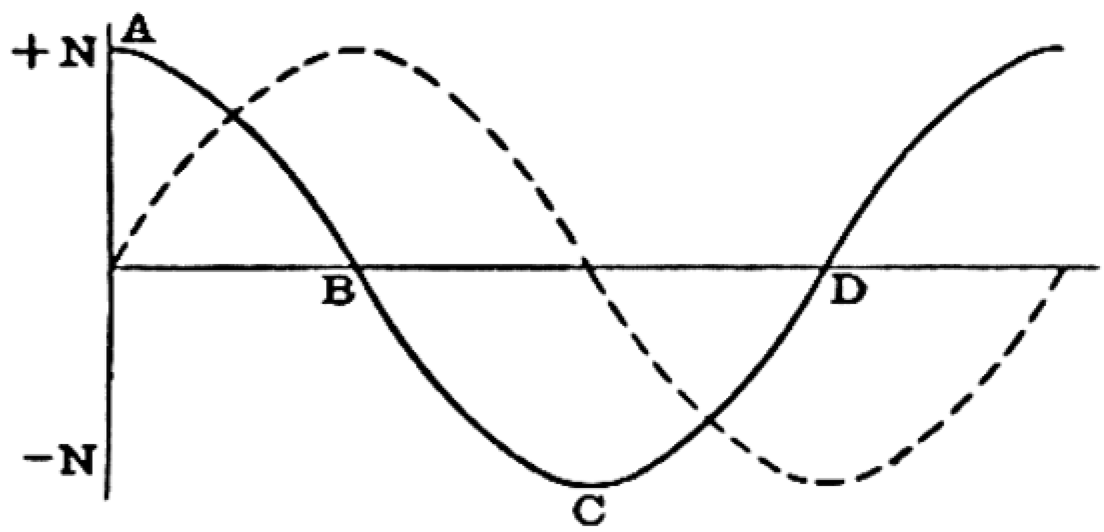


FIG. 257.—Showing the Relative Number of Lines passing through the Coil in Different Positions.

the use of induced currents; a coil connected to a ballistic galvanometer is reversed in each field in succession, the throws produced are proportional to the intensities of the fields.

Ruhmkorff's Induction Coil.—The induction coil is an apparatus for transforming a comparatively strong current at a low E.M.F. into a weaker one at a very high E.M.F. In principle it consists of a primary coil formed of one or two layers of thick wire through which the low voltage current is passed; around this and insulated from it by an ebonite tube is the secondary coil consisting of a large number of turns of fine wire. When the primary current is started or stopped an inverse or direct E.M.F. is produced in the secondary, the magnitude of which varies (1) with the number of magnetic lines of force thrust through the secondary, and (2) with the rapidity with which this number is changed. To increase the number of lines included by the secondary it has a large number of turns, and the primary is wound on an iron core made of a bundle of fine iron wires insulated from each other by shellac varnish. If the core were a solid iron block part of the energy of the primary current would be wasted in producing eddy currents in the metal, and these would

be in such a direction as to oppose any change in the number of lines of force, thereby decreasing the rapidity with which this number can be varied. When the primary circuit is completed the current gradually grows to its final value (p. 418), the induced E.M.F. is therefore small; on the other hand, the circuit may be broken very suddenly if the spark produced by the extra current at break can be suppressed. The efforts of coil designers have therefore been directed to increasing the E.M.F. induced in the secondary at the moment of breaking the primary circuit. A make-and-break arrangement suitable for small coils is shown in Fig. 258 together with a diagram of the rest of the coil. The primary current goes up the vertical

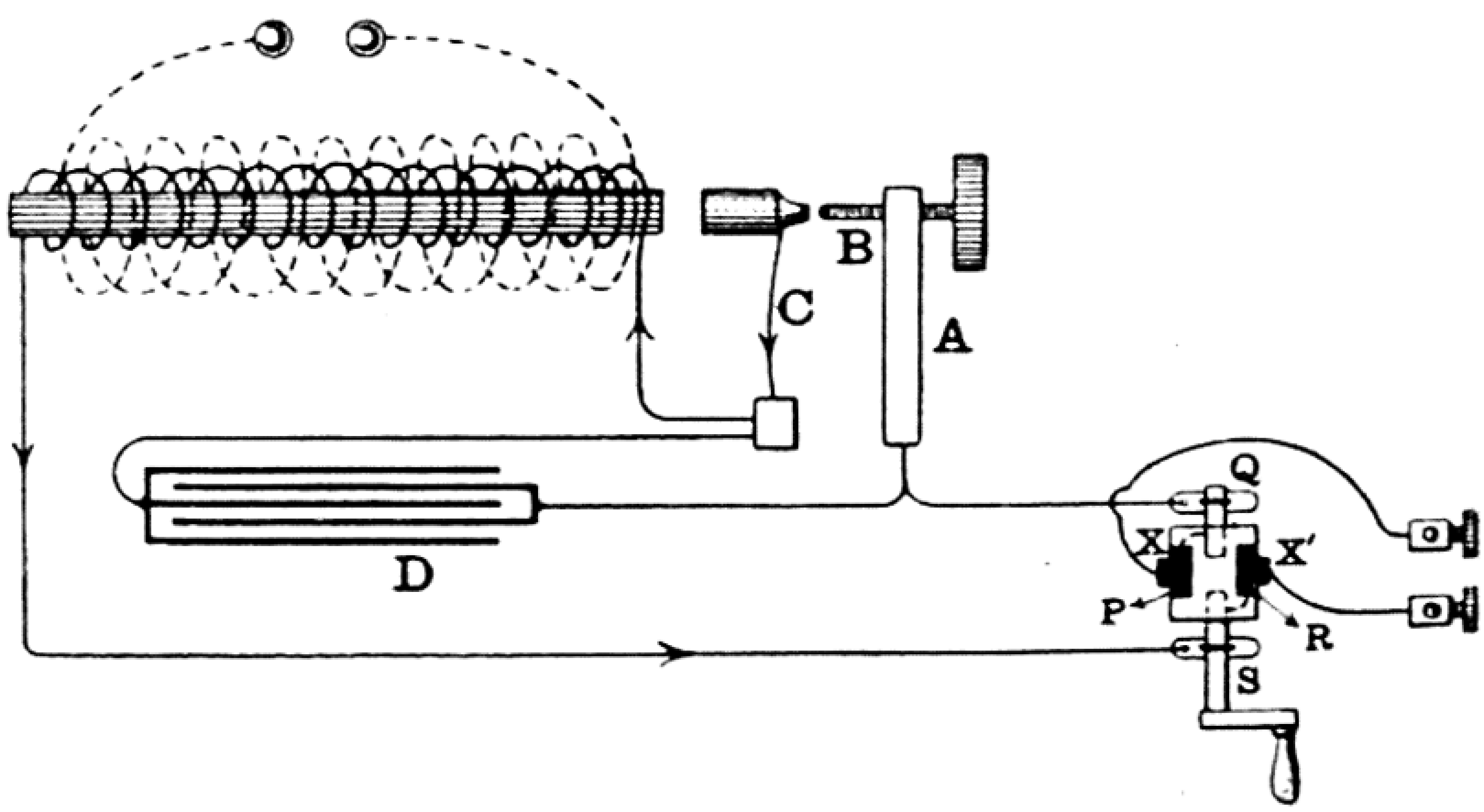


FIG. 258.—Induction Coil and Commutator.

pillar A, along an adjustable screw B, tipped with platinum, which just touches a spring C, also tipped with platinum. It then goes down the spring and through the primary coil. The iron core is thereby magnetised and attracts a piece of soft iron attached to the top of spring C. The circuit is thus broken at the platinum points, the core loses its magnetism and allows the spring to fall back, when the process is repeated. The spark at the point of rupture is largely decreased by connecting to the platinum points a condenser D, made of tinfoil and paraffined paper; before the "extra current at break" can leap the gap it has to charge the condenser plates to the necessary potential difference, and this may not be reached. The use of an infusible metal like platinum at the point of rupture not only reduces the wear, but it makes the formation of a conducting arc

of metallic vapour more difficult. A large coil will readily produce an E.M.F. of some hundreds of thousands of volts at the secondary terminals although the primary source may be only a few accumulators. In order that the current in the primary may be reversed a commutator usually forms part of the apparatus. This is shown in the figure. A block of ebonite has two brass strips P, R, at opposite ends of a diameter, these are joined by inwardly projecting metallic pieces to the metallic axis of the block QS. This axis is interrupted at its centre and is joined directly to the ends of the primary coil. Two brass springs XX′ touch the block at opposite ends of a diameter and are joined to the battery poles. Suppose X is joined to the positive pole and that it also touches strip P, then the current flows in the direction of the arrows. When the block is turned round its axis through 180°, X touches R and the current is reversed.

EXAMPLES ON CHAPTER XXXVIII

1. Suppose a railway line is laid in England on insulating material, and that the two rails are connected at a certain station by a cross-wire; show that a current will flow in the cross-wire and the rails when a train is moving on the line. Draw a figure showing its direction when the train is moving from the station. (L. '96.)

2. A N. seeking pole is suddenly brought down to the centre of a coil lying on a table. What is the direction of the induced current? How would you prove it is in the direction you suppose it to be? (L. '01.)

3. A telegraph wire running magnetic E. is blown down. Calculate the mean voltage induced in the wire per metre length, supposing it to fall freely from a height of 5 metres, and state in which direction the current will flow. ($g = 1000$ cm./sec.^{2}, H $= 0{\cdot}18$ C.G.S.) (L. '08.)

CHAPTER XXXIX

THE MAGNETIC PROPERTIES OF IRON AND STEEL

Intensity of Magnetisation.—So far in the chapters on magnetism we have supposed each unit N. pole to be the origin of 4π lines of force which radiate in all directions and finally end on a S. pole. Let us now consider the lines inside the magnetic material itself. When a bar magnet is broken each half becomes a complete magnet, and at the moment of separation of the two portions lines of force stretch between the additional N. and S. poles so formed. We conceive these lines as previously existing in the magnetic material. Similarly if an iron ring has been magnetised parallel to its circumference by wrapping round it a coil of wire through which a current is sent, neither poles nor lines of force can be detected, but if a gap is made in the ring lines of force run across this breach. As before it is supposed that the lines were present all the time in the magnetised iron, the gap merely makes them evident. Such lines are called lines of magnetisation; in a straight bar magnetised along its length they run from the S. to the N. pole through the iron. When a gap is made in the iron perpendicular to the direction of magnetisation each face becomes a magnetic pole and **the pole strength per cm.^{2} is called the intensity of magnetisation** of the iron. When the intensity of magnetisation is the same at every point the specimen is said to be uniformly magnetised. If S is the section and l the length of a bar in which the magnetisation is uniform and equal to I, then the pole strength at each end is IS, and the magnetic moment of the bar is ISl or Iv, where v is the volume of the material. I can therefore be defined also as the magnetic moment per unit volume. Since each unit pole gives rise to 4π lines, the density of the lines in the gap which we have supposed cut in the iron will be 4πI, so far, that is to say, as the magnetic force there is due to the magnetism of the adjacent molecules.

Magnetic Induction.—Consider now the case of a long, straight, iron bar round which a magnetising solenoid is wrapped. If we imagine a gap cut perpendicularly across it at any point there will be, as before, $4\pi I$ lines per sq. cm. due to the magnetisation of the bar, but in addition there will be lines due to the magnetising current and to the poles at the ends of the specimen. **The total number of lines per cm.^{2} is called the magnetic induction in the iron.** With reference to the lines coming from poles at the ends it must be remembered that these are supposed to radiate in all directions from the N. pole, a certain fraction will therefore run to the S. pole through the iron itself, and will tend to turn the regularly arranged molecular magnets from their positions and so demagnetise the bar. It is to get rid of this self-demagnetising force that keepers are used and that permanent magnets are frequently made in ring or horse-shoe shape; the effect in each case is to provide a shorter or easier path for the lines in the direction in which we wish them to go.

Magnetic Force in the Iron.—We have thus analysed the lines in the gap into two components, (1) The lines of magnetisation whose density is $4\pi I$, (2) Those arising from external currents, or poles which are distant from the gap. **The density of the second set of lines is called the magnetic force in the iron.** Denote this force by H and the induction by B; when both sets are parallel the induction is the sum of the two, and, $B = H + 4\pi I$. We see then that the induction B is measured by the force which a unit N. pole would experience if placed in a narrow crevasse cut perpendicular to the direction of magnetisation; on the other hand, the magnetic force H is measured by the force experienced by the test pole if the magnetism on the faces of the gap is neglected. One method of isolating the H lines from the rest is to suppose the test pole placed in a long and very narrow cylindrical cavity whose length is parallel with the direction of magnetisation; the molecular magnets of which the iron is composed then terminate on the ends of the cavity but not on the sides. The effect of the magnetism at the ends can be rendered negligible by making the cavity sufficiently long and narrow; the force experienced by the test pole at the centre of the cavity is then equal to H. In non-magnetic material, where I is zero, the magnetic force and induction have the same value as each other; this applies also to the air surrounding a magnet. Let us imagine now a small portion of the magnetised iron to be isolated from the remainder by an imaginary cylindrical surface passing between the molecules, the

axis of the cylinder being parallel with the magnetisation. There are equal quantities of N. and S. pole magnetism inside the surface, hence as many lines of induction enter the cylinder as leave it. This will still be true when one end of the cylinder is in the iron and the other in the air outside, or when it is wholly in the air. This can only mean that **lines of induction are closed curves** having neither beginning nor end. The student will see the force of this reasoning if we apply the same idea to a problem in electrostatics. Suppose one end of our imaginary cylinder is inside and the other outside the surface of a charged conductor, then there is an excess of Faraday lines leaving the cylinder equal to the electrical charge enclosed. This is due to the fact that electrical lines of force have a definite beginning and end unlike lines of magnetic induction. It has already been noticed that the inductance of a solenoid is increased by the presence of a soft iron core ; it is seen from the foregoing that this is due to the increase in the density of the lines of induction rather than the lines of force.

Permeability.—The ratio of the intensity of magnetisation to the field producing it is called the susceptibility, writing k for this, $I = kH$. Similarly the ratio of the induction to the field producing it is called the permeability, if μ is this quantity, $B = \mu H$. Experiment shows that both k and μ vary with H. Substituting for B and I in the equation $B = H + 4\pi I$, we get $\mu = 1 + 4\pi k$. If H and either B or I are measured experimentally all the other quantities in the above equations can be calculated.

Magnetometer Method.—This method is one of several that have been used to determine how the various quantities defined above vary as the magnetic force is altered. The quantities observed directly are H and I. The measuring instrument is a magnetometer, which may be merely the compass box of a tangent galvanometer, or the needle may have a mirror attached to it in order that its deflexions may be read by the lamp and scale method. Due E. or W. of the needle (Fig. 259) is the lower end of a long, vertical solenoid C, in series with this is a small circular coil D called the compensator. The rest of the figure shows a battery, an ammeter A, and a regulating resistance F joined to a Pohl commutator P. The maximum current it is intended to use, say 4 amperes, is sent round the circuit ; the needle is deflected, owing to the magnetic field of the solenoid, but may be brought back to zero by

arranging that the compensator produces an exactly equal deflexion in the opposite direction. This compensation will then hold for any smaller current. The circuit is broken and a thin iron or steel wire, previously demagnetised by heating or other means, is placed along the axis of the solenoid with its lower end on a level with the needle. The current is now increased by small steps from zero to the maximum, and at each stage the reading of the ammeter and the deflexion of the needle are noted; the latter is entirely due to the magnetic pole at the lower end of the wire, if we suppose that the upper end is so distant as to render its effect negligible. Let F′ be the field due to this pole at the compass needle, θ the deflexion, and H′ the earth's horizontal component. Then, since F′ and H′ are perpendicular to each other, $F' = H' . \tan \theta$ (p. 319). If S is the sectional area of the wire, I its intensity of magnetisation, and d the distance of the lower pole from the needle, the pole strength is IS and the field $F' = IS/d^2$. Hence $IS/d^2 = H' . \tan \theta$, from which I can be calculated. It can be shown that the magnetic field H in the interior of a long solenoid is $4\pi n A$, where $n =$ the number of turns per cm. length, and A = the current in E.M. units; if A is given in amperes $H = 4\pi n A/10$. A curve can thus be plotted showing the relation between H and I, such a curve OPQR, typical of those obtained for soft iron, is shown in Fig. 260. It is seen to consist of three portions; during the first stage the magnetism of the iron increases very slowly compared with the magnetic force, in the second it increases much more rapidly, while in the third stage again the effect of an increase of field is very small. The last stage corresponds to a state of affairs where practically all the molecular magnets have had their axes turned parallel to the field; in this condition the iron is said to be magnetically saturated. The curve showing the relation between B and H is

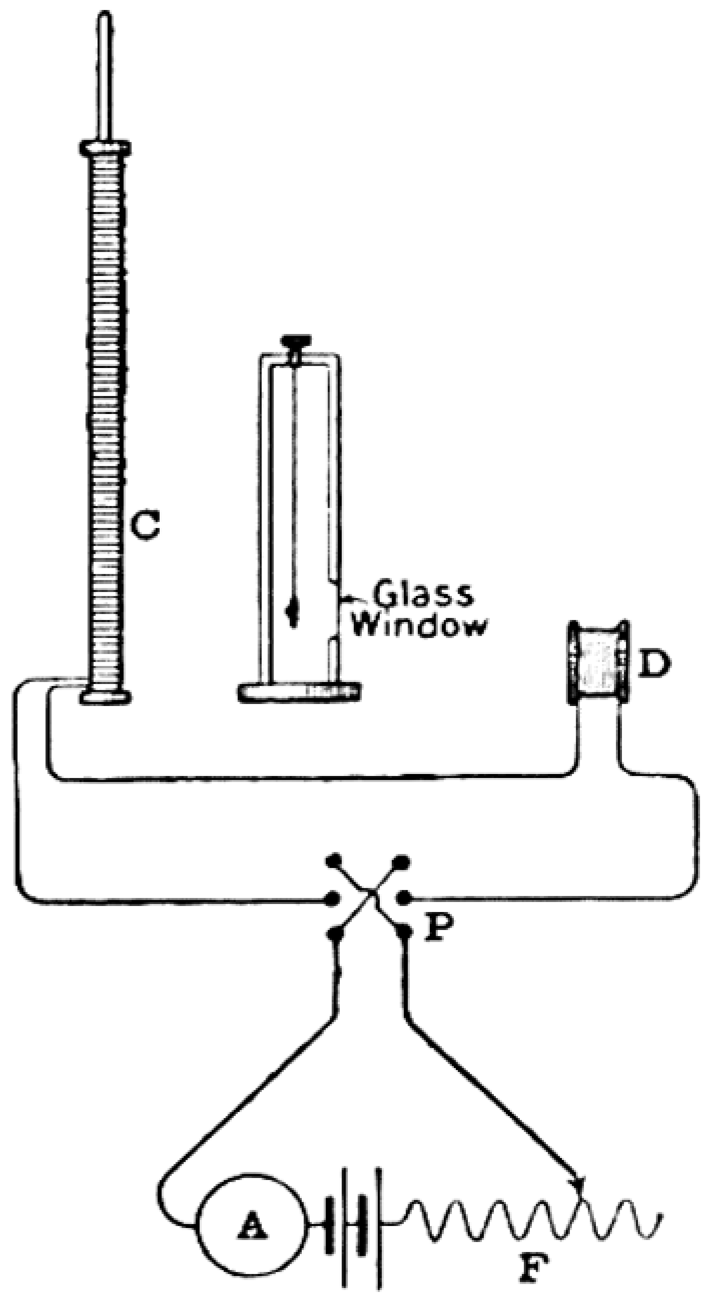


Fig. 259.—Magnetometer Method of Measuring I.

similar to this. The permeability for iron may be as high as 2000, since $B = \mu H$ this shows how enormously the density of the lines in a coil is increased by an iron core. The induction B can increase without limit, this follows from the equation $B = H + 4\pi I$, which shows that an increase in H causes a corresponding increase in B even when I has attained its limiting value.

Magnetic Hysteresis.—If after reaching its maximum value the current is decreased by small steps to zero it is found that the magnetisation does not decrease at the same rate, when the current is zero the iron still retains a considerable amount of magnetism. After being decreased to zero let the current be reversed and gradually increased to the same maximum value in the negative direction, then let it be decreased to zero again and finally increased from this value to its maximum in the positive sense. The complete curve showing how I varies with H is shown in Fig. 260. It is seen that the magnetisation does not respond completely to changes in H but always lags behind it, thus when H has been decreased from its maximum to zero I has the value represented by OT. This lagging effect is called **magnetic hysteresis,** and the closed curve is called a hysteresis curve. It can be shown that the area of the curve is proportional to the amount of work that must be done to take the iron through a complete magnetic cycle. OL shows the value of the negative field required to demagnetise the iron, it is called the coercive force. It gives us a measure of the power of a substance to retain its magnetism under adverse conditions, such as demagnetising fields, and must be large in permanent magnets. The figure shows that the demagnetising force required to destroy the magnetisation is less than that required to produce it, $OL < OX$, hence if a current whose direction can be rapidly alternated is sent through the solenoid while, at the same time, its magnitude is slowly reduced, the iron can be completely demagnetised. The hysteresis curve shows clearly that the magnetic state of iron or steel depends not only on the magnetising

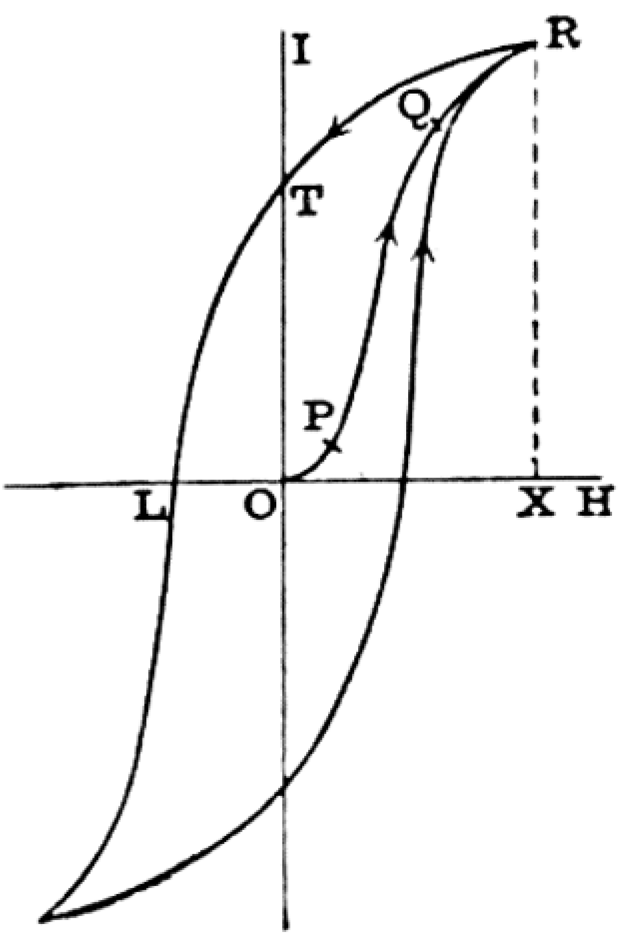


FIG. 260.—Hysteresis Curve.

force applied to it, but also largely on its previous magnetic history.

The Magnetic Circuit.—Since lines of induction form closed curves they bear an analogy to lines of current flow, for these also run round a closed curve, viz. the current circuit. Keeping to this analogy we may call the space through which the lines of induction pass a magnetic circuit; if the path of the lines is entirely within magnetic material the circuit is a closed one. The analogy may be pursued further, as we now proceed to show. Let an iron rod of length l and section S have wound upon it a solenoid of N turns, then suppose it is bent round to form a closed ring. If A amperes flow in the solenoid the magnetic force H in the iron is $H = 4\pi NA/10l$; the product NA is called the ampere-turns. When a unit magnetic pole traverses the whole length of the iron the work done by the force $H = Hl = 4\pi NA/10$. But if a unit charge of electricity is taken round a circuit in which the E.M.F. is E the work done is E (p. 365); the quantity $4\pi NA/10$ is therefore analogous to an E.M.F., it is called **the magnetomotive force.** If B is the induction in the iron $B = \mu H$, and the total number of lines crossing the section $S = \mu HS = \frac{\mu S}{l} \cdot \frac{4\pi NA}{10}$; this is called the **magnetic flux.**

Hence
$$\text{flux} = \frac{4\pi NA/10}{\dfrac{l}{\mu S}}$$

If we take the flux as corresponding to the electric current we see that this equation is analogous to the Ohm's law equation $A = E/R$, thus $l/\mu S$ corresponds to a magnetic resistance. Instead of speaking of a resistance this quantity is called **the reluctance**, and the equation may be written

$$\text{flux} = \frac{\text{magnetomotive force}}{\text{reluctance}}$$

This expression is useful in calculating the flux produced in a circuit by a given magnetomotive force, but it must be remembered that in some important respects the analogy breaks down, for μ depends on the magnetic field whereas the resistance of an electrical circuit is independent of the E.M.F., also there is not anything of the nature of a current of induction. If the iron ring contains an

air gap the reluctance of the circuit is the sum of the reluctances of the iron and the gap, *i.e.* $(l_1/\mu S + l_2/S)$, where l_1 is the length of the iron and l_2 of the gap. Since μ is very large for iron the first term will usually be small and the chief part of the reluctance will arise from the air gap. The following experiment shows how the induction is increased by doing away with air gaps.

EXPERIMENT.—Wrap a few turns of wire round one limb of a large horseshoe magnet, a few cms. from the end, and connect them to a sensitive galvanometer. Suddenly put on the keeper, the galvanometer shows an induced current due to an increase in the number of lines going through the coil. If the coil is placed between the limbs so that lines crossing from one to the other pass through it, then when the keeper is put on a throw is noticed whose direction indicates a decrease in the number of lines penetrating the coil. The lines prefer to pass from one limb to the other through the keeper, since iron transmits them more readily than air, or, putting it another way, the iron path has the less reluctance.

EXAMPLES ON CHAPTER XXXIX

1. What is approximately the magnetic force inside a solenoid of 300 turns, 15 cms. long, which carries a current of 0·2 amps.? (L. '04.)

2. The maximum permanent intensity of magnetisation in a steel bar 10 cms. long by 1 cm. square has been found to be 225 C.G.S. units. Find the tangent of the greatest deflexion of a magnetometer which such a magnet would cause if the centre of the needle were 30 cms. E. of the centre of the magnet. $(H=0{\cdot}18)$ (L. '08.)

CHAPTER XL

THERMO-ELECTRICITY

The Seebeck Effect.—In 1821 Seebeck showed that a current was produced if a temperature difference was established between the junctions of two dissimilar metals which formed a complete metallic circuit. Such currents are called thermo-currents, the E.M.F.'s which are their immediate cause are called thermo-E.M.F.'s, and the two metals together form a thermo-couple.

EXPERIMENT.—Twist a copper and iron wire together at one end and complete the circuit through a galvanometer; since the coil of the latter is copper we have a circuit composed of iron and copper. When the twisted joint is heated slightly the needle is deflected. By means of a cell find the direction of the current; it will be found to flow from copper to iron at the hot junction.

As thermo-E.M.F.'s are very small, often of the order of a thousandth of a volt, the resistance of the circuit, including the galvanometer, should be low in order that the current may be large. If the galvanometer resistance is 100 ohms that of the wires may easily be made so small that any resistance change caused by the heating can be neglected. The thermo-E.M.F. will then be proportional to the current it produces and this can easily be measured. When it is desired to measure the E.M.F. between, say, iron-manganin the galvanometer is joined up as before, but there will now be copper-iron and copper-manganin junctions and we must first decide how these affect the E.M.F. in the circuit.

EXPERIMENT.—Read the galvanometer deflexion with a copper-iron couple when the junction is in steam, then sever the iron near its middle point and interpose a manganin wire. It will be found that the E.M.F. is the same as before, provided the ends of the manganin have the same temperature as each other.

It will suffice, therefore, in the case of the iron-manganin couple

if we connect it to the galvanometer by copper wires and place the copper-iron and copper-manganin junctions in a constant temperature bath. The iron-manganin junction is enclosed in a test-tube, and immersed in a bath that can be heated to a suitable temperature. Fig. 261, A, shows how the E.M.F. in a copper-iron couple varies with the temperature of the hot junction when the cold junction is kept in ice. It is seen that the E.M.F. first rises until it reaches a maximum value; the temperature corresponding to this is called **the neutral temperature.** Beyond this there is a decrease in E.M.F. and finally it is in the opposite direction. The temperature at which the reversal takes place is called the temperature of inversion. If the cold junction is kept at 50° and a new set of observations are made a similar curve is obtained, B in the figure. The neutral temperature is the same in each experiment, and in every case it is midway between the temperature of the cold junction and that of inversion. Raising the temperature of the cold junction 50° therefore lowers by an equal amount the temperature of inversion. If T is the neutral temperature, t_2 and t_1 the temperatures of the hot and cold junctions respectively, the E.M.F. is given by

$$E = k(t_2 - t_1)\left(T - \frac{t_1 + t_2}{2}\right)$$

where k is a constant depending on the couple used.

Experiment.—Heat the copper-iron junction of the first experiment in a Bunsen flame; the deflexion at first increases, then decreases, and is finally reversed. The neutral temperature for this couple is about 270°.

The Thermopile.—These thermo-E.M.F.'s are made use of in the thermopile to detect small temperature differences in the study of radiations. A number of bars, alternately bismuth and antimony, about 2 cms. long, are joined together at their ends as in Fig. 262. They are insulated from each other along their lengths by thin mica strips, and are arranged so that the junctions form the opposite faces of a small cube. One of these faces is covered with lamp-black so as to absorb readily any radiant energy that falls upon it; the other set of junctions is kept bright so as to be non-absorbent, and is further protected from temperature change by a brass cap. The extreme bars of the series are joined to a galvanometer. Such an arrangement evidently constitutes n thermo-couples arranged in series, and the E.M.F. caused by a given temperature difference

between the faces of the cube will be n times that of a single couple. Its method of use is given in Chap. XII; the galvanometer deflexion is proportional to the temperature difference between the faces. When the instrument is to be used in the study of the infra-red end of the spectrum the junctions are arranged to form a very narrow rectangle, in order that the effect of a very narrow portion of the spectrum may be measured. If a thermo-couple which is protected by a porcelain cover is placed in a mass of molten metal it may be used to obtain a cooling curve for the substance and hence its freezing-point (see Fig. 38). They are greatly used by metallurgists for this purpose; their chief advantages are (1) the high tempera-

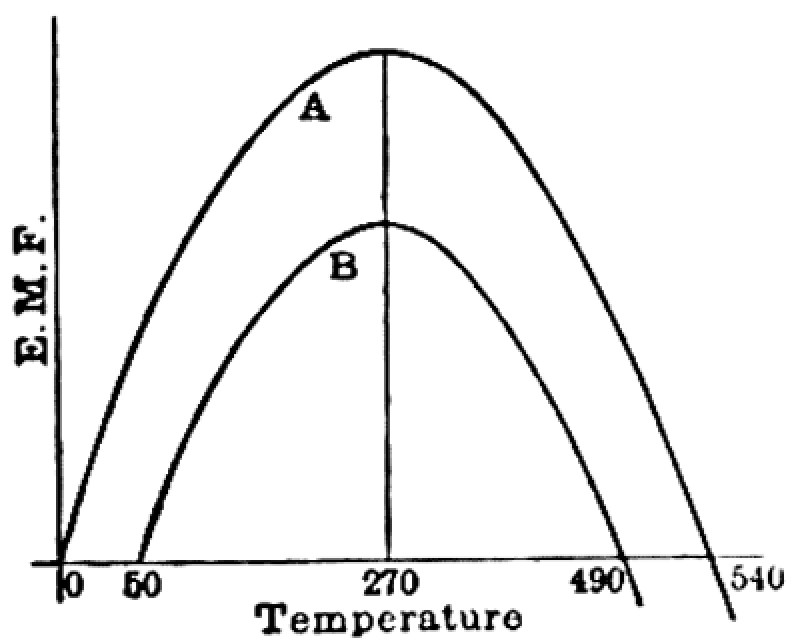


FIG. 261.—Thermo-E.M.F. at Different Temperatures.

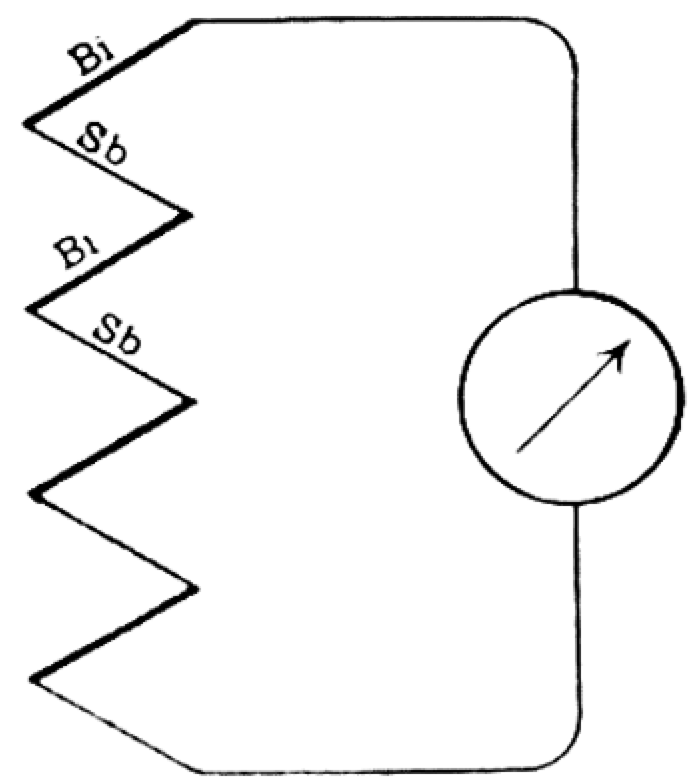


FIG. 262.—Diagrammatic Construction of a Thermopile.

tures for which they can be employed, and (2) the indicating instrument, a galvanometer, may be at some distance from the furnace.

Peltier Effect.—The question now arises, Whence do these thermo-currents derive the energy necessary for their maintenance? The answer was partially supplied by some experiments of Peltier in 1834, which showed that when a current crosses the junction of two dissimilar metals there is an absorption or evolution of heat according to its direction. This is called the Peltier effect. The quantity of heat absorbed or evolved is *proportional to the current*, but varies with the metals used and with the temperature. Thus when a current from an outside source is sent across a copper-iron junction there is an absorption of heat if the direction is from copper to iron, and an equal evolution when the current is reversed, the

temperature in each case being below the neutral temperature. There is thus a cooling effect in the first case and a heating in the second. But we have already seen that the thermo-current flows from the copper to the iron at the hot junction, hence it must cause an absorption of heat at this point. Thus **the thermo-couple absorbs heat from the flame at the hot junction and transforms part of it directly into electrical energy.** If we wish to demonstrate the Peltier effect it must be remembered that there is an evolution of heat in a given portion of the circuit proportional to A^2R (p. 404). This is independent of the current direction and may mask the effect we are looking for. To render this disturbing factor negligible thick bars and small currents must be used; for halving the current reduces the Joule effect to one-quarter its original value, while the Peltier effect is only halved. Keeping this in mind the experiment may be arranged as follows :—

Experiment.—Replace the condenser of Fig. 238 by a thermopile, use a low resistance galvanometer, and put in series with the battery about 400 ohms resistance. Depress the Morse key for 5 seconds; owing to the Peltier effects at the two faces of the pile one set of junctions is cooled and the other heated. Raise the key so that the galvanometer is in series with the pile and note the first throw of the needle. Reverse the poles of the battery and repeat, the throw should be exactly reversed. If the Joule effect enters largely the throws will be very unequal and may be in the same direction. (See below.)

The amount of heat absorbed or developed per second at a junction when a current of one E.M. unit flows across it has been measured in a Bunsen's ice calorimeter. A number of junctions are arranged in series, as in the thermopile, and the alternate ones are immersed in the calorimeter. A current A is passed through them for a time t and the heat developed is measured in the manner explained in Chap. III. Let P be the cals. developed per second by one E.M. unit on account of the Peltier effect at one junction, n the number of junctions immersed, Q_1 the total heat developed in the calorimeter in t secs., A the current in E.M. units, and J the mechanical equivalent. Then

$$JQ_1 = A^2Rt + nPAt \,.\, J$$

If the current is now reversed the sign of the Peltier effect is changed and

$$JQ_2 = A^2Rt - nPAt \,.\, J$$

$$\therefore (Q_1 - Q_2) = 2nPAt$$

from which P can be found.

The Thomson Effect.—Lord Kelvin has shown that the Peltier effect is zero at the neutral temperature (hence the latter term); at higher temperatures it is reversed. Hence at the neutral temperature the couple gives the maximum current (Fig. 261) and yet absorbs no heat. There must therefore be some other source of energy available. This he has located in the conductors themselves. He showed that when a current is sent through a conductor whose ends are at different temperatures there is an absorption or evolution of heat proportional to the current strength. This is called the Thomson effect; like the Peltier effect it is reversed with the current.

EXPERIMENT.—Hang up a U-shaped piece of thin iron wire with its lower portion in a dish of mercury, send a current down one limb and up another so that it just glows in a darkened room. The mercury keeps the lower part cool, hence the current is flowing from hot to cold in one limb and from cold to hot in the other; the Thomson effects are opposed in the two limbs and this causes them to glow unequally.

CHAPTER XLI

POTENTIAL. ENERGY. DIELECTRIC CONSTANT

Potential due to a Charge.—We will now study in greater detail the field surrounding charged conductors; the units used will be those already defined in Chaps. XXXI and XXXII. The difference of potential between two points being measured by the work necessary to move a unit positive charge from one to the other against the electric field, let us calculate this quantity when the field arises from a charge Q at a given point. Let a charge Q units be concentrated at O, and let the points whose P.D. is required be A, B. Divide AB (Fig. 263) into a large number of very short lengths BC, CD, etc.

FIG. 263.

The intensity of the field at B is Q/OB^2, that at C is Q/OC^2, the average field throughout the short distance BC may therefore be taken to have the intermediate value $Q/(OB \,.\, OC)$. Hence the work done in taking unit charge from B to C

$$= \left(\frac{Q}{OB \,.\, OC}\right) . BC$$

$$= \frac{Q}{OB \,.\, OC}(OB - OC)$$

$$= Q\left(\frac{1}{OC} - \frac{1}{OB}\right)$$

Similarly the work from C to D

$$= Q\left(\frac{1}{OD} - \frac{1}{OC}\right)$$

and so on through all the steps up to A. Adding all these expressions together it is clear that all but the two end terms cancel out, and the P.D. between A and B is

$$V_A - V_B = Q\left(\frac{1}{OA} - \frac{1}{OB}\right)$$

If B is very distant 1/OB is zero, and the work done in bringing unit charge from a great distance up to A is Q/OA. This is the potential at A due to a charge Q at O.

Field at the Surface of a Conductor.—Since a charged conductor is an equipotential surface the lines of force cut it normally (p. 353). If the surface density of the charge round a certain point on the conductor is σ there are σ lines starting from a sq. cm. of the surface, hence the density of the lines close to the surface is σ. But the electric field is 4π times the density of the lines (p. 346), hence $F = 4\pi\sigma$ near the conductor. If any portions of the surface are sharply curved the surface density will be correspondingly great (p. 353), and the field may become so intense that the insulating power of the air is destroyed. When this happens the charge escapes and the conductor is discharged. Charged bodies must therefore be kept free from dust, otherwise each particle will act as a discharging point.

Experiment.—Hold in the hand a pin with its point near a positively charged conductor. The surface density of the negative charge induced on the pin is so large that a stream of electricity passes from it to the conductor, which is thereby discharged.

Lightning conductors are merely pointed rods connected to earth; when, during a thunderstorm, an electrically charged cloud comes over them electricity of the opposite kind escapes from the points and neutralises the charge on the cloud. They thus prevent the disruptive lightning spark.

Case of a Spherical Conductor.—When a charged conducting sphere is far removed from other bodies its lines of force radiate equally in all directions; the surface density is uniform and equal to $\sigma = Q/4\pi R^2$, where Q is the charge and R the radius of the sphere. But the field near the surface is $F = 4\pi\sigma$, or, substituting for σ, $F = Q/R^2$, just the same as it would have been if the charge had been concentrated at the centre and the conductor removed. Hence for external

points a charged sphere acts as if the electricity were concentrated at its centre. The field inside is of course zero, unless there are other charges within; the potential of all interior points is therefore that of the surface. It follows from the above that the potential at a point outside, distant x from the surface, is $V = Q/(R + x)$; if x is made very small the potential of the conductor itself is obtained, this is $V = Q/R$. But for any charged conductor $V = Q/C$ (p. 355), where C is the capacity. Hence $C = R$, or **the capacity of a sphere is equal to its radius**. Capacities are therefore measured in cms. in the electrostatic system of units. The unit of capacity is that of a sphere of unit radius which is far removed from other bodies. When dealing with current electricity this is too small, another unit called the micro-farad is then used. If a charge of one coulomb raises the potential of a conductor one volt its capacity is called a farad; the micro-farad is one-millionth of this. One micro-farad $= 9 \times 10^5$ cms.

Spherical and Plate Condensers.—Let us calculate the capacity of a spherical condenser formed of two concentric spheres. Let the radius of the inner be R_1, that of the outer R_2, and suppose the larger sphere is earthed while a charge Q is given to the smaller. There will then be an induced charge $-Q$ on the inside surface of the large sphere; if this were the only charge in the field the potential of all points within would be $-Q/R_2$. If the charge on the small sphere were the only one present the potential of this conductor would be Q/R_1. Hence the potential of the inner sphere under the effect of both charges is $V = Q/R_1 - Q/R_2$ or $V = Q\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. But for any conductor $V = Q/C$, therefore

$$\frac{1}{C} = \frac{1}{R_1} - \frac{1}{R_2}, \text{ or } C = \frac{R_1 R_2}{R_2 - R_1}$$

Next let us find the capacity of a plate condenser like that in Fig. 211. Suppose the area of each plate is S and their distance apart is d. Let one plate be earthed and the other charged with Q units. Neglecting the effect at the edges, where the surface density is greater than elsewhere, the charge is uniformly distributed on the inner side of the plate, and the lines of force between the plates are equidistant and parallel. The field $F = 4\pi\sigma = 4\pi Q/S$, and the work

done in taking unit charge from one plate to the other is F*d*. Hence the P.D. between the plates

$$V = Fd$$
$$= \frac{4\pi Q}{S} \cdot d$$
$$= Q \Big/ \frac{S}{4\pi d}$$

Also $V = Q/C$, hence $C = S/4\pi d$. This shows how the capacity is increased by putting the plates near together.

Specific Inductive Capacity (Dielectric Constant).—Cavendish and Faraday showed independently that the capacity of a condenser varies with the nature of the dielectric or substance between the plates. Thus the capacity is increased when air is replaced by ebonite, sulphur, or an insulating liquid. **The ratio of the capacity when a certain substance is used as dielectric to the capacity with air as dielectric is called the specific inductive capacity of the substance.** It is now more commonly called the dielectric constant.

EXPERIMENT.—Earth one of the plates of the condenser, Fig. 211, and charge the other. Now hold between them a thick sheet of ebonite; the electroscope leaves which are connected to the insulated plate partially collapse. The charge is unaltered but the potential is lowered, hence the capacity must be increased.

Since the potential is lowered the field in the dielectric must be less than it is in air for an equal density of the lines of force. If a substance whose specific inductive capacity is K fills the space between the metallic coatings of a plate condenser the capacity is $KS/4\pi d$. Exactly the same result is obtained by repeating the calculation of the last paragraph if it is assumed that the field in a dielectric is $1/K$ of what it is in air, the charges, and therefore the density of the lines of force, being the same in the two cases. If N is the density of the lines it has been shown that the field $F = 4\pi N$ (p. 346), hence in a dielectric $F = \frac{4\pi}{K} \cdot N$. Other formulæ which have been given also require revision when the medium is other than air and its dielectric constant is K. Thus the force of repulsion between two charges Q, Q′, at a distance apart d in a dielectric becomes $F = QQ'/Kd^2$; comparing this with the fundamental equation on p. 315, it is seen that the constant K there given is the specific

inductive capacity of the medium. Similarly the potential due to a charge Q at a distance R becomes $V = Q/KR$, since the work done in bringing up the unit charge from a great distance is $\frac{1}{K}$th of what it is in air. Also the field near a charged conductor is $4\pi\sigma/K$.

The Leyden jar is perhaps the commonest form of condenser (Fig. 264). The lower half of a glass jar is coated inside and outside with tinfoil; these sheets form the condenser plates, and contact is

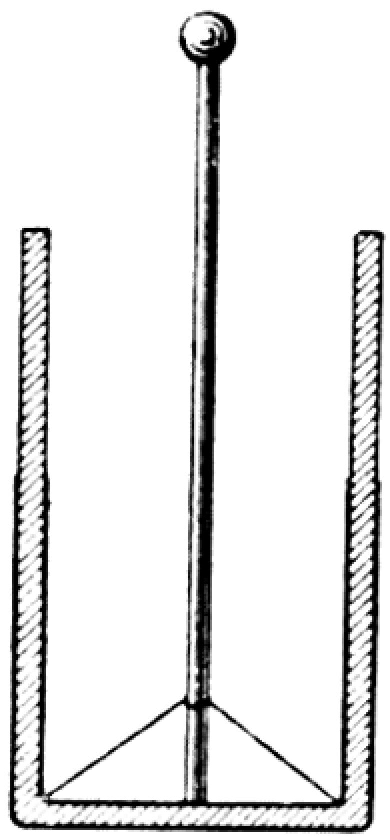


Fig. 264.—Leyden Jar.

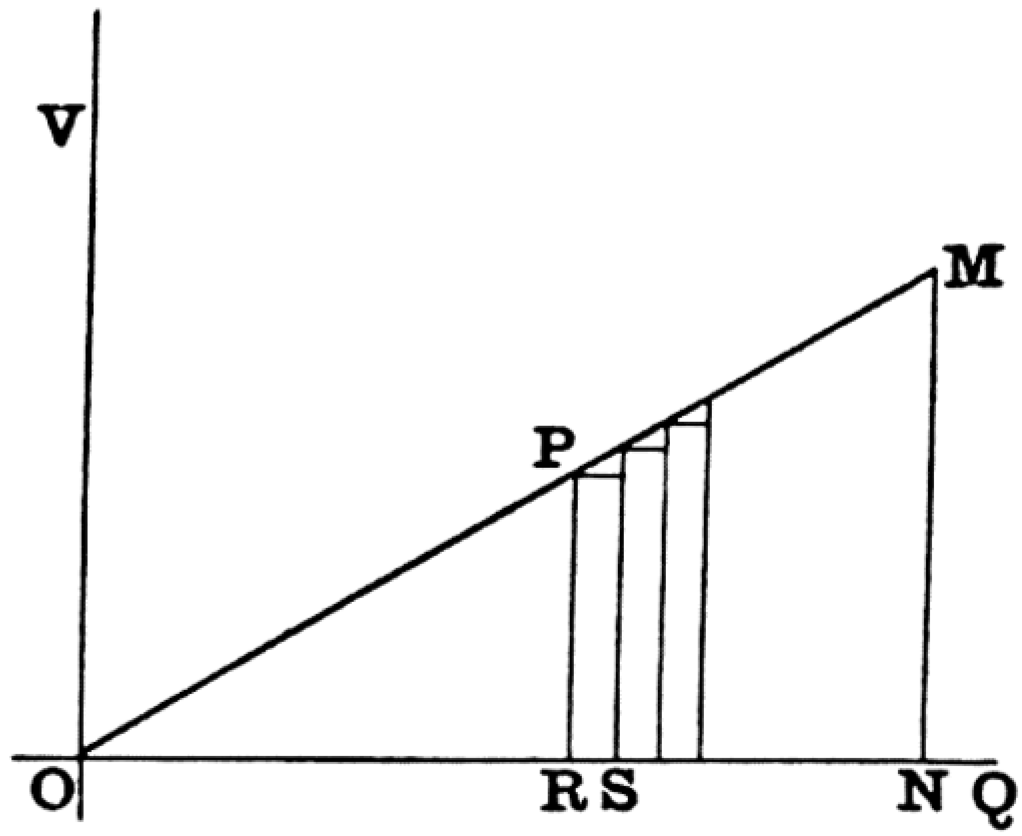


Fig. 265.

made with the inner coating by a projecting brass rod. The whole arrangement may be regarded as a plate condenser with glass as the dielectric.

Energy of a Charged Conductor.—The potential of a conductor being proportional to the charge on it the curve showing the relation between these quantities is a straight line; let OM, Fig. 265, be such a line. When the potential is v let a small additional charge q be carried to the conductor from the earth, during this process v may be treated as constant and the work done is vq. If PR represents v and RS the charge q, the work is represented by the rectangle PS. When further small quantities are brought up the work done will be represented by similar small rectangles; hence the whole work expended in charging the conductor from potential zero to that corresponding to MN is represented by the area of the triangle ONM, *i.e.* by $\frac{1}{2}$ON . NM. If the final charge and potential are Q and V respectively the energy accumulated is therefore $\frac{1}{2}QV$ ergs. Since

$Q = VC$, the energy on the conductor may also be written $\frac{1}{2}V^2C$ or $\frac{1}{2}Q^2/C$. In the case of a condenser Q is the charge on the insulated plate and V the P.D. between the plates.

The Energy is in the Medium.—According to the calculation just given the energy of a charge is concentrated on the conductor, but the following experiment shows that this does not completely represent the facts.

EXPERIMENT.—Charge a Leyden jar which is furnished with movable coatings, and place it on a sheet of ebonite. The inner coating and the glass may now be successively removed from the outer conductor without feeling any discharge, but if the jar is put together again it gives a spark when the plates are connected. On the other hand no spark is obtained if the glass is dis-electrified by passing it through a Bunsen flame.

This experiment is interpreted as showing that the energy is located in the dielectric itself, the plates merely serving as electrodes through which energy can run into or out of the intervening medium. Owing to the tension in the lines of force tending to draw together the opposite faces of the glass the latter is in a state of strain, if the condenser is very strongly charged the glass may be broken. Just as the bent rod (p. 248) possesses potential energy when it is strained so does the strained medium between the plates, and the work expended in charging the condenser is spent in creating this strain. If a Leyden jar is discharged by momentarily connecting the plates, a further spark may be obtained after a few seconds, showing that the glass does not at once recover from its strained condition. It is supposed as the result of such experiments that all the energy of the charge is in the strained medium. Let us calculate what must be the amount per cm.^{3} in the case of a plate condenser. With the previous notation the total energy is $\frac{1}{2}QV$ or $\frac{1}{2}V^2C$, where V is the P.D. between the plates. If the field intensity is F

$$V = F \,.\, d$$

and

$$C = \frac{KS}{4\pi d}$$

$$\therefore \text{ energy} = \tfrac{1}{2}V^2C = \tfrac{1}{2}F^2d^2 \cdot \frac{KS}{4\pi d}$$

$$= \frac{KF^2}{8\pi} \,.\, dS$$

$$= \frac{KF^2}{8\pi} \,.\, v$$

where v is the volume of the dielectric. Thus the energy per cm.^{3} is $KF^2/8\pi$. This result can be shown to hold for any conductor.

Tension in the Lines of Force.—We have supposed that the attraction between unlike charges is accounted for by the tension in the lines of force; let us calculate what their tension must be, taking the case of a plate condenser. Let T be the total pull on one of the plates due to this tension, and suppose it moves one plate towards the other through a small distance x. The work done is Tx and this is obtained at the expense of the energy in the condenser. Hence Tx represents the decrease in energy of the condenser. Before the displacement the energy was $\frac{1}{2}Q^2/C = \frac{1}{2}Q^2 \cdot \frac{4\pi d}{KS}$, afterwards it is $\frac{1}{2}Q^2 \cdot \frac{4\pi}{KS}(d-x)$, or the change in energy $= \frac{2\pi}{KS}Q^2x = Tx$. Putting $Q = S\sigma$ we get $T = 2\pi\sigma^2 S/K$. The pull per cm.^{2} is therefore $2\pi\sigma^2/K$. Since $\sigma = KF/4\pi$ (p. 440) this may be written $KF^2/8\pi$. As it depends only on F, the expression gives the stress per cm.^{2} on any conductor when the field at the surface is F. That the sideways repulsion of the lines varies with the medium is shown by an experiment of Quincke's.

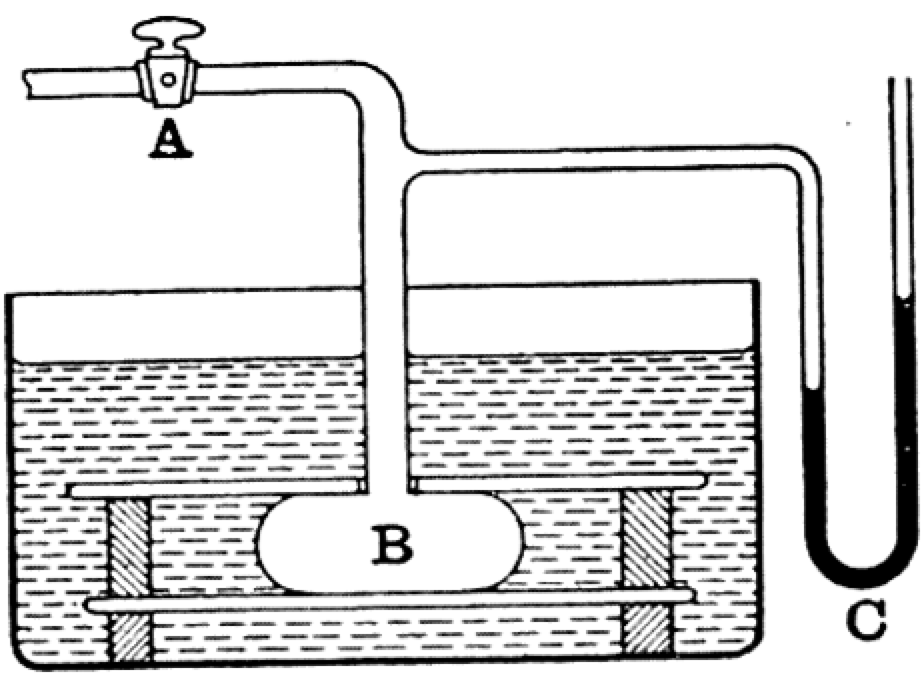


FIG. 266.—Quincke's Experiment.

EXPERIMENT. — Two large metal plates (Fig. 266), insulated from each other by bits of glass, are wholly immersed in paraffin oil. A bubble of dry air B is blown in through calcium chloride, the tap A is then closed and the pressure is read on the gauge C. The condenser is now strongly charged from an electrical machine (p. 449). The repulsion of the lines is greater in paraffin than in air and the bubble is forced to contract, thus increasing the pressure measured by the gauge.

Partition of Charges.—Let two separate conductors whose capacities are C_1 and C_2 and charges Q_1 and Q_2 be joined together by a thin wire; it is required to calculate the charge on each of them and their common potential after they are joined. The joint capacity

is $(C_1 + C_2)$ and the total charge is $(Q_1 + Q_2)$, if V is their common potential

$$Q_1 + Q_2 = V(C_1 + C_2)$$

or

$$V = \frac{Q_1 + Q_2}{C_1 + C_2}$$

If their original potentials were V_1 and V_2 respectively, $Q_1 = V_1C_1$ and $Q_2 = V_2C_2$.

$$\therefore\ V = \frac{V_1C_1 + V_2C_2}{C_1 + C_2}$$

After putting them in contact let Q_1' and Q_2' be the charges.

Then $$Q_1' = VC_1 \quad \text{and} \quad Q_2' = VC_2$$

$$\therefore\ Q_1'/Q_2' = C_1/C_2.$$

The charges therefore distribute themselves proportionately to the capacities.

Capacities in Series.—When the inner coatings of a number of Leyden jars are joined together and the outer coatings are similarly connected the condensers are said to be in parallel. If the outer coating of one is joined to the inner coating of the next, and so on, they are said to be in series or in cascade. In the former case the joint capacity is the sum of each; let us calculate what is the joint capacity in the second arrangement. Suppose there are three condensers whose capacities are C_1, C_2, C_3 (Fig. 267), and let a charge Q at potential V be given to the first while one plate of the last is earthed. Then $+Q$ units run from the lower plate of the first into the upper plate of the second, a similar transfer takes place at the second and third, and finally a charge Q runs to earth. Let $(V - V_1)$ be the P.D. between the plates of the first condenser, $(V_1 - V_2)$ the corresponding quantity for the second, $(V_2 - 0)$ for the third.

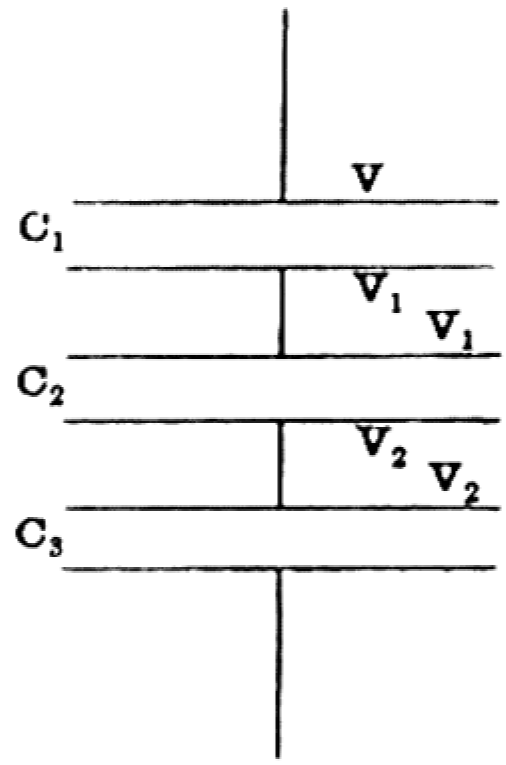


Fig. 267.—Condensers in Series.

Then since the charge on each condenser is Q

$$V - V_1 = \frac{Q}{C_1}$$

$$V_1 - V_2 = \frac{Q}{C_2}$$

$$V_2 - 0 = \frac{Q}{C_3}$$

$$\therefore V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

If a single condenser is charged to a potential V by the charge Q its capacity C is given by

$$V = \frac{Q}{C}$$

$\therefore$ The capacity C of the three in series is thus given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

EXAMPLES ON CHAPTER XLI

1. Two equal small insulated spheres, placed 10 cms. apart, are charged respectively with 5 and 20 units of electricity. The spheres are made to touch and are then replaced; what are the forces between them before and after contact respectively? (L. '88.)

2. Two spheres, of diameters 9 and 3, are connected by a long thin wire and 144 units of electricity are shared between them. Compare their charges. From which sphere would a brush discharge first occur if their joint charge were gradually increased?

3. A condenser A has plates area 1000, and a dielectric of thickness 4; another condenser B has plates area 800 and the same dielectric of thickness 5. Compare the charges and energy in A and B when they are connected, A to a source of potential 4, and B to a source of potential 5. (L. '96.)

4. What is electrostatic capacity? What is the capacity of a condenser made of sheet glass 2 mm. thick, with tinfoil coatings each 30 cms. square, if the specific inductive capacity of glass is 7·5? (L. '04.)

5. Explain what is the meaning of the term "lines of electric force," and

what inference may be drawn from their distribution. How many lines of force approximately will there be per cm.^{2} in the space between the parallel plates, 10 cms. in diameter, of an air condenser charged with 250 electrostatic units of electricity? (L. '04.)

6. Two hollow conducting spheres, of radii 3 and 10 cms. respectively, are each completely insulated, the centre of the larger sphere being on the surface of the smaller. The smaller sphere has a charge of 2 electrostatic units and the larger a charge of 4. Find the force (*a*) at a point outside the larger sphere, (*b*) at a point inside the smaller sphere. (L. '08.)

CHAPTER XLII

ELECTRICAL MEASUREMENTS AND MACHINES

Comparison of Capacities.—In addition to the method given on p. 388, capacities may be compared with the help of a quadrant electrometer. Let the capacities be C_1 and C_2; earth one pair of quadrants and connect the other pair to the conductor whose capacity is C_1. Charge it to a suitable potential V_1 and note the deflexion θ_1. Connect the second conductor to the first so that the charge is shared between them; the potential falls to V_2 and the deflexion to θ_2. If Q is the charge initially given to the first conductor $V_1 = Q/C_1$; after the division of the charge, $V_2 = Q/(C_1 + C_2)$.
Hence, dividing one by the other,

$$\frac{C_1 + C_2}{C_1} = \frac{V_1}{V_2} = \frac{\theta_1}{\theta_2}$$

or

$$\frac{C_2}{C_1} = \frac{\theta_1 - \theta_2}{\theta_2}$$

When it is the capacities of condensers that are being compared one pole of each is connected to earth during the measurement. It has been assumed that the capacity of the quadrants themselves is negligible, if this is not allowable C_1 must be taken to be the joint capacity of the quadrants and the first conductor (see ex. 7, p. 451).

Attracted Disc Electrometer.—It has already been shown for a plate condenser that the pull on one of the plates due to the electrical charges is $T = 2\pi\sigma^2 S/K$. If V is the P.D. between the plates $V = Fd = \frac{4\pi\sigma \, . \, d}{K}$, where F is the field in the space between the plates and d the distance between them. Hence

$$\sigma = KV/4\pi d \text{ and } T = \frac{K}{8\pi} \cdot \frac{V^2}{d_2} \cdot S \qquad \left(i.e. \ \frac{KF^2S}{8\pi}\right)$$

or

$$V = d\sqrt{\frac{8\pi T}{KS}}$$

Thus if the pull T in dynes is measured when the substance between the plates is air, for which $k = 1$, the potential V may be found. This is the principle of the attracted disc electrometer. The formula has been obtained on the assumption that the field between the plates is uniform; to fulfil this condition the attracted circular disc B (Fig. 268) on which the pull is to be measured forms the central portion of a much larger plate, the two portions being in the same plane and separated from each other by a narrow air gap. The outer part is called the guard-ring. To ensure that no lines reach the upper side of the plate, B, together with the guard-ring, forms the bottom of a cylindrical metal box A. The lower plate C can be moved up and down by a micrometer screw. When the plates are uncharged B is pulled slightly above the plane of the guard-ring by a spring D, but when there is an electrical field between them B is pulled downwards by the tension in the lines of force and may be brought into the plane of the guard-ring. This position is reached when a small pointer (not shown in the figure) stands at its zero. The force required to produce the requisite extension of the spring is measured once for all by placing weights of m gms. on B until the zero position is reached, then $T = mg$ dynes. To determine the potential of a conductor the cylindrical box is connected to a Leyden jar and charged to a high potential V_1; on account of the large capacity the potential will remain constant in spite of small leakages. The lower plate is earthed and its position adjusted until B is in the plane of the guard-ring; let d_1 be the distance apart of the plates. C is next insulated, connected to the conductor whose potential V is required, and the distance again altered until the pointer is at zero. Let d_2 be the new distance between the plates.

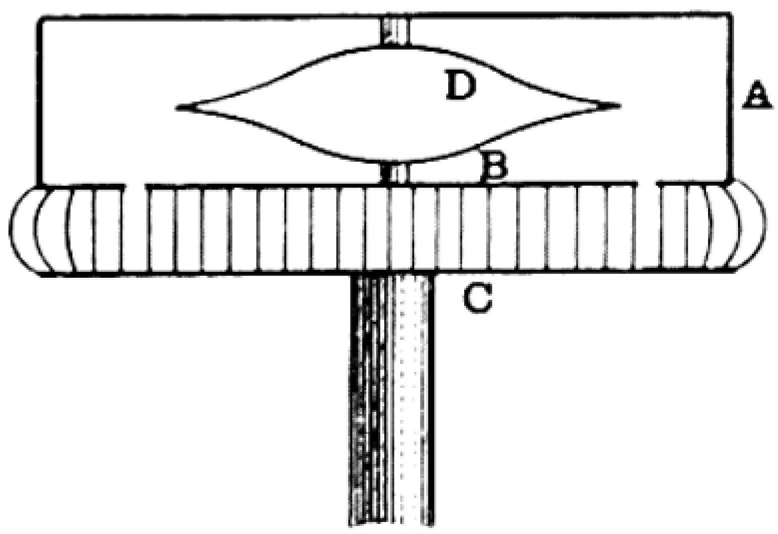


FIG. 268.—Attracted Disc Electrometer.

Then $$V_1 = d_1\sqrt{\frac{8\pi T}{KS}},$$ where S is the area of plate B,

and $$V_1 - V = d_2\sqrt{\frac{8\pi T}{KS}}$$

Hence $$V = (d_1 - d_2)\sqrt{\frac{8\pi T}{KS}}$$

The distance between the plates would be difficult to determine, but the difference in distance $(d_1 - d_2)$ can be found accurately by means of the micrometer screw. The instrument is chiefly of historical interest, potentials are now more usually measured by comparison with a standard cell.

Measurement of Dielectric Constant.—Any method of comparing two capacities accurately will suffice to measure dielectric constants. The capacity C_1 of an air condenser is measured by comparing it with a standard, the space between the plates is then filled with the dielectric and the new capacity C_2 is found. The ratio C_2/C_1 is the dielectric constant. The methods of pp. 388 and 446 may be used. In the case of a liquid the principle of the attracted disc electrometer is also available. The two plates are separated about 2 mm. by three strips of glass and are placed in a horizontal position in a glass vessel. The upper plate hangs from the arm of an ordinary balance and is weighed. The plates are then charged from a battery and the additional weights found which are necessary to balance the electrical attraction. Liquid is next poured in to cover the plates and the new pull due to the charges is found, the P.D. being the same as before. From the last paragraph $T = \frac{k}{8\pi} \cdot \frac{V^2}{d^2} \cdot S$, thus the ratio of the attraction in the second case to that in the first is the dielectric constant. After pouring in the liquid the weights must be adjusted with the plates uncharged, on account of the apparent loss of weight of the suspended plate.

Electrical Influence Machines.—On p. 351, a method has been described of electrifying a conductor by carrying to it a succession of charges obtained by electrical influence. This illustrates the principles of electrical influence machines. If we analyse the process it is seen that an insulated conductor, the carrier, is placed near a charged body, the inductor, and is then earthed; it thus receives by influence a charge opposite in sign to that on the inductor while an equal quantity of electricity of the other kind runs to earth. The electricity on the carrier is then delivered to the body whose charge it is desired to increase. These processes are carried out

very elegantly in the Wimshurst machine and in a simpler manner in the electrophorus.

The Electrophorus.—This consists of a thin disc of ebonite resting on an earth-connected metal plate and a carrier formed of a metal plate supported by an insulating handle. The upper surface of the disc is negatively electrified by friction with a woollen rubber and the carrier is placed on it; owing to irregularities of the surfaces contact occurs at only a few points so that we may regard the two as merely placed close together. The carrier is momentarily earthed by touching it with the finger, when negative electricity flows to earth; it may now be raised by the handle and is found to be positively charged. A series of charges may be obtained in this manner without rubbing the ebonite afresh. Let us study the process of charging from the potential point of view with the help of a gold-leaf electroscope.

EXPERIMENT.—Insulate the metallic base by placing it on a block of paraffin and connect it to an electroscope (Fig. 269). Gently rub the ebonite with catskin; the negative charge produced lowers the potential of the base and positive electricity runs into it from the electroscope as the rubber is raised, the leaves therefore diverge with negative electricity. Place the carrier on the ebonite, the leaves collapse slightly and the charges are distributed as in the figure. Owing to the negative charge on the ebonite the potential of the carrier is less than that of the earth, hence when it is earth connected a positive charge runs into it and raises its potential to zero at the same time raising the potential of the base. Positive electricity therefore runs from the latter into the leaves and causes them to collapse. Remove the carrier, its potential rises as it gets further from the charge on the ebonite; also the potential of the base decreases and positive electricity flows to it from the electroscope causing the leaves to diverge with a negative charge.

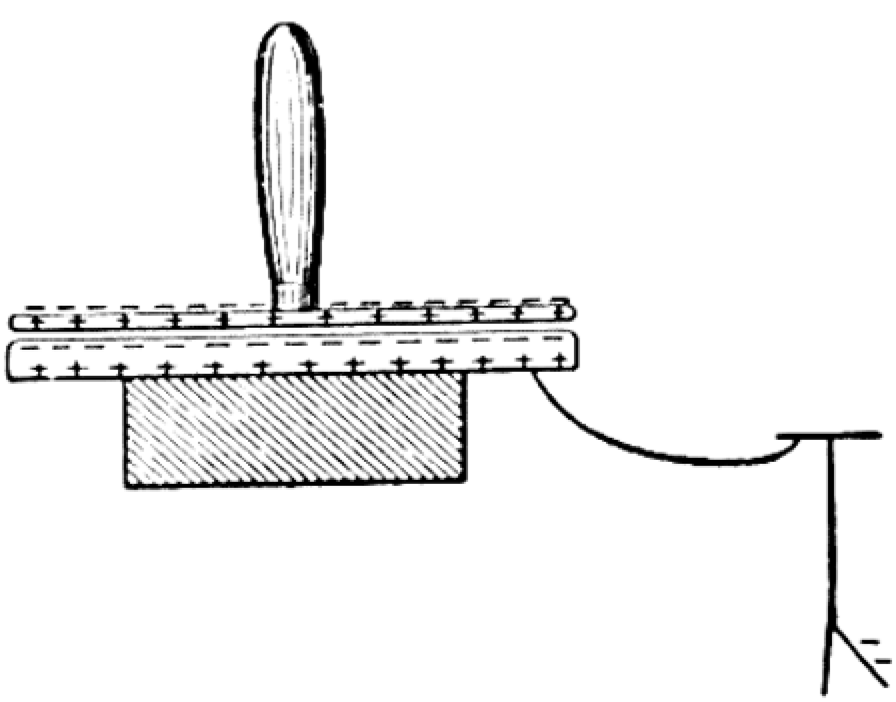


FIG. 269.

The Wimshurst Machine.—In this apparatus the carriers take the form of tinfoil strips stuck radially on the outer sides of two glass discs which can be rotated in opposite directions by pulleys and strings. Each strip when charged is also made to act as an

inductor to the strips which are near it on the other plate. The conductors to be charged carry a comb of fine points, called the collecting brushes; when a charged carrier passes these its charge is neutralised by a stream of electricity of the opposite sign coming from the points (p. 437), thus leaving on the collecting conductor a charge of the same sign as was on the carrier. The earthing of the carriers is brought about by small wire brushes which touch them twice during a revolution.

In order to render explanation easier we will suppose the plates are replaced by cylinders turning the one within the other (Fig. 270).

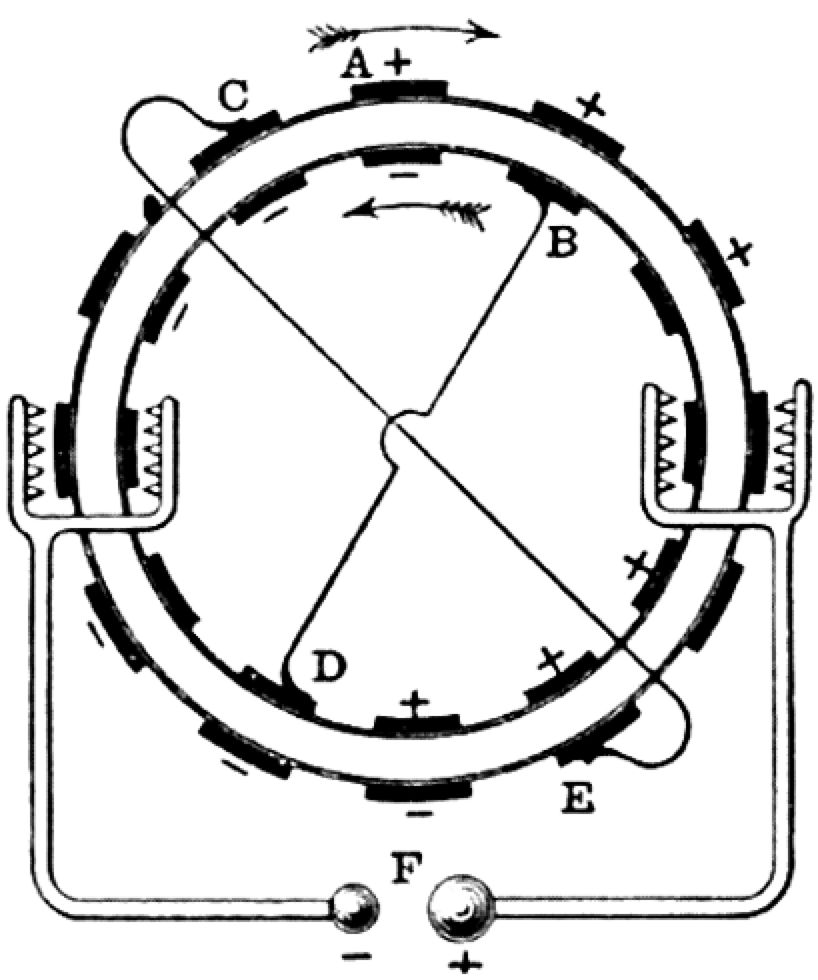


FIG. 270.—Wimshurst Machine (diagrammatic).

Let the rotation be in the direction of the arrows and suppose a small positive charge is given to one of the carriers at A. As this moves to the right it induces a negative charge on any strips that are in contact with the brush at B. These negative charges moving to the left will, in a similar manner, induce a positive charge on carriers in contact with C. Thus all the strips on the upper half of the outer cylinder become positively charged in turn while the corresponding ones on the inner cylinder carry negative charges. A similar process takes place on the lower halves of the cylinders. When a negative charge is induced on a strip at B an equal quantity of positive electricity runs along the brush holder and is shared with a strip at D. Hence all the carriers passing the latter point are positively electrified, and they in turn induce negative charges on the strips which pass E. The result is that all the carriers approaching the collector on the right are positively charged, while those moving to the left take with them negative electricity. These charges are collected by the combs and raise the P.D. between the conductors attached to them to such an extent that sparks pass between the discharging knobs F. With a large machine, electrolytes can be decomposed and conductors heated exactly as with the current produced by cells, the electricity is of

the same nature in each case. The only differences are in the potentials produced, and the quantity of electricity in motion. A Wimshurst may produce a potential of many thousand volts, but the current will be small owing to the large internal resistance that it encounters in the machine.

EXAMPLES ON CHAPTER XLII

1. A, B, and C are three Leyden jars, equal in all respects. A is charged, made to share its charge with B, and afterwards to share the remainder with C. The three jars are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge. (L. '84.)

2. A battery of 4 ohms resistance is sending a current through an external resistance of 6 ohms. The poles are connected to a quadrant electrometer and the deflexion of the needle is 100 divisions. What will be the deflexion when, everything else remaining the same, the external circuit is broken? (L. '87.)

3. Two hollow, concentric, conducting spheres of radii 4 and 12 cms. are insulated and the outer given a charge of 20 and the inner of 5 E.S. units. Find the intensity of the electric field at points distant from the centre 3, 9, and 15 cms. respectively. (L. '10.)

4. How would you compare the charges on two small, irregularly shaped, conductors?

5. An air condenser with plates 10 cms. square and $\frac{1}{2}$ cm. apart is charged with 100 electrostatic units of electricity. Find the loss of energy when it is plunged under oil of specific inductive capacity 2. (L. '08.)

6. Find in the last question the force of attraction between the plates in the two cases.

7. One pair of quadrants of an electrometer are earthed and the other pair are charged until the deflection is θ_1. This charge is now shared with a condenser of capacity C and the deflection falls to θ_2; find the capacity of the quadrants.

CHAPTER XLIII

TECHNICAL APPLICATIONS OF ELECTRICITY

IN the remaining pages a brief account will be given of various electrical appliances, so far as they can be understood from the principles already explained.

Incandescent Lamp.—This consists of a very fine filament which is heated to incandescence when a suitable current is sent through it (Fig. 271). The object aimed at is to ensure that a large fraction of the energy supplied shall be converted into the radiations of the visible spectrum; the accompanying infra-red radiations of course require energy for their production, they should be reduced as far as possible. It is found best for this purpose that the temperature of the filament should be high, hence substances of high melting-point, such as tantalum and tungsten, are now generally used. To hinder cooling by convection and conduction the filament is enclosed in a highly exhausted glass bulb, connection being made with the exterior by means of platinum wire fused through the walls; this arrangement also prevents possible oxidation. The resistance of the rest of the circuit is kept low. The lamps in a building are connected in parallel and the lighting company is required to maintain a nearly constant voltage at the supply terminals. Suppose one lamp is in circuit and a second is then switched on; the resistance is halved and the total current doubled, but the current going through the first lamp is unaltered. The light given out by any lamp is therefore independent of the number that are being used. The unit of energy for supply purposes is the kilowatt-hour, *i.e.* 1000 watts for 1 hour;

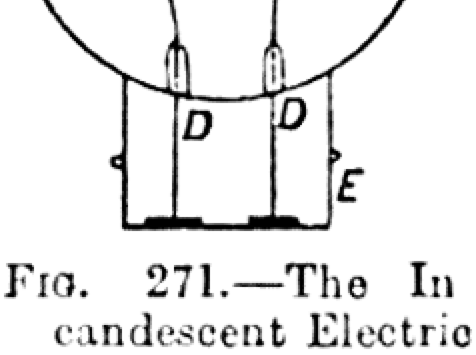


FIG. 271.—The Incandescent Electric Lamp.

it is called the Board of Trade Unit, and costs, according to the locality, from $1\frac{1}{2}d.$ to $6d.$

The Arc Lamp.—In an arc lamp, light is produced by heating two carbon rods by means of energy supplied electrically. When the ends of the rods are made to touch and a current of several amperes is sent along them some of the carbon is vaporised; if now they are separated by a few mms., the current passes through the vaporous arc causing a vigorous emission of light. Owing to its high temperature, between 3000° and 4000° C., the arc is one of the cheapest methods of light production. The rays proceed chiefly from the end of that rod which is connected to the positive pole of the supply. This is a great advantage in the lantern, as it is easier to correct the lens system for what is practically a point source. In a self-regulating lamp the length of the arc is adjusted automatically. One method of doing this is as follows: The upper carbon is attached to a piece of soft iron which projects into a solenoid in series with the arc. When the carbons get too close together more current flows, and the iron is pulled up by the magnetic field of the coil; if the gap is too large the current is reduced and the upper carbon falls again.

Electrical Furnaces.—Electrical furnaces can be divided into two main types, (1) resistance type, (2) arc type. The former are used for annealing tool steel, by jewellers for firing enamel ware, and generally for raising the temperature of small bodies not higher than about 1500° C. In construction they consist of a heating coil of wire wound round a tube of fire-clay, the whole being well lagged with material of poor thermal conductivity to prevent heat losses. The substance to be heated is placed in the fire-clay tube. The second type is used for the production of the highest possible temperatures. They consist essentially of an arc capable of carrying a large number of amperes; the substance to be treated is placed immediately below the arc and the whole is surrounded with fire-brick. Electrical ovens are merely a special form of the resistance type. For either kind to be worked economically they must be used at places where power is cheap; a number are in use near Niagara Falls—the great head of water which is available there provides a source of energy which is easily tapped. The kinetic energy of the stream is made to turn water turbines which are yoked to suitable dynamos (p. 458).

Electric Bells.—The construction of these will readily be understood by referring to Fig. 272. F is the bell gong and E the hammer,

the latter is attached to a piece of soft iron D which carries a light spring C. B is a screw which is just in contact with the spring, H is an electromagnet. S, R, are two strips of metal which can be brought into contact by pressing the "bell-push" P. This contact completes the electrical circuit of one or more Leclanché cells and a current flows round the electromagnet, through GCB, and back to the battery. The iron keeper D is consequently attracted by the magnet and the hammer strikes the gong. But the movement of D breaks the circuit at C, H therefore loses its magnetism and the keeper falls back; the cycle of operations is thus repeated.

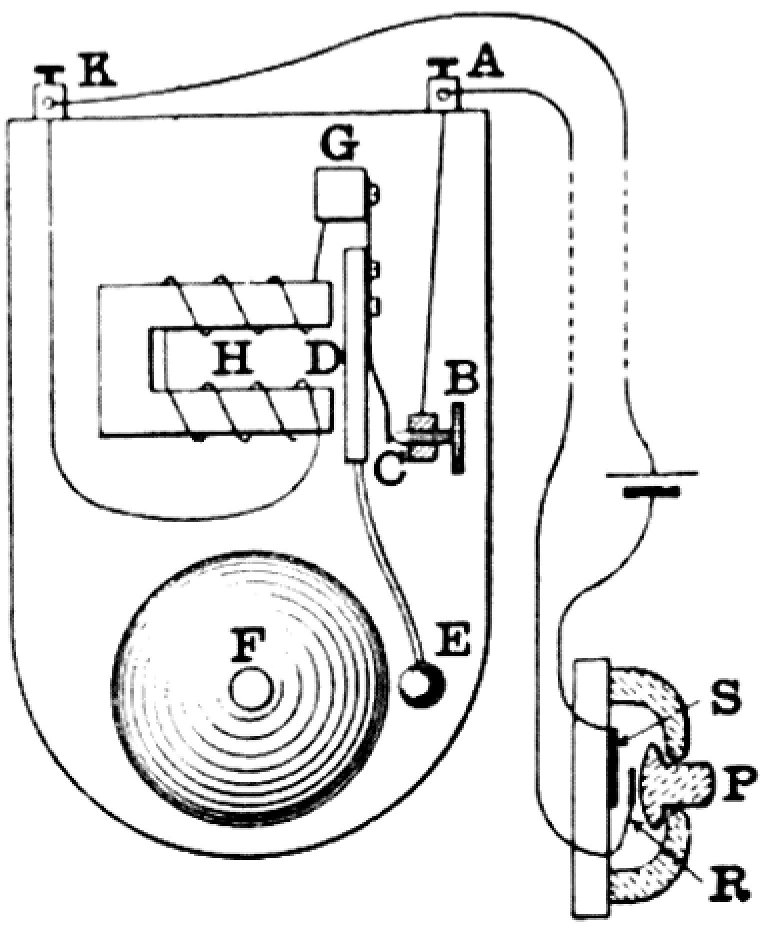


FIG. 272.—Electric Bell.

Telegraphy. — Modern telegraphic and telephonic apparatus is too complicated for us to do more than indicate briefly the actual methods in use. A telegraphic installation includes a sending and receiving apparatus at each station and a conducting wire, or "line," connecting the two places. It was discovered in 1837 that a return wire was unnecessary as it was found that the earth is a good enough conductor for the purpose. The receiving instrument takes various forms. In some cases it consists of a vertical galvanometer whose needle turns round a horizontal axis after the manner of a dip needle. When the current flows it deflects the needle to the right or left and a metallic pointer, situated on the outside of the instrument, strikes sharply on one or other of two small stops; such an apparatus is called a sounder. By a suitable code, depending on the direction of the deflexion and the interval between successive sounds, the signals can readily be interpreted as letters or numbers. A special key AC (Fig. 273) is used at the sending station to reverse the current; one method of arranging the circuit is shown in the figure. The equipment at each station is similar and therefore one only need be described. F is the vertical galvanometer, E a large metal plate sunk in the earth, L the line connecting the two stations. The battery is connected to the cross bars A, B, of the key. Normally

the brass strips C, D, are in contact with A, and one pole of the battery is insulated ; but if one of them, say C, is depressed so as to touch B the contact with A is broken and a current flows in the direction ALA′F′E′EFC ; the operator at F′ thus receives a signal. If D is pressed down instead the current flows in the reverse direction. In the Morse receiver, which is frequently used, the current flows round the coils of an electromagnet ; this attracts an inked wheel and causes it to print long or short dashes on a moving sheet of paper. If the line is long the received current may be too weak to work the inking mechanism. In such cases the electromagnet is made to attract a

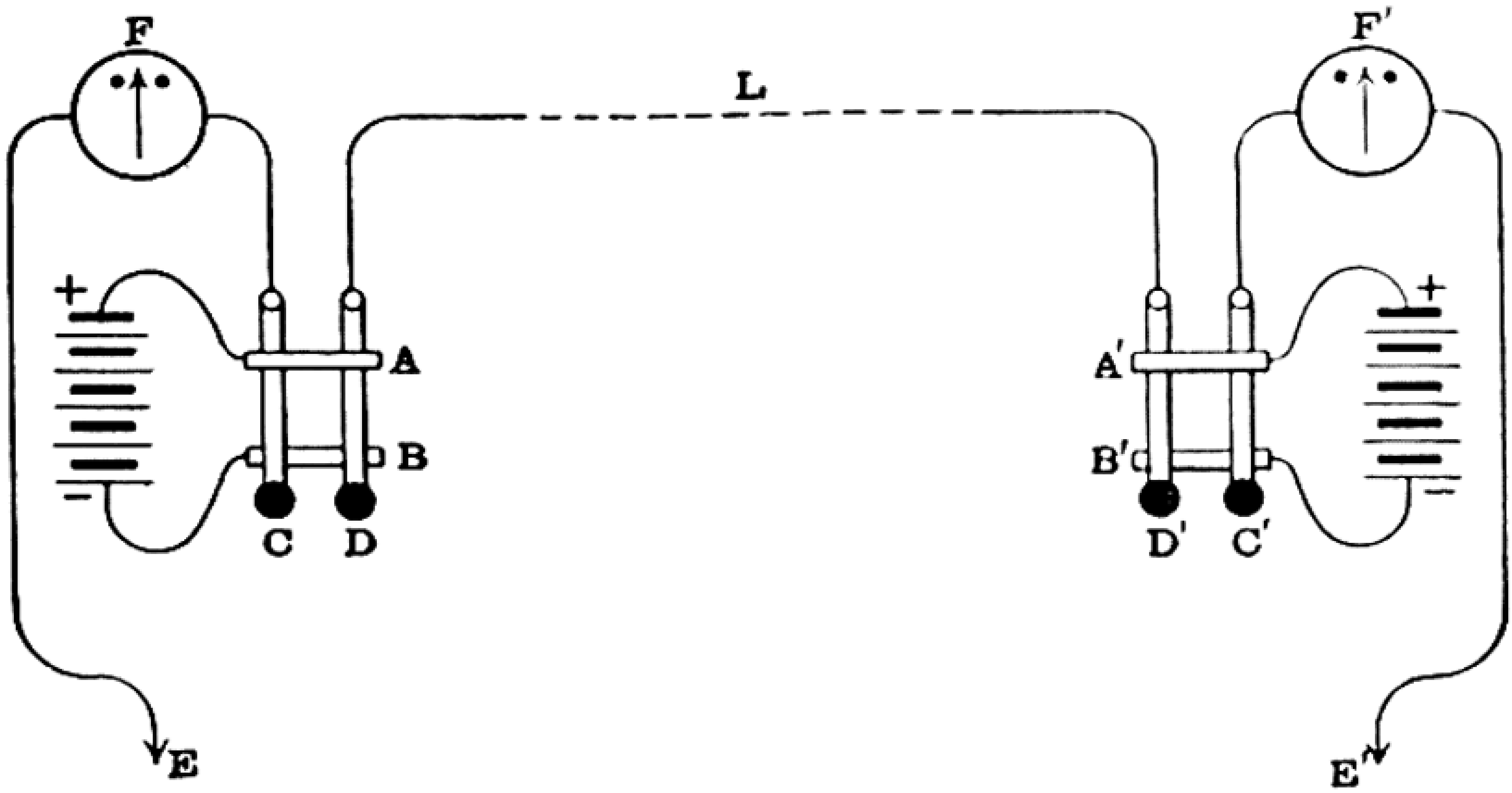


FIG. 273.—Telegraph Circuit with Reversing Key and Sounder.

light keeper, and the movement of the latter completes a circuit consisting of a local battery and the Morse or other receiving instrument. The necessary energy for working the receiver is thus derived from the local battery, which can be made powerful enough for the purpose. An arrangement of this kind is called a relay. Fig. 274 shows a method of arranging the circuit, the relay being omitted. K is a Morse key which, at rest, is connected with the Morse inking apparatus M ; one pole of the battery is then insulated. If the key is depressed the connection with M is broken and the battery is connected to the line ; current then flows to the station on the right, through M′ and back to E *viâ* the earth. G is a sensitive galvanometer to show the sender that the current is actually passing. For submarine work the receiver is the moving coil of a galvanometer

(Fig. 253) which carries a special kind of pen in place of a pointer. By its means the signals are traced on a moving sheet of paper. This apparatus, invented by Lord Kelvin, is called a siphon recorder.

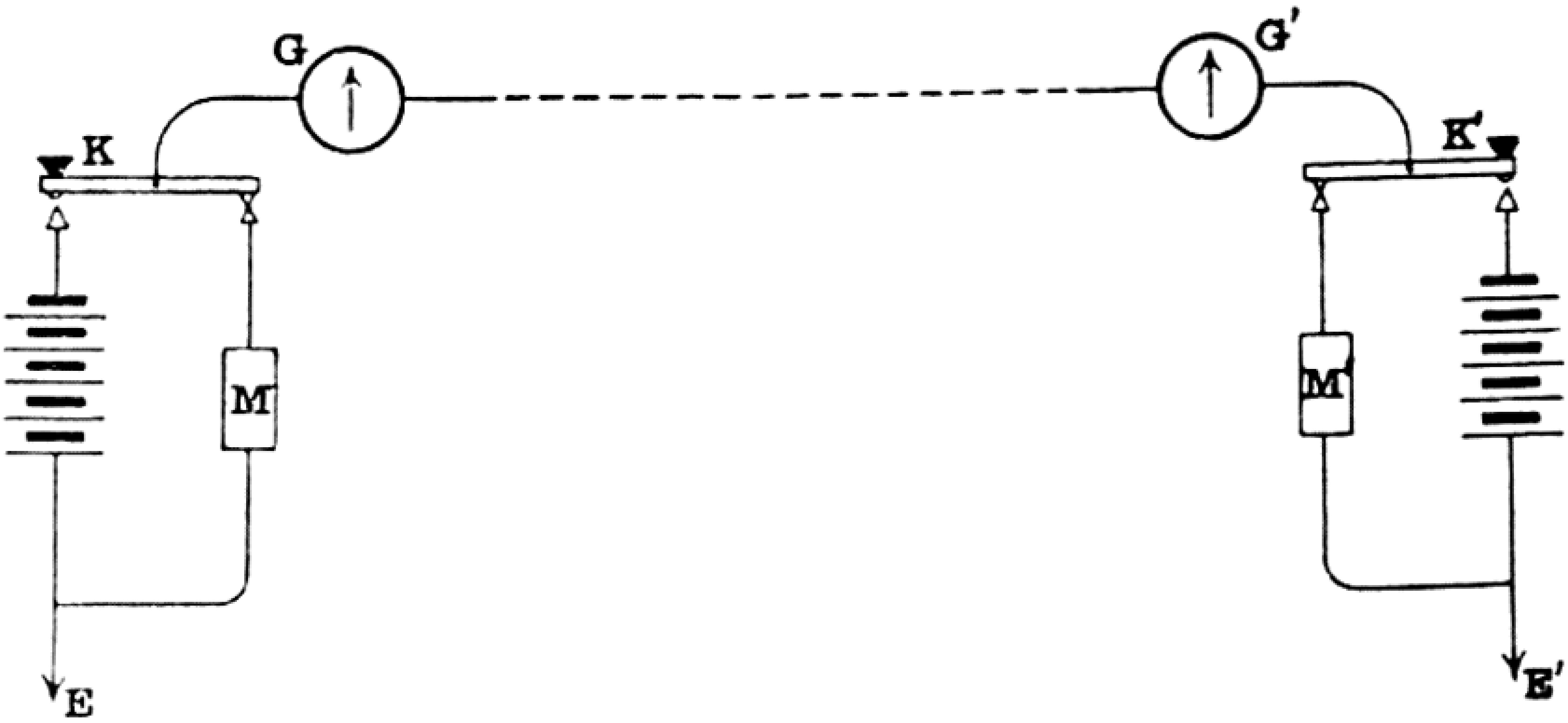


FIG. 274.—Telegraph Circuit with Morse Key.

Telephony.—A modern form of telephone receiver is shown in Fig. 275. It consists of a mouthpiece M closed by a thin sheet of iron, the diaphragm, D; immediately behind this are soft iron pole pieces, P, fastened to the poles of a permanent horseshoe magnet L. A number of turns of fine wire are wrapped round the pole pieces and their ends are brought to the terminals T. S is a screw which passes through the ebonite cover to hold the horseshoe in position. By contact with the magnet the pole pieces are kept magnetised in a condition corresponding to the steep part of the curve in Fig. 260; hence any small change in the magnetic field causes a considerable variation in their pole strength. In the early days of telephony, an apparatus of this kind was employed both as transmitter and receiver. Let two such instruments be connected by a pair of wires, and suppose words are spoken into one of them. The pressure variations in the air cause the diaphragm to vibrate, and it alternately approaches

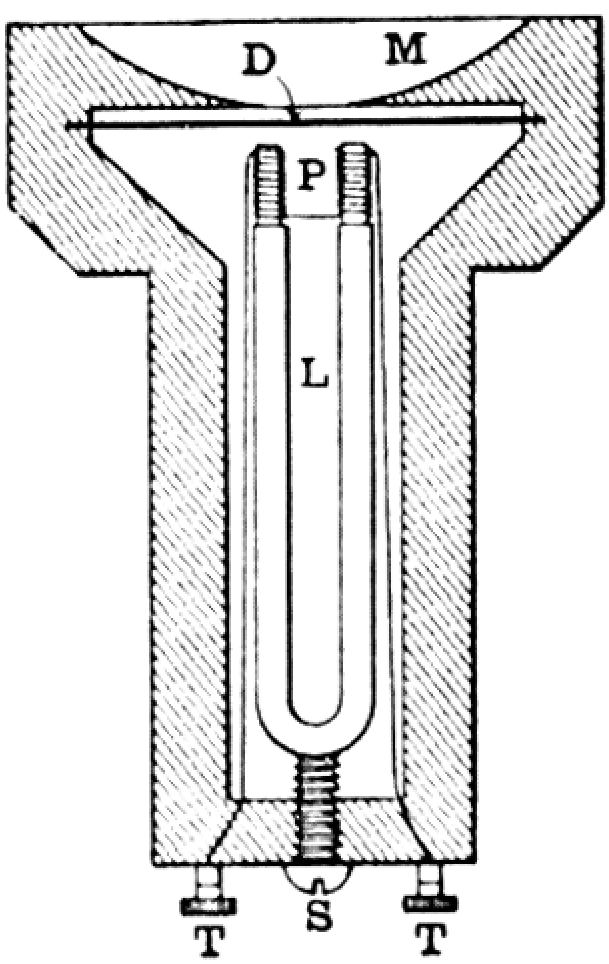


FIG. 275.—Telephone Receiver.

to and recedes from the pole pieces; as D is made of iron this is equivalent to moving a keeper to and from the poles, and the density of the lines of magnetic force is changed in the region occupied by the coils. Induced currents are therefore set up, which are transmitted to the second instrument. But a current passing round the pole pieces at the distant station changes their magnetism appreciably, and the second diaphragm is also made to vibrate. These vibrations set the air in motion and the sounds are reproduced at the receiving station. As the currents produced by such means are very feeble it has been found necessary to devise some other method of transmission, the original instrument being retained as a receiver. The construction of a transmitter will be understood from Figs. 276 and 277. A pointed piece of carbon B (Fig. 276) rests in two notches cut in two

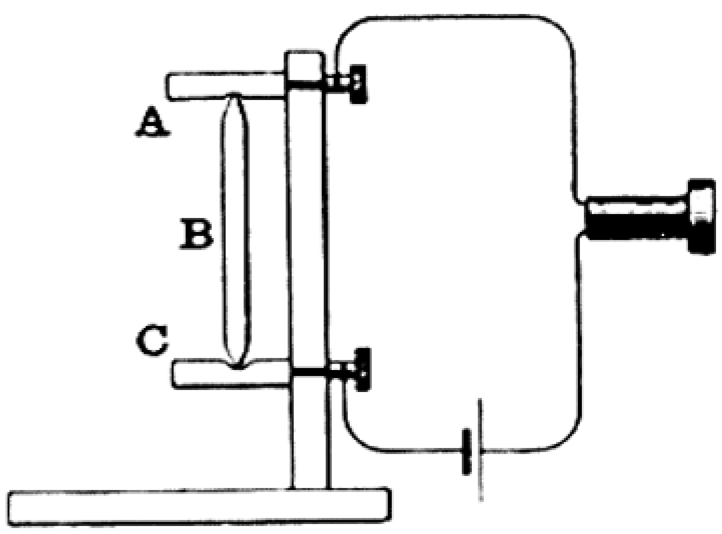


Fig. 276.—Microphone.

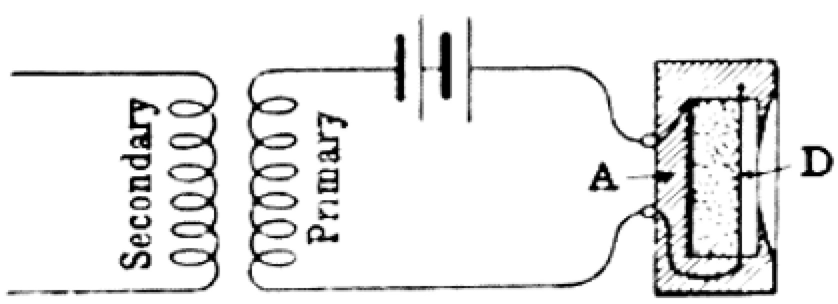


Fig. 277.—Hunnings' Transmitter.

carbon plates A, C; a single cell and a telephone receiver complete the circuit. It is found that any slight vibration causes the resistance at the carbon contacts to vary between wide limits, hence the current undergoes corresponding variations. The ticks of a watch placed on the table are heard like strokes on an anvil if the ear is applied to the telephone. This apparatus, called the Hughes microphone, illustrates the principle on which modern transmitters are based. A diagrammatic representation of the Hunnings transmitter, which depends on the microphone principle, is shown in Fig. 277. D is the thin diaphragm as before, A is a metal plate; the 2 mm. space between them is filled with granular carbon, and both A and D are connected to wires. The whole forms a sensitive microphone, the resistance of the carbon being very sensitive to vibrations. A local circuit is formed which includes the microphone, two or more cells, and the primary of a small induction coil. The coil secondary is connected directly to the two line wires.

The vibrations of the diaphragm cause the current in the primary coil to vary, and by the well-known action of the coil a smaller current at a higher E.M.F. is transmitted along the line. For short distances the induction coil can be dispensed with. If an earth return is used it is found that the inductive effects of neighbouring circuits produce such a constant hum in the receiver that intelligible speech is impossible. Disturbances of this nature can be largely reduced by employing a properly placed return wire.

Dynamos.—A dynamo is a machine for converting mechanical into electrical energy. In the form described below the mechanical energy derived from a steam or other engine is used to rotate a coil in a strong magnetic field; this, as we have seen, produces induced currents. As shown on p. 420, the induced E.M.F. changes its direction at each half-revolution; hence if the ends of the coil are connected separately to two insulated metal rings on the axis of rotation, an alternating current can be sent round an external circuit through metal brushes which press on the rings. Such a machine is called an alternating current dynamo. For lighting incandescent lamps an alternating current may be used, but for many purposes, *e.g.* charging accumulators, a direct current is necessary. In these cases the alternating current in the coil must be converted into direct current in the external circuit by means of a suitable commutator. Let the coil in Fig. 278 rotate in the magnetic field with its ends connected to the halves of a brass split ring AB fixed on the revolving shaft. The metal brushes C and D lead to the external circuit and are alternately in contact with A and B as these revolve. The brushes are so arranged that, at the moment the E.M.F. in the coil is changing its sign, A leaves C and reaches D; the external current is therefore always in the same direction. This is the principle of the direct-current dynamo. With such a simple apparatus it is clear that the current falls to zero at each half-revolution: to remedy this defect a number of coils are used, arranged so that when one is least active, another is producing its maximum current. Fig. 279 shows diagrammatically four such coils connected to a four-part commutator; the dotted lines represent the brushes and connections to the external circuit, N and S are the poles of a strong electromagnet. By referring to Fig. 257, it is seen that the current in a coil changes its direction at the moment when the number of lines of force enclosed is a maximum. Hence in Fig. 279 the induced E.M.F. in the top and bottom coils is just on the point

of reversal while the other pair are providing the maximum E.M.F. The result is to produce a current towards one and away from the other brush. In practice a much larger number of coils is used and the commutator is subdivided in a corresponding ratio. In order that the lines of force from the magnet may be concentrated through the coils the latter are wound on an iron core. This is built up of flat iron rings so as to prevent eddy currents; core and coils together form the armature. The method of winding just described was invented by Gramme; in modern machines it

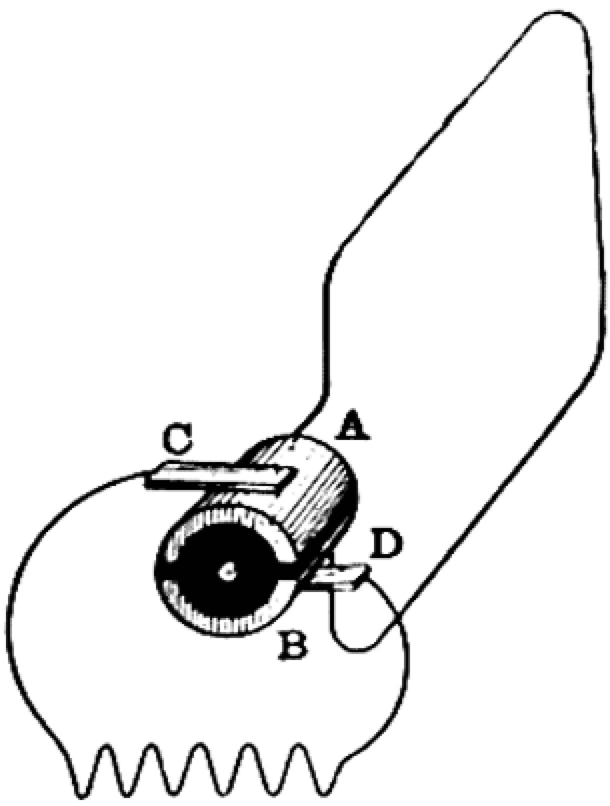


FIG. 278.—Coil and Commutator.

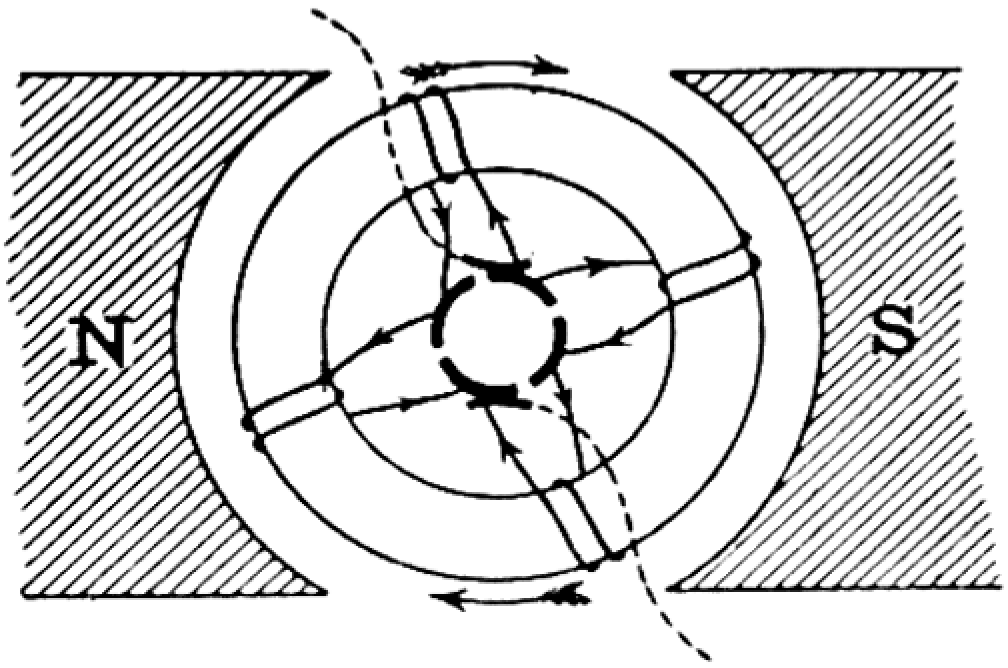


FIG. 279.—Diagrammatic Representation of a Four-Coil Armature.

is superseded by more efficient but complicated methods. Direct-current machines are self-exciting, *i.e.* they themselves provide the current to excite the electromagnet. When a machine is started there is usually sufficient magnetism in the magnet core to produce a feeble current in the armature, the whole or part of this is made to traverse the magnet coils and the field rapidly increases in strength. In Fig. 280, two methods of connecting the magnet coils and armature are represented. In Fig. (*A*) the whole current passes round the external circuit R and the magnet connected in series with it; this dynamo is said to be series-wound. The current from a machine of this type varies greatly with the conditions in the external circuit; when the resistance is increased the current traversing the magnet is decreased and the E.M.F. generated is reduced also, since it depends on the field strength. Both changes contribute to a fall of current. Fig. (*B*) shows a

shunt-wound machine. In this case the external circuit and the magnet coils are in parallel and the current can pass through the one or the other. The coils round the magnet are made up of a large number of turns of fine wire so that little current passes through them and yet a strong field is produced. When the resistance of the external circuit is increased a larger fraction of the current is diverted to the magnet, the field therefore increases and the E.M.F. generated is raised. Hence a shunt-wound machine tends to regulate itself so that a constant current is sent round a circuit of varying resistance. This renders it particularly useful for the charging of accumulators. With either type the output is of course limited by

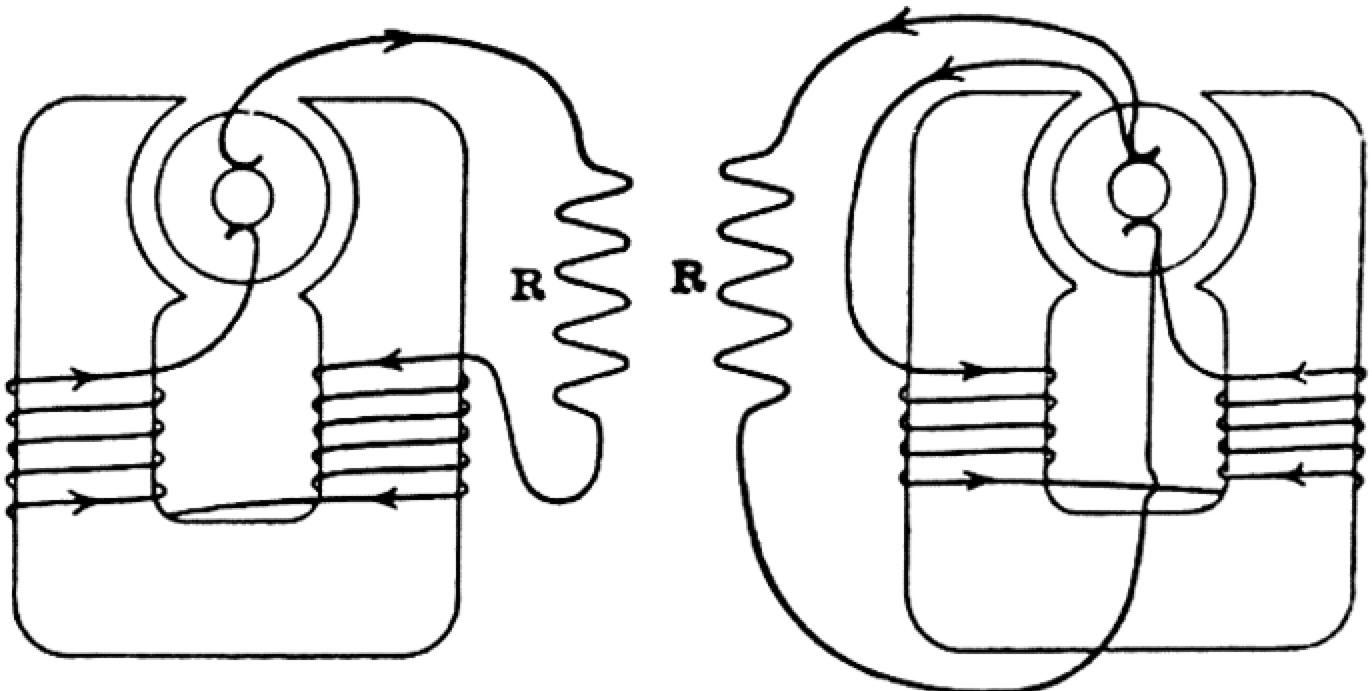


FIG. 280.—(*A*) Series-wound; (*B*) Shunt-wound Dynamo.

the power available for driving it; as the current increases it requires a greater effort to turn the armature, for it must be remembered that the induced currents are in such a direction as to oppose the motion. For lighting purposes the dynamo is required to maintain a constant E.M.F. at the terminals of the lamp circuit, no matter how many lamps are in use. A combination of series and shunt windings is found best for this purpose. The machine is shunt wound, but a few turns of the external circuit are also wrapped round the magnet. The advantage of a dynamo over a primary battery as a producer of current on a large scale lies in the cost. If cells are used the current is produced by oxidising, *i.e.* burning, zinc, while the dynamo derives its energy from the fuel used in the engine which drives it.

Electro-motors.—Suppose in Fig. 280 that the external circuit is replaced by a battery or other source capable of driving a current through the armature and magnet. From the principles explained

in Chap. XXXVII, it is clear that, owing to the action of the field on the current, there will be forces called into play tending to make the armature rotate. The direction of rotation, by Lenz's law, is such that the E.M.F. induced in the armature is opposed to the passage of the current. This is the principle of the electro-motor. If E volts is the back E.M.F. generated when the current forced through is A amperes, then the energy required to drive the current is EA watts, and this, neglecting frictional and other losses, is the activity, of which the motor is capable. Hence to be efficient it must be capable of generating a large back E.M.F. In both dynamos and motors the resistance of the armature coils is kept low in order that the energy dissipated as heat may be small (Chap. XXXVI). Suppose now that a motor having a resistance of a fraction of an ohm is suddenly switched into a 200-volt circuit; there will be initially a very large current, but as the speed of the armature increases the back E.M.F. rises and the current is reduced. To protect the machine from injury at the moment of starting the current is introduced through a starting resistance; this is gradually switched out as the speed increases.

CHAPTER XLIV

CONDUCTION OF ELECTRICITY IN GASES

In the preceding pages such terms as "charge of electricity" and "electrical current" have frequently been used, but no attempt has been made to form a mental picture of what constitutes a "charge," or what it is that moves when a "current" passes along a wire. Conduction in electrolytes has been definitely ascribed to the motion of charged ions (p. 395). During the last thirty years the ionic theory has been extended to gases and metals, thus permitting much more definite pictures to be formed of electrical phenomena and leading to a greatly extended knowledge of atoms and the constitution of matter. We proceed to describe experiments on the conduction of electricity in gases.

McLeod Gauge.—A means is required of measuring gas pressures of the order of 1 mm. of mercury and less. A glass apparatus called a McLeod guage, shown in Fig. 281, is commonly used. The reservoir B, containing mercury, is connected by rubber tubing to a tube AM about 80 cms. long. This is surmounted by a bulb N and a closed capillary tube having a mark at C. A side tube DE of the same diameter as the capillary goes to the apparatus where the gas pressure is required to be known. To measure this pressure, which we will assume is x cms. of mercury, the reservoir B is raised until mercury flows past A, traps the gas in the bulb and compresses it to C. Mercury also rises in the side tube to E. Suppose the volume of the capillary to be v and of the bulb from A to C to be V. (These are determined before the apparatus is put together.) The pressure at C is, by Boyle's law, greater than x and is equal to that at D in the same horizontal plane. But the pressure at D is equal to DE cms. of mercury plus the gas pressure above E, *i.e.* to $(\mathrm{DE}+x)$ cms. Applying Boyle's law to the gas in C before and when it is

compressed; initial pressure was x and volume $(V+v)$; final volume is v and pressure $(DE+x)$.

$$\therefore\ x(V+v)=(DE+x)v$$

$$\therefore\ x=DE.\frac{v}{V}\ \text{cms.}$$

The fraction v/V can be made 1/1000 or less, so that even if x is small DE is still large enough to be measured easily.

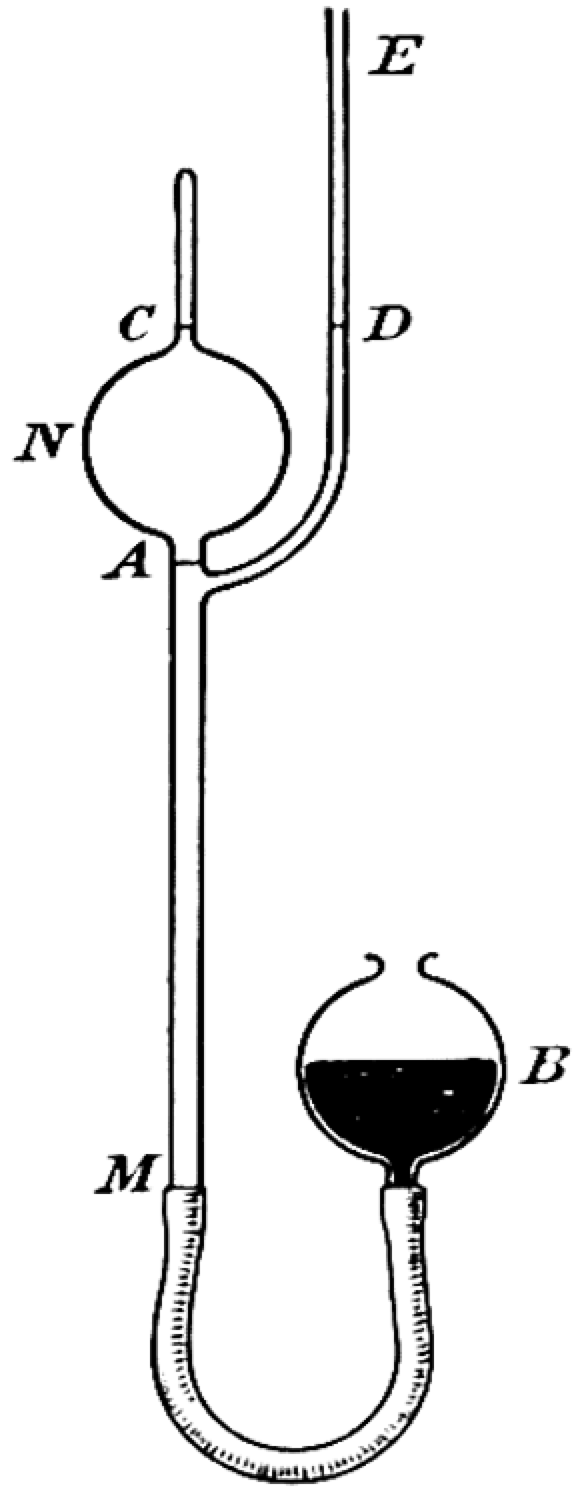


Fig. 281.—McLeod Gauge.

Discharges in Vacuum Tubes.—For the experiments to be described a source of high potential is required, such as an induction coil (p. 422) capable of giving a spark of 6 in. or more in air; also an air pump like the Töpler (p. 46*) capable of reducing the pressure in a closed tube to 0·001 mm. of mercury or less.

A glass tube (Fig. 282) is connected to another containing phosphorus pentoxide, which dries the contained gas, and to the pump and

gauge. Platinum wires are fused through the glass and carry, inside the tube, discs of aluminium for electrodes. When the secondary terminals of the coil are connected to A, B, and a current is passed from the anode A to the kathode B, sharp, disruptive sparks are seen. If air be pumped out until the pressure is between 5 and 10 mm. of mercury, the greater part of the tube from the anode is filled with a pinkish discharge called the positive column. Beyond this is a dark space C, named the Faraday dark space, bounded on the kathode side by a violet glow—the negative glow—reaching up to and covering the kathode surface. When the pressure is still further reduced, the positive column divides into a set of saucer-shaped bands of light, called the positive striæ, the negative glow extends further

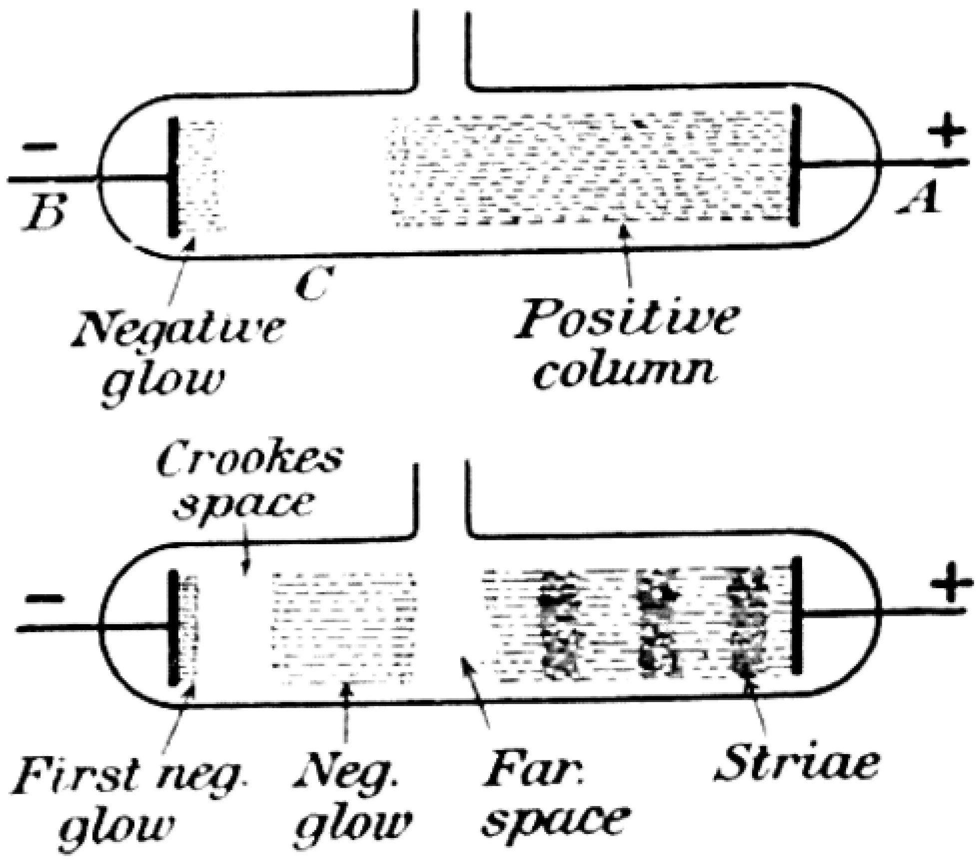


Fig. 282.—Discharge in Vacuum Tubes.

from the kathode, while the Faraday space and the positive column retreat before it to the anode. At about 1 mm. pressure there appears a dark space between the negative glow and the kathode—this is called the Crookes dark space or simply the dark space—and another bluish glow, named the first negative glow, covers the surface of the kathode. This stage of the discharge is shown in the lower part of Fig. 282. At about this pressure the voltage between the terminals is a minimum. As the pressure is still further reduced the striæ become hazier and retreat into the anode, the negative glow extends, becomes fainter, and finally disappears, but the Crookes dark space becomes larger and larger until the luminosity of the discharge vanishes. The voltage required to pass the current at length becomes so high that sparks pass between the electrodes

outside the tube. In the meantime, at a pressure near 0·1 mm., the walls of the tube, if made of soda glass, begin to fluoresce with a brilliant, apple-green light, which becomes more pronounced as the pressure is lowered. It is with the tube in this highly exhausted condition that the most important discoveries have been made.

Kathode Rays.—Crookes made a tube of the shape shown in Fig. 283, where the anode and kathode are marked. Fixed in the tube is a light mica cross. When a discharge is passed at a pressure of 0·001 mm. or less the glass fluoresces green, but the cross throws its shadow on the end of the tube and the glass there does not fluoresce.

From this and similar experiments Crookes concluded that there are rays coming from the kathode normally to its surface, which travel in straight lines independently of the position of the anode, and which cannot pass through solid substances. These rays are

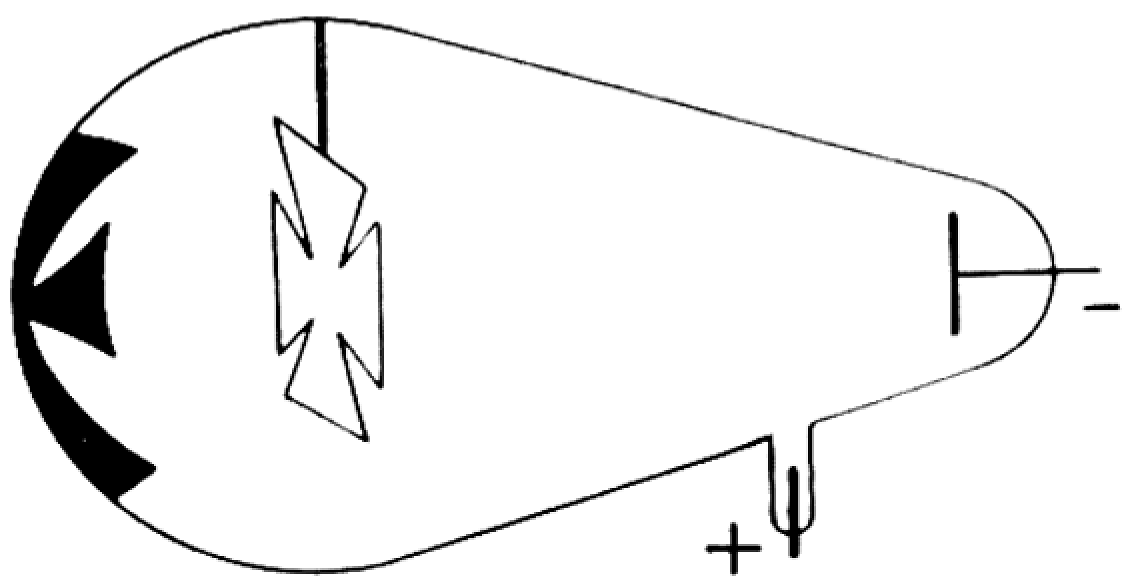


FIG. 283.—Shadow cast by Kathode Rays.

called **kathode rays.** They were regarded by Crookes as a fourth state of matter. If the rays are caused to fall on various mineral substances, such as calcspar, fluorspar, etc., these minerals fluoresce strongly with colours characteristic of each; some substances also phosphoresce after the rays have ceased. As an illustration of the fact that the rays leave the kathode in a direction normal to its surface, it is found that if this electrode be made concave the rays are brought to a focus at the centre of curvature. A thin sheet of copper placed at this point is melted by the rays in a few seconds, showing that they carry a large amount of energy which is mostly converted into heat when they strike a solid target.

Kathode rays are deflected by electric and magnetic fields. This can be shown in a tube like that depicted in Fig. 284. B is the kathode, C is a metal slit $\frac{1}{2}$ a mm. wide; when it is connected to earth there is no field to the right of it. D and E are metal plates a

few cms. long. If they are connected together and are therefore at the same potential, the rays coming through the slit C cause a green patch to be seen on the glass at P. If E is now connected to the positive and D to the negative pole of a battery of 100 or more volts, the rays are bent downwards and the patch appears at Q. This is what would be expected if the rays carry a negative charge, for they would then be repelled by D and attracted by E. If D and E are again connected and the N. pole of a magnet is placed near the tube anywhere between D and P, so that, in the figure, the lines of force are running into the paper, the rays are again deflected towards Q.

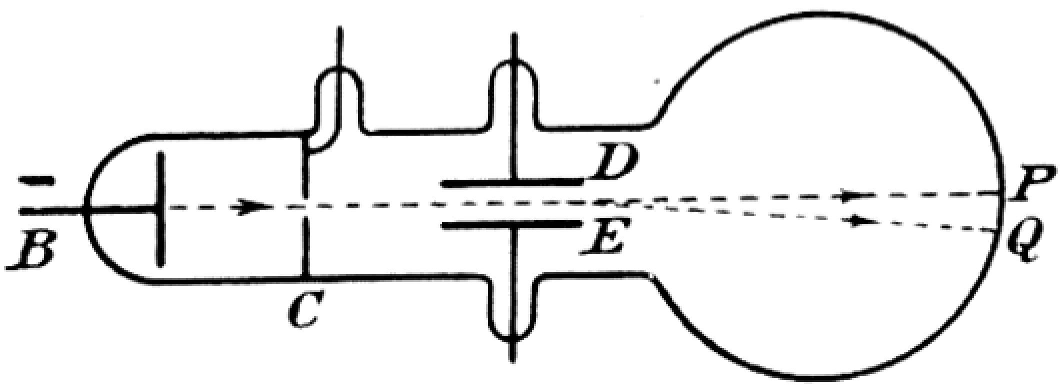


FIG. 284.—Electrostatic deflection of Kathode Rays.

If, following Fleming's rule (p. 410), the left hand is held with the first finger pointing in the direction of the magnetic field (into the paper), the thumb pointing in the direction towards which the rays are bent (downwards), the second finger points in the direction in which a positive current is flowing, viz. from P to B. This is the same as a negative current flowing from B to P, and thus agrees with what has been deduced from the electrical deflection.

The negative charge on the rays may be demonstrated directly. The kathode is placed as shown in Fig. 285, so that the rays pass through a narrow slit. A metal cylinder G has a corresponding slit facing the kathode. It is insulated from but completely surrounds an inner cylinder E. G is connected to earth and screens E from all electrical fields. Such an arrangement is called a Faraday cylinder. E is connected to an electroscope by the wire F. When the tube is worked, the electroscope shows no disturbance until the kathode rays are bent by a magnet to enter E, the rays then give up their charge to the cylinder and the electroscope shows a negative charge.

By measurements which it is beyond the scope of this book to describe, it has been found that kathode rays consist of negatively charged particles, whose mass is 1/1845 the mass of a hydrogen atom, hitherto the smallest particle known. The charge on each is of the

same magnitude as that on a hydrogen or other univalent ion in electrolysis, approximately $4 \cdot 77 \times 10^{-10}$ electrostatic units. Such particles are called **electrons**. It appears that electricity, like matter, is atomic, since the smallest charge that can be given to or taken from a body is the electronic charge, no fractions of it have been isolated, and all other small charges which have been measured are exact multiples of it. The velocity of the electrons in a vacuum tube may be from 1/30 to 1/3 the velocity of light. Even such small masses, moving with this enormous velocity, will carry a large amount of energy, since the kinetic energy is $\frac{1}{2} mv^2$, and $v = 3 \times 10^{10}$ cms./sec.

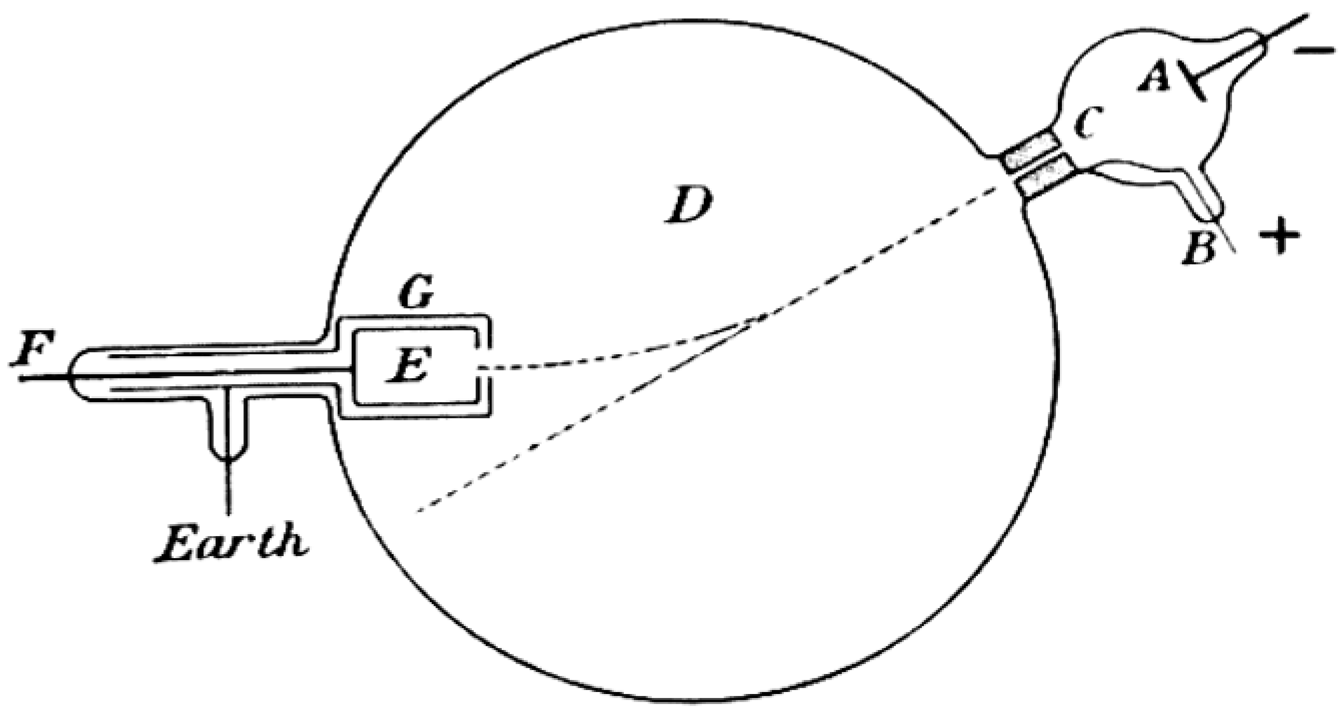


FIG. 285.—Charge on Kathode Rays.

Electrons.—Electrons can be produced by other means than discharges in vacuum tubes; for example, if a metal wire be made white hot it emits large numbers of electrons. But the important fact emerges that no matter what their source or method of production they have always the same charge and mass. *They are, therefore, a common constituent of all matter.* It is now fairly certain that all chemical atoms are built up of electrons surrounding a heavy nucleus. The nature of the latter is not yet completely known, but it carries a positive charge just sufficient to neutralise the negative charges of the electrons surrounding it. If, then, an electron be removed from an atom by any means, the part which is left will carry an excess of positive electricity and will behave as a positively charged body. Electrification of conductors in many cases means they have either lost or gained electrons; in the first event they are positively and in the second negatively charged. In insulators the electrons are supposed to be fixed in the atoms of which they form a part, but in metallic conductors they are believed to be more or less

free. When a potential is applied to a conductor, the electrons move in a direction opposed to that of the field, since they are negatively charged. It is this motion of electrons which constitutes an electric current. Electrification by induction is also due to the motion of electrons.

Positive Rays.—It has been shown that a solid obstacle casts a shadow behind it when it is placed in the path of the kathode rays. The presence in the tube of another set of rays can be shown by similar means. If a wire be placed in the *dark space* in a tube which is in the condition shown in the lower part of Fig. 282, it will be found that behind the wire there is none of the first negative glow on the surface of the kathode. This shows that the glow is caused by rays of some kind coming to the kathode from the Crookes space, and that these rays are stopped by the wire. They are called **positive rays**. The paths of the rays are most easily seen when the gas in

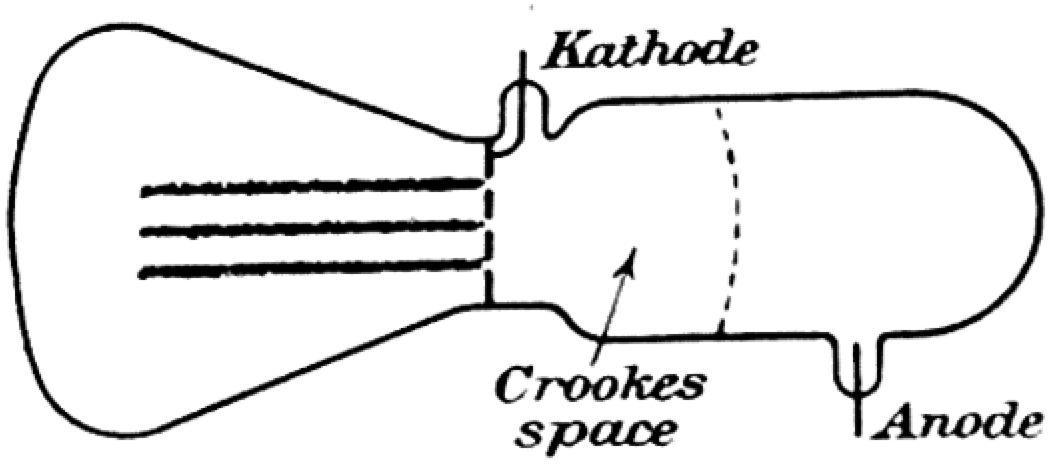


FIG. 286.—Positive Rays.

the tube is helium and the kathode is pierced by a number of holes, as in Fig. 286. The rays then pass through the holes and produce pinkish streamers behind the electrode. It is in this part of the tube that they have been studied. When they are allowed to fall on some powdered willemite, a white mineral substance, they cause it to fluoresce. Positive rays can be deflected by electric and magnetic fields, and the direction in which they are deflected shows they carry a positive charge. The fields required are, however, much greater than those necessary to deflect kathode rays. Positive rays differ from electrons in that the amount of the deflection varies with the nature of the gas in the tube. They have been shown to be atoms, or less frequently molecules, which have lost one or more electrons in the discharge. Their larger mass, compared with electrons, explains the difficulty of deflecting them, while the variety of the weights of atoms shows why the deflection depends on the nature of the gas. By measuring the electrical and magnetic deflections the

relative masses of atoms can be compared ; this gives an extremely accurate method of determining relative atomic weights. It is this which makes the study of positive rays so important. By their use it has been shown that chlorine is a mixture of two kinds of atoms, of relative masses 35 and 37, and similar results have been found for some other substances. Positive rays, like kathode rays, affect a photographic plate ; this is made use of when a permanent record of their deflection is desired.

X-rays.—It was discovered by Röntgen in 1895, that a discharge tube when worked at a very low pressure causes certain substances in its neighbourhood to fluoresce strongly and photographic plates are fogged. These effects occur even when the plates and tube are covered with black paper. They are found to be due to radiations coming from those parts of the tube walls which are struck by

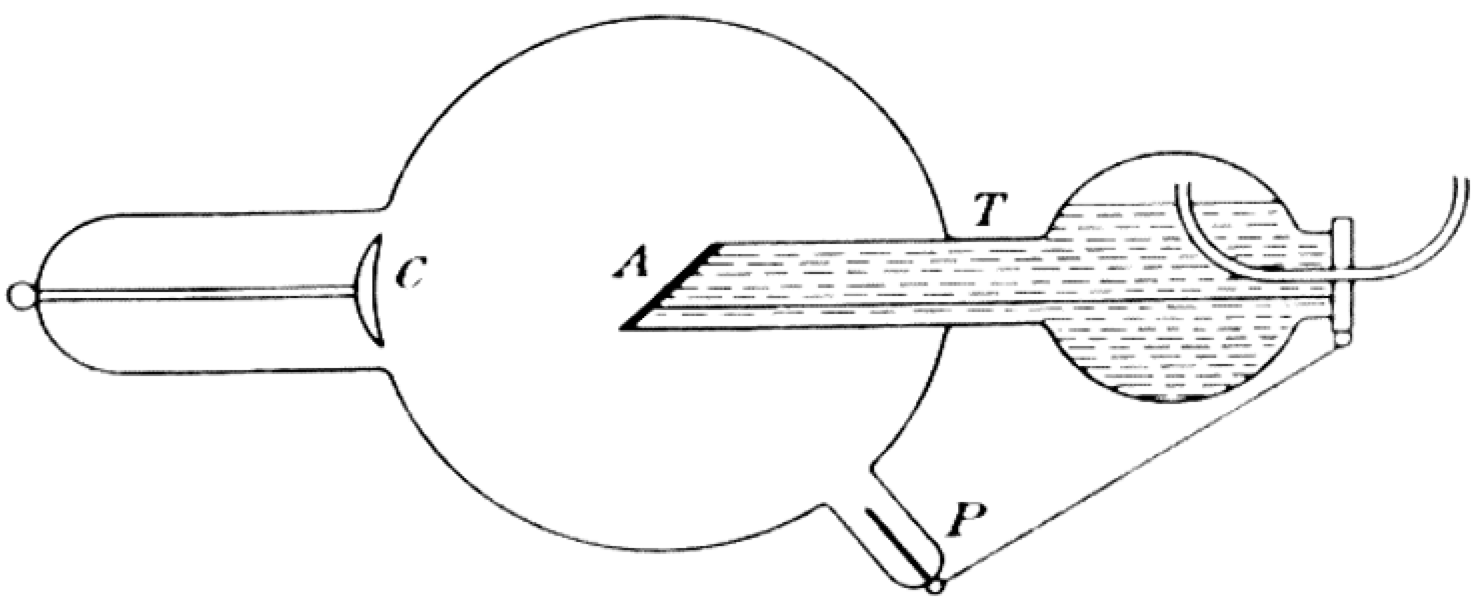


FIG. 287.—X-ray Tube.

the kathode rays. The radiations are called **Röntgen rays** or, more popularly, **X-rays.** They pass readily through light substances like wood, paper, and aluminium, with more difficulty through bone, while lead and the heavy metals are more or less opaque to them. They have been proved to be waves of the same nature as light waves but of much shorter wave length. Yellow light has a wave length about 6×10^{-5} cms., but that of X-rays is of the order 10^{-8} cms., or a thousand times less. This is the reason for a second difference ; X-rays cannot be focussed by a lens. When the hand is placed on a photographic plate enclosed in black paper, and the rays sent through from above, the flesh transmits the rays readily but the shadow of the bones is shown clearly on the developed plate. This gives the rays great importance in surgical work. To obtain sharp shadows the source of the rays must be small (p. 134), also the great

energy carried by the kathode rays would soon melt the tube walls. Hence in the modern tube (Fig. 287) the kathode rays are focussed by means of a concave kathode C on to a heavy, infusible, metal target A, usually made of tungsten. To reduce the heating the target is sometimes in contact with water. The anode P is connected to A. By these means the origin of the rays is confined to a small spot on A, and the radiation travels upwards in the figure in an intense beam. The rays are due to the sudden stoppage by the target of the electrons coming from the kathode.

Conduction of Electricity in Gases—The fact that an electroscope retains a charge is sufficient to show that the air in contact with it is a bad conductor of electricity Gases can, however, be made conductors. If a charged electroscope be exposed to X-rays,

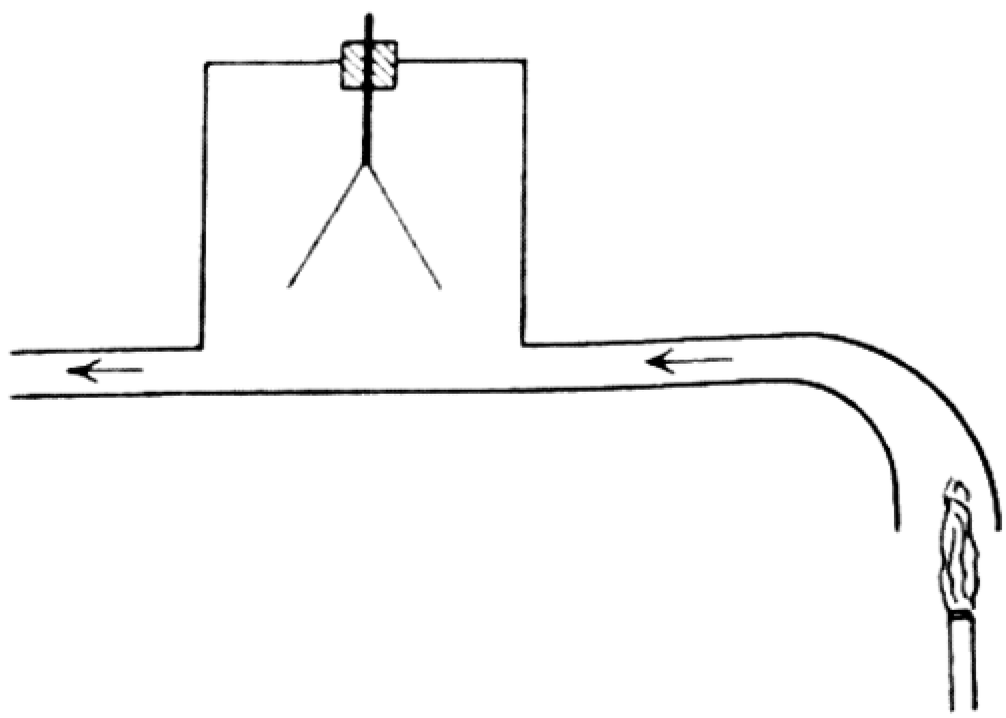


FIG. 288—Ionisation of Air.

or if a radioactive substance like salts of radium, thorium, or uranium be held near it, the leaves collapse more or less quickly. Flames, hot bodies, and electric discharges also cause conductivity. Gases in this condition are said to be ionised. Since the ionisation persists for a short time, the gas may be drawn from one place to another and tested when removed from the ionising source. Fig. 288 shows a simple apparatus for the purpose. By means of a filter pump the gas to be tested is drawn into a chamber containing a charged electroscope. The air may come from the neighbourhood of an X-ray bulb, a flame, or a wire heated to incandescence by a current, or it may be drawn over a radioactive substance. The electroscope will be found to lose its charge whether positive or negative. If, however, the ionised air be pulled through a tight plug of cotton wool or be

bubbled through water before reaching the electroscope, it will be found that its discharging action has disappeared. The conductivity can also be removed by passing the ionised air through a strong electric field. For example, if ionised air be pulled through the space between two concentric metal tubes which are insulated from each other but are connected to the opposite poles of a battery of about 100 volts, the conductivity can be entirely removed. It disappears spontaneously if the gas be allowed to stand for a few seconds. It has been shown that ionisation of a gas is due to the removal of electrons from some of its atoms or molecules by the ionising agent, X-rays, etc. These molecules have consequently a positive charge. The free electrons soon attach themselves to neutral molecules, which are thereby negatively charged. Such charged molecules are called positive and negative ions respectively. When a gas in this condition is placed in an electric field the ions move in opposite directions, carrying their charges with them; it is the motion of these charges that constitutes the current in an ionised gas. Hence when a conducting gas is drawn through the tube just mentioned, the ions of one sign go to the outer and those of the other sign to the inner tube and the emergent gas is deprived of its conducting power. When a gas loses its ionisation by standing, either the ions have become attached to the walls of the containing vessel or the oppositely charged ions have recombined with each other to form electrically neutral systems. Similarly, when the ions are passed through water or a cotton wool plug they become attached thereto.

Saturation Current.—The currents through ionised gases are usually much smaller than those through an electrolyte and are too small to be measured by a galvanometer. Fig. 289 shows an arrangement which can be used to investigate the relation between applied E.M.F. and the current. A, B are two insulated metal plates between which the current is to be measured. An X-ray bulb is placed in a lead case with a hole in one side which is closed with thin aluminium foil, a substance that allows the rays to pass through freely. This arrangement ensures that the only rays which escape from the box are those which pass between A and B. D is a quadrant electrometer with one pair of diagonally opposite quadrants earthed and the other pair connected to B. A is connected to one pole of a battery C of about 50 cells, the other pole of which is earthed. B and

its quadrants are first connected to earth, so that their potential is zero. Evidently if A be connected to the positive pole of C positive ions will be driven on to B and so to earth. At a given instant B and its quadrants are insulated; the ions they now receive will

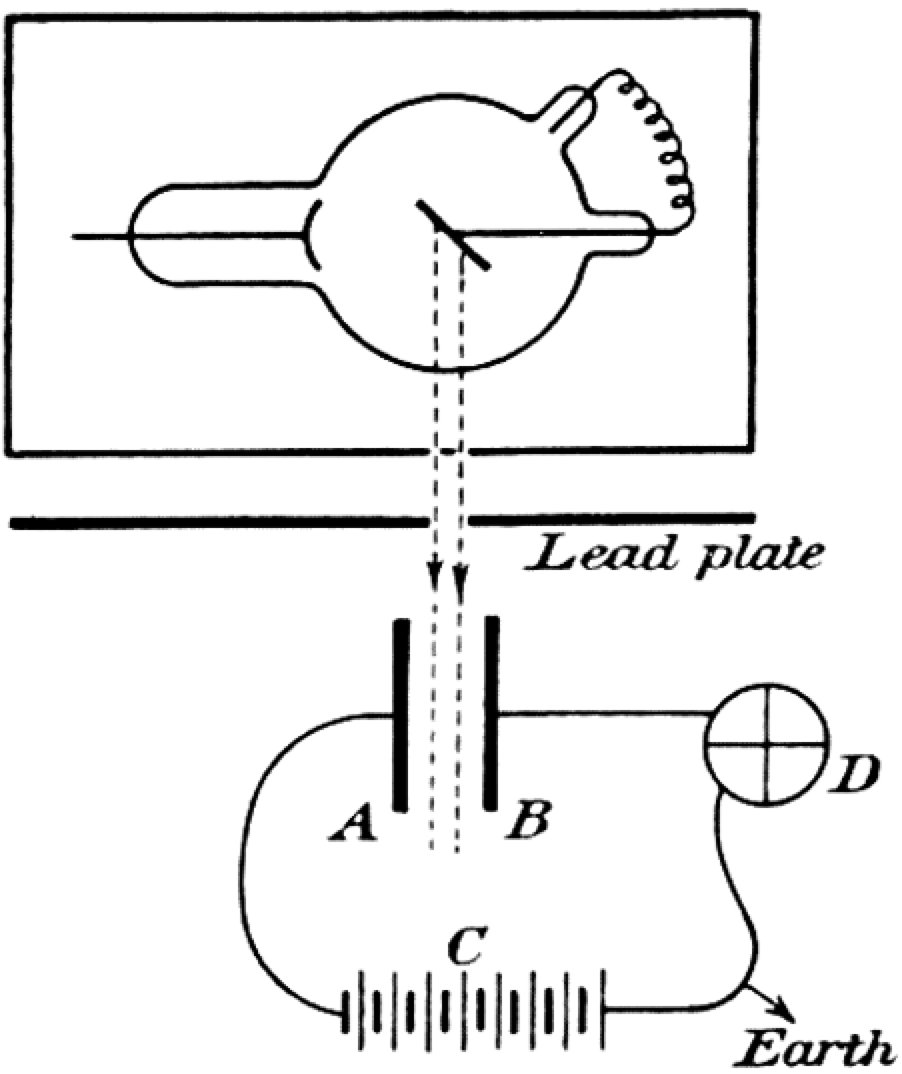


FIG. 289.—Apparatus for Measuring Current in a Gas.

accumulate and raise their potential, and this potential will be measured at every instant by the electrometer. After a time t let their potential be V. If C is their capacity, the accumulated electricity is $Q=VC$, and the average current, which is the charge

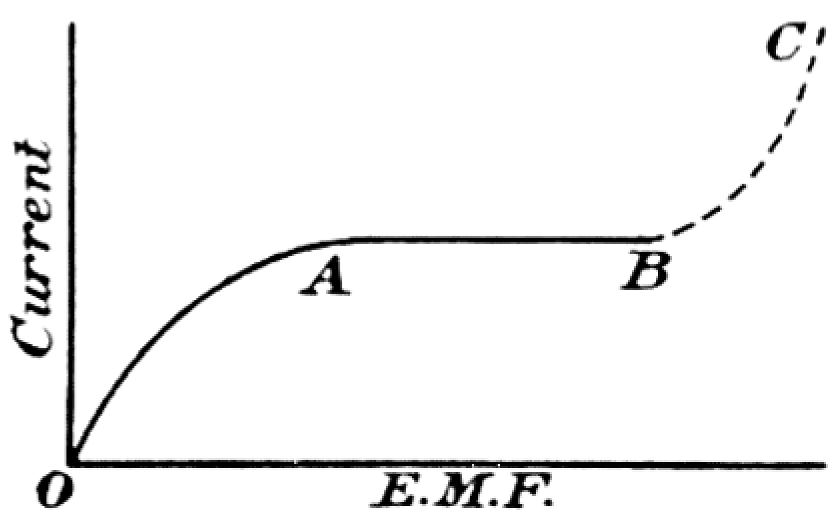


FIG. 290.—Potential-Current Curve.

received per unit time (p. 359), is VC/t. For many purposes only relative currents are required. These are measured by V/t, *i.e.* by the rate at which the potential rises, and this is given by the rate of movement of the electrometer spot across the scale.

The relation between E.M.F. and current is shown by the curve OAB (Fig. 290). At first the current increases rapidly with the potential, but reaches a steady value at and beyond A. This maximum value is called the **saturation current**. It is clear the current does not obey Ohm's law; if it did the curve OAB would be a straight line. The reason for a saturation current is simple. The X-rays produce each second between the plates a definite number of ions. If there is no field these accumulate until the recombination of the oppositely charged ions just balances their rate of production by the rays. When an increasingly large field is imposed the ions are driven on to the plates in greater and greater numbers until they are removed as fast as they are formed. No further increase of current is then possible, unless more ions are produced each second.

Electricity from Hot Bodies.—When inorganic salts are heated certain of them emit ions in large quantities. Thus aluminium phosphate gives a large excess of positive ions, but zinc iodide an

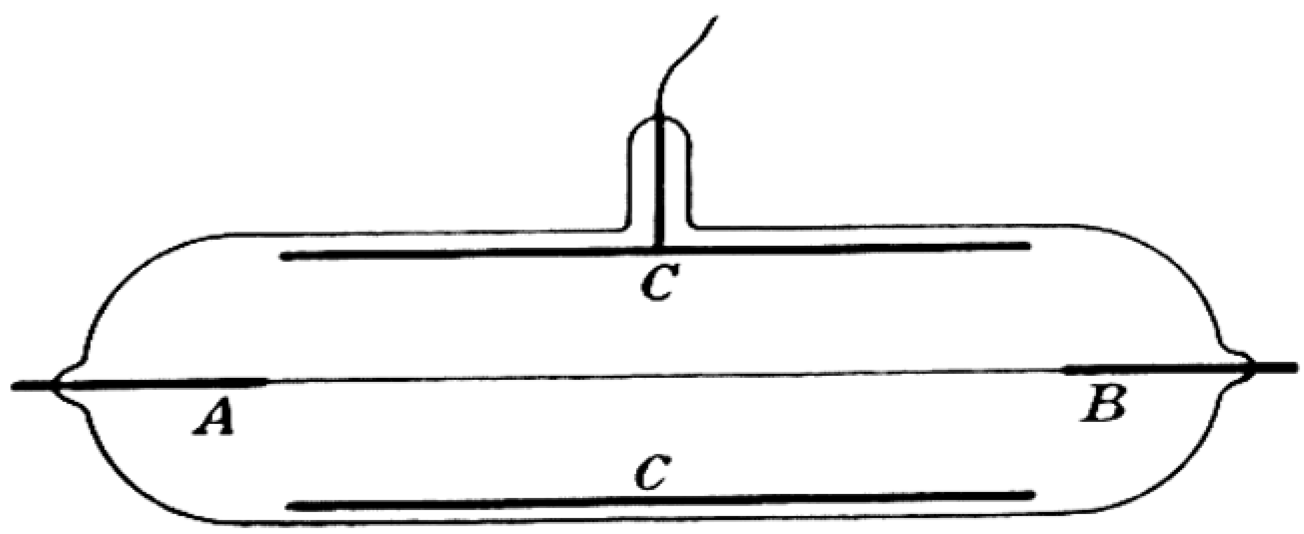


FIG. 291.—Current from a hot Filament.

excess of negative. Heated metal wires give off both kinds, but at temperatures above 1000° C. the negative are largely in excess. **Thermionics** is the name given to this branch of the subject, and the carriers of the discharge are called thermions. Concerning the positive ions little is known, in spite of much research; attention will here be confined to those instances where the carriers are known to be electrons. In Fig. 291 AB is a thin wire of platinum or tungsten which can be heated to incandescence by a current from a few accumulators. C, C is a metal cylinder surrounding the wire. The whole is enclosed in a glass tube which can be evacuated. AB is connected to the negative pole of a second battery, the other pole of which goes to a galvanometer; the second galvanometer terminal is connected to C. When the wire is white hot electrons are emitted

and are driven by the field on to C and through the galvanometer. The relation between E.M.F. and current is given by OAB (Fig. 290). It will be noticed that the current is large enough to be measured by a galvanometer. It is very greatly increased if the wire be coated with lime. Whether the coated or bare wire be used, it is found there is practically no current if the positive pole of the second battery mentioned above is connected to AB. In other words, the tube passes electricity in one direction only. If then the second battery be replaced by a source giving current alternately positive and negative to the wire, only the negative phase will pass, and current in one direction only can be taken from C. A tube with this property is called a valve and is largely used in X-ray work and wireless telegraphy. It is well known that hot bodies lose any charge imparted to them, the reason is now clear—they emit ions which carry away their charge.

Ionisation by Collision.—When the pressure in the tube (Fig. 291) is reduced to a few mm. a new feature appears. The current remains saturated over a certain range of voltage, but with further rise of the latter begins again to increase very rapidly, and soon produces a visible discharge. A new source of ions has therefore been tapped. This part of the curve is represented by BC (Fig. 290). The same feature can be produced with the apparatus shown in Fig. 289, but some thousands of volts would be required if the air were at atmospheric pressure. In their movement through the gas the electrons continually collide with neutral molecules, but between collisions their motion is accelerated by the field. At low pressures, when the distance moved over between collisions is large, the kinetic energy of the electrons finally becomes great enough to enable them to remove electrons from the neutral molecules with which they collide. These electrons in turn produce the same effect, and so the current rapidly increases. The positive ions formed at the same time, being heavier, require greater E.M.F.'s to acquire the energy necessary to produce ions by collision, although eventually this stage can be reached.

Radioactivity.—Since a tube fluoresces strongly when it is producing X-rays, it was thought that certain fluorescent crystals might also emit X-rays. Becquerel placed a photographic plate in black paper, on this a small piece of lead, and above all one of the crystals. After some days it was found the plate was affected except under

the lead. Further experiments showed the rays came from the uranium of the salts and had no relation to fluorescence or external conditions. Further, they were not X-rays. This spontaneous emission of rays is called **radioactivity**. The rays ionise gases into which they pass, and this effect is generally made use of in investigating their properties. Thorium is also radioactive, but certain minerals are more active than either thorium or uranium. This has been traced to the presence of a new element—now called radium—which is very strongly radioactive. It will be taken here as the typical radioactive substance.

Quality of the Rays.—It can readily be shown that the rays are not all of one kind. The apparatus shown in Fig. 289 can be used, except that the X-ray tube is replaced by some radium placed in an open lead capsule immediately below AB. The rays ionise the air between the plates, and the voltage of the battery is adjusted to give the *saturation* current. The open end of the capsule is next covered by successive sheets of very thin aluminium foil ; between each addition the new saturation current is measured. It is found that at hickness of 0·1 mm. of aluminium stops the greater part of the radiation. These easily absorbed rays are called α- (alpha) rays. If the poles of a small electro-magnet are now placed with the gap just above the covered capsule, the magnetic field cuts down the remaining radiation still further. The rays deflected by the field are called β- (beta) rays. The remaining rays—named γ- (gamma) rays—are not deflected by a magnetic or electric field, and are able to penetrate several cms. of lead. They are very penetrating X-rays.

The α-rays.—The α-rays from *all* radioactive substances are helium atoms that have lost two electrons ; their positive charge is consequently $9{\cdot}5 \times 10^{-10}$ electrostatic units. By taking electrons from other atoms they finally become helium. All the α-rays from the same element are ejected with the same velocity, but this varies with the substance. Magnetic and electric fields deflect α-rays, but the fields must be very strong as the mass of the rays is high. The direction of the deflections shows the charge on the rays is positive. They ionise gases very strongly, but photographically they are not very active, as they do not penetrate far enough into the film. The rays cause fluorescence in certain substances ; at every impact of one of the particles a small flash of light is produced. By noting in a

dark room the distance from the radium at which these flashes are no longer created, the range of the fastest α-rays in air has been found to be slightly over 7 cms.

The β-rays.—The β-rays are electrons carrying the usual negative charge of $4{\cdot}77 \times 10^{-10}$ electrostatic units. They are deflected by electric and magnetic fields, but less than kathode rays owing to their higher velocity. The direction of the deflection shows the charge is negative. β-rays are more penetrating than α-rays, their photographic action is stronger, but the ionisation they produce is less. When they are fired through narrow slits and then deflected by a magnetic field, the amount of deflection is not the same for all, as it is for α-rays from a single substance. This is due to their differing velocities, which range from that of kathode rays to the enormous value of 180,000 miles per second.

The negative charge carried by the rays can be shown directly, for if they are shot into an insulated lead plate it gradually shows a negative charge. For this to succeed the plate must be placed in a highly evacuated vessel, or be embedded in paraffin wax, otherwise the β-rays render the surrounding air conducting and the plate loses the charge as fast as it receives it. By a similar experiment the positive charge on α-rays can be shown, but it is more difficult.

The bombardment of the radium salts by the α-rays coming from the interior layers heats the material to a temperature slightly higher than that of its surroundings. When α- and β-rays are ejected from radium a new substance is formed by the remainder of the atom ; this undergoes further changes and in this manner a succession of substances appears ; the final product is lead.

ANSWERS TO THE EXAMPLES

MECHANICS

CHAPTER I*

(1) 0·523 ; 0·872 ; 2·268. (2) $-\sqrt{3}$; $-\frac{1}{\sqrt{3}}$; $\frac{1}{\sqrt{3}}$.

(3) ·05 mm. (4) 1 minute. (6) ·01 deg.

CHAPTER II*

(1) 12·5 ft. per sec. per sec. (2) 250 lbs. ; 281·25 lbs. (3) 3820 kgms.
(4) 187·5 lbs. (5) 50 gms.-cms. (6) 16 cms./sec. ; 163,000 gms.
(7) 42·4 H.P. (8) 52 H.P.

CHAPTER III*

(2) 211·4 gms. making an angle β with the 16 gms. such that $\sin\beta = 5/221{\cdot}4$; 161·1 making an angle β with the 16 gms. such that $\sin\beta = 5\sqrt{3}/161{\cdot}1$.
(3) $\sqrt{3}$ making an $\angle\,\beta$ with the 6, such that $\sin\beta = \sqrt{3}/12$.
(5) 192 gms. in the shorter and 144 in longer string.
(7) 5 lbs. ; 12·5 lbs.

CHAPTER IV*

(1) Vol. of gold : vol. of silver = 4·23 : 1. (2) 4,319 cu. in. approx.
(3) 31 gms approx. (4) 258·4 cms. (5) 4 ; 250 c.cms. (6) 30 in.
(8) 446·1 lbs. ; 4859·7 lbs. approx. ; 687·5 lbs./sq. ft.
(9) 175·1 gms./cm.^{2} more.

PHYSICS

CHAPTER I

(3) $\frac{1}{120}$. (4) 42·42 H.P. [$g = 32$, 1 gal. = 10 lbs.]. (6) 93·82 c.c.
(8) 30 in.

CHAPTER II

(2) 176°, −56·2°, −459·4°, 1832°, −40° (3) 18·89°, 36·67°, 31·11°.
(4) 90·68°.

CHAPTER III

(1) 0·638. (2) Water equivalent = 12, sp. heat = 0·112. (3) 4·76 gms.
(4) 26·35°.

CHAPTER IV

(1) 0·019 per cent. (2) 0·00006, $L_t = 99{\cdot}9\ (1 + 0{\cdot}00002t)$.
(3) 73,200 kilos. (4) 76·025 cms.

CHAPTER V

(1) 0·000066. (2) 680 gms./cm.^{2}, 667,080 dynes/cm.^{2}, 667·9 gms./cm.^{2}
(3) 0·000187. (4) 0·0357 cm. (5) 49·57 gms. (6) $\dfrac{d_t}{d_0} = \dfrac{1 + 4S}{1 + \Delta(t - 4)}$
(7) 757·78 mms.

CHAPTER VI

(2) 762·3 mms. (3) Density at sea-level = 1·74 that on the mountain top.
(4) 452° C. (5) 999·3 gms./cm.^{2}
(6) It weighs 0·343 gm. more at 100°. (8) 0·1819 cub. ft.
(9) (a) 75 cms., (b) 74 cms. (10) 1330° C. (11) 15·4° C.

CHAPTER VII

(1) 0·9047. (2) 537·7 (3) 14·92 kgms. (4) 7·06 gms.
(5) 0·088. (6) 3·15 gms. (7) 8·34 gms. (8) 8·99 cms.
(9) 11·12 gms.

CHAPTER VIII

(1) B will be the higher if the temperature is raised, A if the temperature is lowered. (4) 65·5 cms. (5) 8 cms.

CHAPTER IX

(1) 90·72 c.cms. (2) 12·37 gms.

CHAPTER X

(1) 96·8 per cent. (4) $2{\cdot}871 \times 10^7$ ergs, $2{\cdot}927 \times 10^4$ gms./cms.
(5) 144, 235·5. (6) 0·457°. (8) $\frac{1}{8}$. (9) 38·79°.

CHAPTER XI

(1) 0·88°. (2) $\frac{1}{15800}$ deg. Cent. (3) 1800 kgms.
(4) 53·3 mins., taking the density of ice as unity.

CHAPTER XIII

(3) 0·4 cm., 1·2 cms.

CHAPTER XIV

(2) 3 in each. They coincide. (4) $P_3Q_4 = 24$ ft., $Q_3P_4 = 14$ ft.
(5) 60°. (6) 45°. (7) 0·71°, 0·0071°.

CHAPTER XV

(3) 50 cms. from mirror for real image, 30 cms. for virtual image.
(4) $5\frac{5}{8}$ cms., $-8\frac{3}{4}$ cms. (5) $-38{\cdot}4$ cms.
(7) $15{\cdot}5$ cms. in front of mirror, real, inverted. $18{\cdot}14$ sq. mm.
(9) $-8{\cdot}25$ cms., $11{\cdot}27$ cms. from needle.

CHAPTER XVI

(4) $\frac{1}{\sqrt{3}}$. (10) $64{\cdot}8°$. (11) $\frac{2}{3}$ in.

CHAPTER XVII

(1) $1{\cdot}2$ in. from the surface. (2) $1{\cdot}33$ cms. from centre on same side.
(3) $3{\cdot}22$ cms. from surface. (4) $14{\cdot}55$ ft. from surface.
(6) 60° 27′ approx. (7) $\sqrt{3}$.
(8) $f = 3$ ft. if the image is virtual, $-1\frac{1}{2}$ ft. if image is real. The object is 6 ft. from the lens in each case.
(9) 6 in. diameter. (11) No.
(12) length $= 1{\cdot}6$ ft. with near end $1{\cdot}4$ ft. from lens; $1{\cdot}66$ ft. from lens, length $= 1\frac{1}{3}$ ft.
(14) $2{\cdot}6$ in. from the eye. (17) $1{\cdot}33$.
(18) (1) $-35{\cdot}2$ in. from lens, (2) $+ 86{\cdot}39$ in. from lens.
(21) 30 in., concave. (24) 30 cms. (25) $1{\cdot}54$.
(26) At a distance greater than $2f$ from 1st lens.

CHAPTER XVIII

(1) $\frac{\mu - 1}{\mu' - 1} \cdot A$. (2) $(\mu - \mu')A$. (6) $1{\cdot}5°$.

CHAPTER XIX

(2) $1{\cdot}18$ and $1{\cdot}26$ metres. (3) $1 : 1{\cdot}77$, $23{\cdot}5$ per cent. (4) 79 cms.
(5) 40 cms. from smaller if between the two, or 120 cms. from it if the lamps are on the same side.

CHAPTER XX

(2) $\frac{4}{51}$. (6) 17. (7) Side $= 64{\cdot}5$ in., $462{\cdot}2 : 1$.
(8) 20. (9) $3{\cdot}75$ cms., 4. (10) $3\frac{1}{3}$ in., 3. (11) $5\frac{1}{7}$ in.
(12) Concave, $6\frac{2}{3}$ in. (13) Concave, $\frac{5}{8}$.
(14) $\frac{1}{20}$ diopters. From 20 cms. to infinity.

CHAPTER XXI

(3) 3×10^{10} cms./sec. (4) 3×10^{10} cms./sec.

CHAPTER XXII

(1) $90{\cdot}2$. (2) $100{\cdot}08$, $99{\cdot}91$, $400{\cdot}16$, $399{\cdot}83$.

(3) $y = A \sin(\omega t - \alpha)$, where $\tan \alpha = \dfrac{a_2 \sin \beta}{a_1 + a_2 \cos \beta}$ and
$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \beta$.

CHAPTER XXIII

(2) 56·1°. (5) 1285·8 metres/sec.

CHAPTER XXIV

(1) 220, 224, . . . 436, 440. (2) 336·7 metres/sec.
(3) 939·4, 681·3, 869·7, 740·6.

CHAPTER XXV

(3) 600 per min.

CHAPTER XXVI

(1) All the same. (2) 1600 lbs. (3) No change. (4) 8 per sec.
(5) 1·13 : 1. (6) Frequency is raised in the ratio 20 : 17.
(8) 3·98 per sec. (9) Assuming a temperature not far from 0° C. there are 6 nodes 6·37 in. apart, the outer ones being half this distance from the ends of the tube. These are the points of maximum density variation. (10) 110·63. (11) 70·6 cms.

CHAPTER XXVIII

(1) 257·1°, 319·8°. (4) 108 dynes/cms. (5) 1·96 dynes.

CHAPTER XXIX

(1) 14. (2) 2 per min. (4) 11,250 C.G.S. units.

CHAPTER XXX

(1) 160 ergs. (2) 986 ergs, $986\sqrt{3}$ dynes/cms.
(3) (*a*) 1 : 1·306, 1 : 1·84.

CHAPTER XXXII

(2) 1000. (3) 3, $\frac{3}{4\pi}$.

CHAPTER XXXIII

(2) (1) 30·98, (2) 13·85. (3) $\frac{2}{3}$. (4) (*a*) 0·057, (*b*) 0·57.

CHAPTER XXXIV

(1) 3 rows of 12 in series and these rows in parallel.
(2) 45°. All four in parallel. (3) 0·5 amp., 0·158 amp.
(4) 3·6 ohms. (5) 1·72 amps., 5·16 volts.
(6) Battery = $\frac{11}{13}$ amp., short arc = $\frac{8}{13}$, long arc = $\frac{3}{13}$ amp.
(7) 37 ohms. (9) 20·49 ohms. (10) 0·2 volt.

CHAPTER XXXV

(1) 0·5 amp. in each case, $a : b = 2 : 1$.
(2) 0·505 amp., A (amps.) = 0·505 tan θ.
(3) 62·5 gms. (approx.), $2 \cdot 4 \times 10^4$ cals. (4) 0·656 gm. (5) 0·30 gm.
(6) 0·05 gm., 0·15 gm.

CHAPTER XXXVI

(1) (*a*) 3 : 5, (*b*) 5 : 3. (2) 1 : $\frac{1}{2}$: $\frac{1}{101}$.
(3) Currents are as 4 : 1, rates of working of battery 4 : 1.
(4) 0·617 and 0·308 amps., 2 : 1.
(5) 1 : 10 : 3 : 30 in the same order as in question.
(6) The copper conductor costs less, and the heat losses, which vary as A^2R are also reduced. (7) 47·6°. (8) 196 pence (approx.).
(9) $\frac{l_1}{l_2} = \frac{d_1^2}{d_2^2}$.

CHAPTER XXXVII

(1) 4242·6 dynes/cms. (2) 7·2 dynes. (3) 1635 Gausses.

CHAPTER XXXVIII

(3) 9×10^{-6} volts. Towards the E.

CHAPTER XXXIX

(1) 5·03 Gausses. (2) 0·93.

CHAPTER XLI

(1) 1 and 1·56 dynes. (2) 108, 36. The smaller.
(3) (*a*) 5 : 4. (*b*) Equal. (4) 2685·7 cms. (5) $\frac{10}{\pi}$.
(6) (*a*) $\frac{6}{x^2}$ where x = distance in cms. from the centre of the larger sphere, (*b*) Zero.

CHAPTER XLII

(1) A = C = $\frac{1}{18}$, B = $\frac{1}{4}$. (2) 166·6. (3) 0, $\frac{3}{61}$, $\frac{1}{6}$. (5) 157 ergs.
(6) (*a*) 628·2, (*b*) 314·1 dynes. (7) $\frac{\theta_2}{\theta_1 - \theta_2} \cdot C$.

INDEX

[*The numbers refer to the pages; those followed by an asterisk to the sections on Mechanics, the others to the sections on Physics.*]

THE END